Comments on "Impedance Control with Adaptation for Robotic Manipulations"

Darren Dawson and Zhihua Qu

Abstract—**This comment points out an error in the adaptive impedance control approach for robotic manipulators given in the short paper by Lu and Meng. In that work, the authors assert that their adaptive impedance control scheme can compensate for uncertainty in the force measurements; however, we show that it is possible for the uncertainty in the force measurements to cause the parameter estimates to go unbounded. To remedy this possible instability problem, we suggest an additional auxiliary controller.**

I. Introduction

In recent years, there has been much activity in the adaptive control of robotic manipulators. For an excellent review paper on the subject, the interested reader is referred to [2]. It has been noted by several researchers [8], [9] that the robustness of adaptive controllers to sensor noise, additive bounded disturbances, and unmodeled dynamics is a complicated issue. In general, when one considers these robustness issues, one must be very careful to ensure that the overall system remains stable. That is, one should ensure that the parameter estimates in the adaptive controller remain bounded. In this comment, we point out some stability issues that were inadvertently overlooked when the authors in [1] extended previous adaptive motion robot controllers [3], [4] to force control applications. Specifically, we show that it is possible for the uncertainty in the force measurements to cause the parameter estimates to go unbounded. To remedy this possible instability problem, we suggest an additional auxiliary controller based on the work given in [5].

II. Comments on Algorithm 1

In Section III of [1], the authors propose an adaptive impedance control algorithm that compensates for parametric uncertainty and force measurement uncertainty. The details of the indirect adaptive controller used to compensate for the parametric uncertainty in the robot manipulator equation are well established [3], [4]; however, in [1], the authors claim that an auxiliary controller (i.e., [1, eq. (22)]) can compensate for imperfect force sensor measurements. We believe that this claim is questionable. For the following development, we will use the same notation in [1] whenever possible to keep this work as brief as possible.

To justify our belief that [1] is in error, we start with the stability proof. Specifically, from [1, eqs. (17) and (23)], the Lyapunov function is given by

$$V_c(X, \phi) = X^T \mathscr{P} X + \phi^T \Gamma_c^{-1} \phi \tag{1}$$

while the associated time derivative is given by

$$\dot{V}_c(X, \phi) \le -\lambda_{\min}\{\mathscr{Q}\} \|X\|^2 - \gamma \|\mathscr{B}^T \mathscr{P} X\|^2 + \|\mathscr{B}^T \mathscr{P} X\| \bar{\delta} \tag{2}$$

where

$$2\|\hat{H}_x^{-1}\| \, \|\Delta F_e\| \le 2h^* \delta = \bar{\delta}. \tag{3}$$

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In [1], the authors seem to claim that $\dot{V}_c(X, \phi)$ defined (2) can be made strictly negative by adjusting the control parameter γ to compensate for the constant uncertainty represented by $\bar{\delta}$ (note from [1, eqs. (21) and (27)] that $\|\Delta F_e\| \le \delta$ and $\|H_x^{-1}\| \le h^*$, respectively) and thereby obtain the standard asymptotic tracking error result for adaptive motion robot control [3].

We believe that this claim can be shown to be false by using the following argument. First, we rewrite (2) in the form

$$\dot{V}_c(X, \phi) \le -\lambda_{\min}\{\mathscr{Q}\} \|X\|^2 + \bar{\delta} \|\mathscr{B}^T \mathscr{P} X\| \cdot \left[1 - \gamma \|\mathscr{B}^T \mathscr{P} X\| / \bar{\delta}\right]. \tag{4}$$

If $\|\mathscr{B}^T \mathscr{P} X\| \ge \bar{\delta}/\gamma$ in (4), then an upper bound for (4) is given by

$$\dot{V}_c(X, \phi) \le -\lambda_{\min}\{\mathscr{Q}\} \|X\|^2. \tag{5}$$

However, if $\|\mathscr{B}^T \mathscr{P} x\| < \bar{\delta}/\gamma$ in (4), then an upper bound for (4) is given by

$$\dot{V}_c(X, \phi) \le -\lambda_{\min}\{\mathscr{Q}\} \|X\|^2 + \bar{\delta}^2/\gamma. \tag{6}$$

From (6), it is clear that $\dot{V}_c(X, \phi)$ may never be strictly negative; however, the region in which $\dot{V}_c(X, \phi)$ could be positive can be made arbitrarily small by increasing the control parameter γ. This fact is also obvious from the condition established for γ in [1] (i.e., (24) and (26)). Specifically, [1, eq. (24)] can approximated by

$$\gamma > \bar{\delta} / \|\mathscr{B}^T \mathscr{P} X\|. \tag{7}$$

From both (6) and (7), it is obvious that the control parameter γ must be infinitely large to ensure that $\dot{V}_c(X, \phi)$ is strictly negative.

