Exam #2
EEL 3657

Name: Solutions

SS#: _____________

Please show all work for partial credit. (100 points total)

Problem 1. (20 points) A unity feedback system has a forward transfer function

\[ G(s) = \frac{30}{(s + 2)(s + 30)} \]

a). What are the position and velocity error constants? (10 points)

b). What are the steady state errors for a unit step input and for a unit ramp input? (10 points)

Solution:  

\[ k_p = \lim_{s \to 0} G(s) = \frac{30}{2 \cdot 30} = 0.5 \]

\[ k_v = \lim_{s \to 0} sG(s) = 0 \]

b) For a unit step input,

\[ e_{ss} = \frac{1}{1 + k_p} = \frac{1}{1 + 0.5} = \frac{2}{3} \]

For a unit ramp input,

\[ e_{ss} = \frac{1}{k_v} = \infty \]
Problem 2. (60 points)

For a unity feedback system shown in the figure, where

\[ G(s) = \frac{K}{s(s + 6)(s + 9)} \]

a). Sketch the root locus by finding:
   1). asymptotes; (15 points)
   2). breakaway points; (15 points)
   3). jw axis crossings. (15 points)

b). Find \( K \) at the jw axis crossings, and then determine for what range of \( K \) the closed-loop system is stable. (15 points)

Solution: a) open-loop poles: 0, -6, -9, no open-loop zeros.

1). asymptotes:

\[ \alpha = \frac{\sum \Phi_l - \sum \Phi_u}{n - m} = \frac{0 + (-6) + (-9)}{3 - 0} = -5 \]

\[ \Phi_l = \frac{\pm 180^\circ + 360^\circ l}{n - m} \quad (l = 0, 1, 2) \]

\[ = \begin{cases} 60^\circ & \text{ } \\ 180^\circ & \text{ } \\ 300^\circ & \text{ } \end{cases} \]

2). 

\[ \frac{dk}{ds} = 0 \quad \ldots \quad \circ \]

Since 

\[ 1 + \frac{k}{s(s+6)(s+9)} = 0 \]

we get:

\[ k = -s(s+6)(s+9) = -(s^3 + 15s^2 + 54s) \]

Use equation 0,

\[ 3s^2 + 30s + 54 = 0 \]

\[ s^2 + 10s + 18 = 0 \]

\[ s = \frac{-10 \pm \sqrt{10^2 - 4 \cdot 18}}{2} = -5 \pm \sqrt{7} = \{-7, 6\} \]
Since \(-7.646\) cannot be on the root locus, the actual breakaway point is \(-2.354\).

3). At the \(jw\) axis crossings, \(s = jw\), substitute \(s = jw\) into the characteristic equation

\[ 1 + G(s) = 0 \quad \Rightarrow \quad s^3 + 15s^2 + 54s + k = 0 \]

we get \((jw)^3 + 15(jw)^2 + 54(jw) + k = 0\)

\((-15w^2 + k) + j(-w^3 + 54w) = 0\)

Equate both real part and imaginary part to zero,

\[ s - 54w^2 + k = 0 \]
\[ -w^3 + 54w = 0 \]

\[ \Rightarrow \quad \begin{cases} w = 0, \pm 7.348 \\ k = 0, 810 \end{cases} \]

So the \(jw\) axis crossings are at \(\pm j7.348\).

b). We got \(k = 810\) at the \(jw\) axis crossings in the previous step, so the stable range of \(k\) is \(0 < k < 810\).
Problem 3. (20 points) For the transfer function \( G(s) = \frac{s + 20}{(s + 1)(s + 7)(s + 50)} \), sketch asymptotic Bode magnitude plot for each individual factors, and mark the slopes in your diagram.

Solution:
\[
G(j\omega) = \frac{j\omega + 20}{(j\omega + 1)(j\omega + 7)(j\omega + 50)}
\]

\[
= \frac{20}{1.750} \cdot \frac{1 + \frac{j\omega}{20}}{(1 + \frac{j\omega}{1})(1 + \frac{j\omega}{7})(1 + \frac{j\omega}{50})}
\]

Individual factors:
1. 0.057
2. \( 1 + \frac{j\omega}{20} \)
3. \( \frac{1}{1 + j\omega} \)
4. \( \frac{1}{1 + \frac{j\omega}{7}} \)
5. \( \frac{1}{1 + \frac{j\omega}{50}} \)