Exam #1
EEL 3657

Name: Solutions
SS#: 

Please show all work for partial credit. (100 points total)

Problem 1. (40 points) In the diagram shown, the dynamics of the block labeled "Plant" are described by the differential equations

\[ 2 \frac{dy(t)}{dt} + x(t) = u(t) \]
\[ x(t) = 3 \int y(t) dt \]

1). Determine the transfer function from \( u \) to \( y \). (20 points)
2). Using Mason's rule, or otherwise, determine the transfer function from \( r \) to \( y \). (20 points)

\[ \begin{array}{c}
    r \\
    \rightarrow \frac{0.5}{s} \\
    + \\
    + \\
    u \\
    \rightarrow \text{Plant} \\
    + \\
    1 + 0.5s \\
    y
\end{array} \]

Problem 2. (60 points) The block diagram of a linear control system is shown:

\[ \begin{array}{c}
    R \\
    + \\
    \Sigma \\
    - \\
    Gc \\
    - \\
    \rightarrow \text{Gp} \\
    - \\
    H \\
    - \\
    \rightarrow C
\end{array} \]

where \( Gc(s) = 1, H(s) = 1, Gp(s) = \frac{24}{s(s^3 + 10s^2 + 35s + 50)} \)

1). Find the closed-loop transfer function. (15 points)
2). Determine the characteristic equation. (10 points)
3). Use the Routh criterion to determine whether or not the closed-loop system is stable. (20 points)
4). If \( Gc(s) = K \), for what range of \( K \) is the closed-loop system stable? (15 points)
Solutions:

Problem 1: 1). \[ 2sY(s) + X(s) = U(s) \]
[\( X(s) = \frac{3Y(s)}{S} \)]
\[ \Rightarrow G_p(s) = \frac{Y(s)}{U(s)} = \frac{-1}{2s^2 + 3} \]

2). Forward path: \[ \frac{0.5}{s} G_p(s) \]
Loop: \[ -(1 + 0.5s) G_p(s) \]
Use Mason's rule,
\[ G_{ry}(s) = \frac{(1 + 0.5)}{s} G_p(s) \]
\[ 1 + (1 + 0.5s) G_p(s) = \frac{10s + 0.5}{-2.5s^2 + s + 3} \]

Problem 2: 1). \[ G_{c}(s) = \frac{G_c(s) G_p(s)}{1 + G_c(s) G_p(s) H(s)} \]
\[ = \frac{24}{s^4 + 10s^3 + 35s^2 + 50s + 24} \]

2). \[ s^4 + 10s^3 + 35s^2 + 50s + 24 = 0 \]

3). \[
\begin{array}{c|ccc}
\text{s}^4 & 1 & 35 & 24 \\
\text{s}^3 & 10 & 50 \\
\text{s}^2 & 30 & 24 \\
\text{s}^1 & 42 \\
\text{s}^0 & 24 \\
\end{array}
\]

No sign changes in the first column. So all roots are on \( \mathbb{H} \).
System is stable.
4). Closed-loop characteristic equation:

\[ s^4 + 10s^3 + 35s^2 + 50s + 24k = 0 \]

Roots, table:

<table>
<thead>
<tr>
<th>( s^4 )</th>
<th>1</th>
<th>35</th>
<th>24k</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s^3 )</td>
<td>10</td>
<td>50</td>
<td>0</td>
</tr>
<tr>
<td>( s^2 )</td>
<td>30</td>
<td>24k</td>
<td></td>
</tr>
<tr>
<td>( s^1 )</td>
<td>-8k+50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( s^0 )</td>
<td>24k</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

To meet the stability requirement, every element in the first column should be positive,

\[-8k+50 > 0\]
\[24k > 0\]

\[\Rightarrow 0 < k < \frac{25}{4}\]