To examine the actual performance of the tracking error and parameter error, we can use (6) to obtain a sufficient condition on the negativity of $\dot{V}_c(X, \phi)$. That is, $\dot{V}_c(X, \phi)$ will be negative if

$$\|X\|^2 > \frac{\bar{\delta}^2/\gamma}{\lambda_{\min}\{\mathscr{Q}\}}. \tag{8}$$

If (8) is satisfied, $\dot{V}_c(X, \phi)$ is negative and $V_c(X, \phi)$ will decrease. If $\dot{V}_c(X, \phi)$ decreases, then by the definition of the Lyapunov function given in (1), X must eventually decrease. However, if X decreases such that

$$\|X\|^2 \le \frac{\bar{\delta}^2/\gamma}{\lambda_{\min}\{\mathscr{Q}\}} \tag{9}$$

then $\dot{V}_c(X, \phi)$ *may* become positive, which means that $V_c(X, \phi)$ will start to increase. If $V_c(X, \phi)$ starts to increase, we gain insight into the problem by examining two possibilities. One, the increase in $V_c(X, \phi)$ causes X to increase such that (8) is satisfied. This means that $V_c(X, \phi)$ will start to decrease, and hence X will eventually decrease. If X continually increased and decreased in this fashion, then X and ϕ both would be bounded. The other possibility is that the increase in $V_c(X, \phi)$ causes ϕ to increase while X stays small enough such that (9) is satisfied. For this case, $\dot{V}_c(X, \phi)$ may remain positive; therefore, ϕ could continue to increase. If $V_c(X, \phi)$ continued to increase in this fashion with X satisfying (9), then ϕ and hence $\hat{p}$ could become unbounded.

The above argument reveals that the parameter estimates *may* go unstable in the presence of a bounded disturbance. That is, the parameter estimate *may* diverge under the assumption that there is uncertainty in the force measurement. If the parameter estimates become too large, we can see from the control given in [1] (i.e., (10)) that the input torque will start to grow and possibly saturate the joint motors; therefore, it would be desirable to modify the adaptive controller to eliminate the possibility of torque saturation.

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In the next section, we illustrate how the torque controller for Algorithm 2 can be modified to ensure that the parameter estimates remain bounded in the presence of force measurement uncertainty.

Remark 1: It should be noted that if the parameter estimates are "reset" to remain within a known region, then the stability of the system is preserved [3]. However, this still does not change the fact that $\dot{V}_c(X, \phi)$ is not strictly negative. Therefore, even if we reset the parameter estimates, we cannot obtain asymptotic tracking performance by the method outlined in [1]. That is, for the method given in [1] with parameter reset, we only obtain bounded tracking error performance.

Remark 2: In [1, Remark 2], the authors also state that the restriction on the inverse of the estimated inertia matrix (i.e., $\hat{H}_x^{-1}$) can be eliminated by following the method outlined in [6]. It should be noted that it was shown in [7] that this restriction was not entirely eliminated. Indeed, to ensure that the joint acceleration is bounded, the inverse of an estimated inertia-related matrix was still required to exist.

Remark 3: In Section III of [1] the authors claim a robustness property with regard to actuator saturation. It should be noted that this robustness property is not clear at all. That is, [1, eq. (28)] can be written in the form

$$\dot{V}_c(X, \phi) \le -\lambda_{\min}\{\mathscr{Q}\}\|X\|^2 + \bar{\delta}_0\|\mathscr{B}^T\mathscr{P}X\| \cdot \left[1 - \gamma\|\mathscr{B}^T\mathscr{P}X\|/\bar{\delta}_0\right] \quad (10)$$

where

$$2\|\hat{H}_x^{-1}\|(\|\Delta F_e\| + \|\Delta F_s\|) \le 2h^*(\delta + \|\Delta F_s\|) = \bar{\delta}_0 \quad (11)$$

and

$$\Delta F_s = F_{\text{sat}} - F. \quad (12)$$

For the typical actuator saturation problem, $\|\Delta F_s\|$ and hence $\bar{\delta}_0$ will both be functions of the state and the control parameters. Therefore, the upper bound on the Lyapunov derivative in (10) becomes

$$\dot{V}_c(X, \phi) \le -\lambda_{\min}\{\mathscr{Q}\}\|X\|^2 + \bar{\delta}_0^2/\gamma. \quad (13)$$

Since $\bar{\delta}_0$ is dependent on the control parameters (in particular γ) and the states, it is not clear at all from (13) that the control algorithm is robust to actuator saturation.

