Problem 1. (40 points) Consider the unity feedback system shown below (H(s)=1):

![Block Diagram]

where \( G_p(s) = \frac{K}{(s+2)^2(s+3)} \).

1) Find the location of the dominate poles to yield a 1.6-seconds settling time and an overshoot of 25%.

2) If a compensator \( G_c(s) = \frac{K(s+a)}{s+b} \) with a zero at \(-1\) (i.e., \(a=1\)) is used to achieve the conditions of 1), what must the angular contribution of the compensator pole be?

3) Find the location of the compensator pole from your answer to 2), i.e., \(b=?\)

4) Find the gain required to meet the requirements of 1), i.e., \(K=?\)

Solution:

1. \[ \zeta \omega_n = \frac{4}{T_s} = 2.5; \quad \zeta = \frac{-\ln \left( \frac{\%OS}{100} \right)}{\sqrt{\pi^2 + \ln^2 \left( \frac{\%OS}{100} \right)}} = 0.404. \] Thus, \(\omega_n = 6.188 \text{ rad/s}\) and the operating point is \(-2.5 + j5.67\).

2. Summation of angles including the compensating zero is \(-170.22^\circ\). Therefore, the compensator pole must contribute \(180^\circ - (-170.22^\circ) = 9.78^\circ\).

3. Using the geometry shown below, \(\frac{5.67}{p_c - 2.5} = \tan 9.78^\circ\). Thus, \(p_c = 35.39\).
Adding the compensator pole and using -2.5 + j5.67 as the test point, $K = 1049.41$. 

\[ \theta \]
Problem 2. (30 points) For the system shown below,
1) Find $K_p$, $K_v$, $K_a$.
2) Find the steady-state error for an input of $50u(t)$, $50tu(t)$, and $50t^2u(t)$, where $u(t)$ is a unit step function.
3) State the system type.

Solution:

\[
G_e(s) = \frac{5}{s(s+1)(s+2)} = \frac{5}{s^3 + 3s^2 + 7s + 15}
\]

Therefore, $K_p = 1/3$; $K_v = 0$; and $K_a = 0$.

For $50u(t)$, $e(\infty) = \frac{50}{1 + K_p} = 37.5$; For $50tu(t)$, $e(\infty) = \infty$; For $50t^2u(t)$, $e(\infty) = \infty$

Type 0
Problem 3. (30 points) For the transfer function \( G(s) = \frac{2(s + 10)}{(s + 1)(s + 2)} \), sketch asymptotic Bode magnitude plot for each individual factors separately, and then add them together. Mark the slopes in your diagram.

Solution:

\[
G(s) = \frac{(\frac{10}{s}) (\frac{5}{s} + 1)}{(s + 1)(\frac{3}{2} + 1)} = \frac{10(\frac{5}{s} + 1)}{(s + 1)(s/2 + 1)}
\]

\[G(j0) = 10 \angle 0 = 20 \text{ dB}\]

\[G(j\infty) \rightarrow \frac{10}{\pi} + 90^\circ\]