Lecture 1
Electric Circuit Variables

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Aims and Plan

- Introduction to the subject.
- Electric circuit variables.
- Electric circuit element: resistor.
The Subject...

- *Electrical engineering* is the profession concerned with systems that produce, transmit, and measure electric signals.
- It combines the physicist’s models of natural phenomena with the mathematician's tools for manipulating those models to produce systems that meet practical needs.
Five Major Classifications of Electrical Systems

- Communication systems;
- Computer systems;
- Control systems;
- Power systems;
- Signal-processing systems.
Circuit Theory

- An electric circuit is a *mathematical model* that approximates the behavior of an actual electrical system.
- The models, the mathematical techniques, and the language of circuit theory will form the intellectual framework for your future engineering endeavors.
Problem Solving Process

Figure 1.4 A conceptual model for electrical engineering design

From: Nilsson/Riedel, Electric Circuits, 6e, July 2000 Prentice Hall, Inc.
What is Electric Circuit?

- An electric circuit is an interconnection of *electrical elements* linked together in a *closed* path so that an *electric current* may flow continuously.

A simple circuit:
Basic Circuit Variables

- **Charge**: the quantity of electricity responsible for electric phenomena, denoted by $q$, Unit: Coulomb, C.
- **Current**: the time rate of flow of electric charge past a given point, denoted by $i$.
  - Mathematical representation: $i = \frac{dq}{dt}$
  - Unit: ampere, A.
Current

Two different types:

- **DC (direct current)** is a current of *constant magnitude*.

  ![Diagram of constant current](image)

- **AC (alternating current)** is a time-varying current \( i(t) \) that has a sinusoidal form.
Time-Varying Current

(a) A ramp with a slope $M$.
(b) A sinusoid.
(c) An exponential. $I$ is a constant. The current $i$ is zero for $t < 0$. 
Voltage

- **Definition:** energy required to move a unit positive charge from the – terminal to the + terminal across an element.

- **Mathematical representation:**
  \[ v = \frac{dw}{dq} \]

- **Unit:** volt, V.
Voltage

The direction of a voltage is given by its polarities:

- The voltage $v_{ab}$ is proportional to the work required to move a positive charge from terminal a to terminal b.
- The voltage $v_{ba}$ is proportional to the work required to move a positive charge from terminal b to terminal a.

$v_{ab}$ is read as “the voltage at terminal a with respect to terminal b”; or, “the voltage drop from terminal a to terminal b”.

\[ v_{ab} = -v_{ba} \]
Power

Definition: time rate of expending or absorbing energy.

Mathematical representation: $p = \frac{dw}{dt}$

Unit: watt, W.

Relation with current and voltage: $p = vi$
Passive convention: the assigned direction of the current is directed from the + terminal to the – terminal.

a). Power absorbed (or dissipated) by the element, as \( v \) and \( i \) adhere to the passive convention.

b). Power supplied (or delivered) by the element, as \( v \) and \( i \) do not adhere to the passive convention.
Work-Out

- Relationship between energy, power, voltage and current...
Example 1:

Find the charge that has entered the terminal of an element from $t=0s$ to $t=3s$ when the current is as shown below.
Example 2:

Find the charge, $q(t)$, and sketch its waveform when the current entering a terminal of an element is as shown below. Assume $q(0)=0$. 
Example 3:

Consider the circuit shown in the figure with \( v=8e^{-t} \) V and \( i=20e^{-t} \) A for \( t\geq 0 \). Find the power supplied by this element and the energy supplied by the element over the first second of operation. Assume that \( v \) and \( i \) are zero for \( t<0 \).
Example 4:

The average current in a typical lightning thunderbolt is $2 \times 10^4$ A and its typical duration 0.1s. The voltage between the clouds and the ground is $5 \times 10^8$ V. Determine the total charge transmitted to the earth and the energy released.
A linear element satisfies the property of superposition and homogeneity.

IF \[ i_1 \rightarrow v_1 \]
\[ i_2 \rightarrow v_2 \]

THEN \[ i_1 + i_2 \rightarrow v_1 + v_2 \]
\[ ki_1 \rightarrow kv_1 \]
\[ ki_2 \rightarrow kv_2 \]
A passive element absorbs energy. 

\[ w = \int_{-\infty}^{t} vid \tau \geq 0 \quad \text{for all } t. \]

An active element is capable of supplying energy.

\[ w = \int_{-\infty}^{t} vid \tau > 0 \quad \text{for at least one } t. \]
(a) The entry node of the current $i$ is the positive node of the voltage $v$,

(b) the entry node of the current $i$ is the negative node of the voltage $v$. The current flows from the entry node to the exit node.
Resistors

Resistors impedes the flow of current. Unit: Ohm.

Ohm’s law: \( v = Ri \)

\( v = Ri_a \)

\( v = -Ri_b \)
Power delivered to a resistor:

\[ p = vi = v(v/R) = v^2/R \]

\[ p = vi = (iR)i = i^2R \]
Example 5:

Model of a car battery and the headlight lamp:

Work out the energy supplied by the battery for a four-hour period.
Summary

Circuit variables:
- Charge
- Current
- Voltage
- Power
- Energy
Concepts:
- Linearity
- Passive elements
- Active elements
- Passive convention

Circuit element:
- Resistor
  - Ohm’s law