III. Comments on Algorithm 2

The second control algorithm presented in [1] is an extension of the work presented in [4] to impedance control. As in the previous section, it is possible for the bounded uncertainty in the force measurement to cause the parameter estimates in Algorithm 2 to go unbounded. This can easily be established by examining the Lyapunov function (see [1, eqs. (32)-(35)]

$$V_s(s, \phi) = \tfrac{1}{2}(s^T H_x s + \phi^T \Gamma_s^{-1}\phi) \quad (14)$$

and the associated upper bound on the time derivative

$$\dot{V}_s(s, \phi) \le \underline{k}_d\|s\|^2 + \delta\|s\|. \quad (15)$$

That is, from (14) and (15), we can show that the parameter estimates (i.e., $\hat{p}$ defined in [1, eq. (33)]) in the control law

$$F = \hat{H}_x x_r + \hat{C}_x x_r + \hat{g}_x + \hat{f}_x + \hat{F}_{\text{ext}} - K_d s \quad (16)$$

may become unbounded and thereby possibly cause the actuators to saturate.

As in [5], the above problem can be remedied by modifying the control given by (16) to be

$$F = \hat{H}_x \ddot{x}_r + \hat{C}_x \dot{x}_r + \hat{g}_x + \hat{f}_x + \hat{F}_{\text{ext}} - K_d s - k_\delta \operatorname{sgn}(s) \quad (17)$$

where $\operatorname{sgn}(\cdot)$ is the vector signum function defined as

$$\operatorname{sgn}(s) = \left[\operatorname{sgn}(s_1), \operatorname{sgn}(s_2), \cdots, \operatorname{sgn}(s_n)\right]^T \quad (18)$$

and k_δ is a scalar positive constant that satisfies

$$k_\delta > \delta. \quad (19)$$

Applying the control given in (17), the derivative of the Lyapunov function defined in (14) becomes

$$\dot{V}_s(s, \phi) \le -\underline{k}_d\|s\|^2 + \delta\|s\| - k_\delta s^T \operatorname{sgn}(s). \quad (20)$$

By noting that

$$s^T \operatorname{sgn}(s) = \sum_{i=1}^{n} |s_i| \quad \text{and} \quad \sum_{i=1}^{n} |s_i| \ge \|s\| \quad (21)$$

we can place the following upper bound on (20)

$$\dot{V}_s(s, \phi) \le -\underline{k}_d\|s\|^2 + (\delta - k_\delta)\sum_{i=1}^{n} |s_i|. \quad (22)$$

By utilizing (19), we can obtain a new upper bound on $\dot{V}_s(s, \phi)$ in (22) as

$$\dot{V}_s(s, \phi) \le -\underline{k}_d\|s\|^2. \quad (23)$$

We now detail the type of stability for the tracking error. The bound on the Lyapunov time derivative delineated by (23) informs us that $s \in L_2^n$ (see [10]), which means that the filtered tracking error s is bounded. To establish a stability result for the position tracking error (i.e., e), we establish the relationship between the position tracking error and the filtered tracking error (i.e., s). From [1, eq. (30)] we can state that

$$e(\Omega) = H(\Omega)s(\Omega) \quad (24)$$

where

$$H(\Omega) = (\Omega I + \Psi_s)^{-1} \quad (25)$$

I is the $n \times n$ identity matrix, and Ω is the Laplace transform variable. Since $H(\Omega)$ is a strictly proper, asymptotically stable transfer function matrix [10], and $s \in L_2^n$, we can state that [2]

$$\lim_{t \to \infty} e = 0. \quad (26)$$

The above result informs us that the position tracking error is asymptotically stable. In accordance with the above theoretical development, all we can say about the velocity tracking error (i.e., $\dot{e}$) is that it is bounded [2]. It is also obvious that since $\dot{V}_s(s, \phi)$ is strictly negative definite as delineated by (23), ϕ and hence $\hat{p}$ will both remain bounded.

IV. Conclusion

This comment pointed out an error in the adaptive impedance control approach for robotic manipulators given in [1]. In [1], the authors asserted that their adaptive impedance control scheme could compensate for uncertainty in the force measurements; however, we have shown that it is possible for the uncertainty in the force measurements to cause the parameter estimates to go unbounded. To remedy this possible instability problem, we suggested an additional auxiliary controller.

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Reply to "Comments on 'Impedance Control with Adaptation for Robotic Manipulations'"

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We thank Dr. Dawson and Dr. Qu for their interest in our paper [1] and are pleased to provide our reply as follows.

I. Algorithm 1

The quantity γ in [1, eq. (22)] is chosen according to [1, eq. (25)]. For instance, if $X \neq 0$, $e^* \neq 0$ and $\alpha \leq 2\mu\delta$ (i.e., case (c)), one may choose

$$\gamma = \frac{2\mu\delta}{\beta^2} = \frac{2\|\hat{H}_x^{-1}e^*\|\delta}{\|e^*\|^2}. \tag{1}$$

Note that γ in (1) is always *finite* whenever $e^* \neq 0$. It is true that γ becomes unbounded as $\|e^*\| \to 0$. However, it is *not* the single γ itself by F_c in [1, eq. (22)] that participates in the determination of the control torque, as is seen from [1, eqs. (8) and (10)]. As a matter of fact, with γ chosen as in (1), [1, eq. 22)] gives

$$F_c = -\frac{\delta\|\hat{H}_x^{-1}e\|\hat{H}_x e^*}{\|e^*\|^2} \tag{2}$$

whose norm is always *bounded* by $\delta \cdot \mathrm{cond}(\hat{H}_x)$ where $\mathrm{cond}(\hat{H}_x)$ is the condition number of $\hat{H}_x$ and δ is the noise bound given in [1, eq. (21)]. Therefore, the above choice of F_c, in conjunction with cases (a) and (b), leads to a *bounded* control F_c given by

$$F_c = \begin{cases} -\dfrac{\delta\|\hat{H}_x^{-1}e^*\|\hat{H}_x e^*}{\|e^*\|^2}, & \text{if } X \neq 0,\ e^* \neq 0 \text{ and } \alpha \leq 2\mu\delta \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

From [1, eq. (20)] it now follows that

$$\dot{V}_c = \begin{cases} -X^TQX - 2\left(\delta\|\hat{H}_x^{-1}e^*\| + \Delta F_e^T\hat{H}_x^{-1}e^*\right) \leq -X^TQX, \\ \quad \text{if } X \neq 0,\ e^* \neq 0 \text{ and } \alpha \leq 2\mu\delta \\ -X^TQX, \quad \text{otherwise.} \end{cases} \tag{4}$$

Hence, $\dot{V}_c \leq 0$ whenever $X \neq 0$.

By (4), V_c is bounded and $X \in L^2$. By [1, eq. (17)], the boundedness of V_c implies $X \in L^\infty$. Furthermore, since u defined by [1, eq. (12)] is bounded, [1, eq. (15)] implies that $\dot{X} \in L^\infty$. Therefore, Barbalat's lemma leads to $X(t) \to 0$ as $t \to \infty$ [2, p. 19].

Note that the above analysis is valid only if both $\|\hat{H}_x\|$ and $\|\hat{H}_x^{-1}\|$ are uniformly bounded. As in the unconstrained motion control case, this can be achieved in practice by restricting the estimates of the parameters, i.e. $\hat{p}$, to lie within bounds [3, p. 54].

Based on the argument given above, we fail to find any error in the Lyapunov analysis of Algorithm 1.

II. Robustness of Algorithm 1

It is true that $\Delta F_s = F_{\text{sat}} - F$ depends on control parameters and the state variables, and these dependences can be seen analytically from [1, eq. (10)]. When a contact between the robot hand and its environment occurs, the robot motion is usually slow so that the terms $\hat{C}_x$, $\hat{g}_x$, $\hat{f}_x$, x, and $\dot{x}$ can be assumed to be bounded without loss of generality. In addition, we note that the force measurement from the force sensor and the term F_c determined by (3) are also bounded. As a result, $\|\Delta F_s\|$ is bounded, and the validity of the robustness analysis presented in [1, sec. III-D] follows. By reading through Remark 3 made by Dawson and Qu in their comments, it seems that they were confused by the unboundedness of γ—it is *not* the γ itself but the *bounded* F_c that participates in the determination of F.

III. Algorithm 2

It has long been recognized that the convergence or boundedness of unknown parameters and the convergence of tracking error are two different issues [2]. In robotics, parameter reset has been a simple and effective means to prevent the unboundedness of the parameter estimates [3], and we have done so for both algorithms of our paper [1].

The control suggested by the authors of the comments in their (17) is *discontinuous* with respect to s. It has been known for some time that control of this type leads to control chattering that may excite high-frequency dynamics neglected in the course of modeling and, consequently, cause system instability [4, ch. 6]. In [1, sec. IV-A], it has been made clear that with the availability of a set of sufficiently powerful actuators, the continuous control [1, eq. (32)] can be used to keep both $\|e\|$ and $\|\dot{e}\|$ sufficiently small. We are therefore not convinced that one should seek a theoretically asymptotically stable solution with a controller that has a highly undesirable feature.

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