

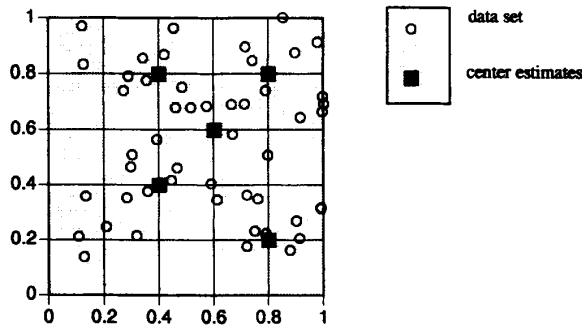


Fig. 10. Example 2: Original data set and estimated centers.

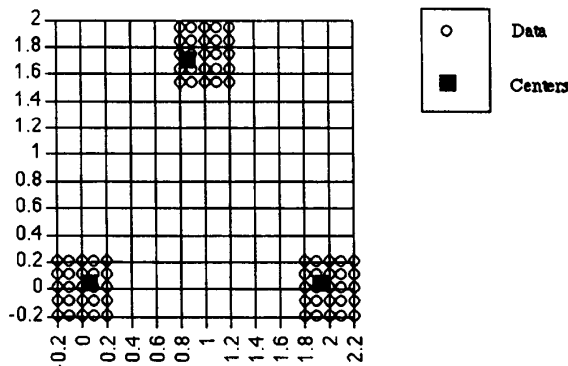


Fig. 11. Three clusters focused on vertices of an Isosceles triangle.

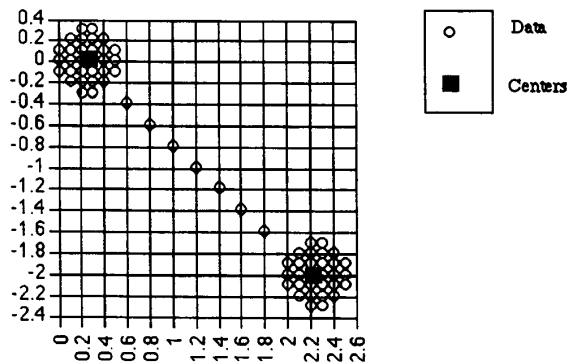


Fig. 12. Two clusters connected by a bridge.

the algorithm to the selection of the parameters α and β , the effect of the granularity of the gridding and the selection of δ are some of the most important. The answer to these questions can best be obtained by experience using this approach.

In summary our initial investigations with this mountain method of clustering seem to indicate that it is a promising approach to fuzzy clustering and worth further investigation by those interested in clustering techniques.

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A New Identification Jacobian for Robotic Hand/Eye Calibration

Hanqi Zhuang and Zhihua Qu

Abstract—Hand/eye calibration is the process of identifying the unknown position and orientation of the camera frame with respect to the robot hand frame, when the camera is rigidly mounted on the robot hand. While computationally slightly more involved, one-stage iterative algorithms have two distinguished advantages over traditional two-stage linear approaches: (a) They are less sensitive to noise, and (b) they can handle cases in which the camera orientation information is not available.

A more compact and lower dimensional Identification Jacobian is derived in this correspondence. The Jacobian, which relates measurement residuals to pose error parameters of the unknown hand/eye transformation, is a crucial component of one-stage iterative algorithms. The derivation procedure for the new Jacobian is straightforward and simple, owing to an alternative mathematical formulation of the hand/eye calibration problem. Observability conditions of the pose error parameters in the unknown hand/eye transformation are also provided based on this Identification Jacobian.

I. INTRODUCTION

A hand/eye configuration refers to the setup in which a camera is rigidly mounted on the hand of a robot manipulator. Hand/eye calibration is the process of computing the position and orientation of the camera frame with respect to the robot hand frame. The camera frame refers to the 3D coordinate frame used by the camera. The robot hand frame, also called the end-effector frame, refers to the 3D coordinate frame used by the robot controller software.

Shiu and Ahmad [1] mathematically formulated the hand/eye calibration problem as solving a system of homogeneous transformation equations of the form,

$$AX = XB \quad (1)$$

where X is the 4×4 homogeneous transformation from the robot tool coordinate frame to the sensor coordinate frame, A is the measurable

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H. Zhuang is with the Robotics Center and Department of Electrical Engineering, Florida Atlantic University, Boca Raton, FL 33431.

Z. Qu is with the Department of Electrical Engineering, University of Central Florida, Orlando, FL 32816 USA.

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4×4 homogeneous transformation of the robot gripper from its first to second location, and $\mathbf{B}$ is the measurable 4×4 homogeneous transformation of the sensor, also from its first to second location.

Since then, a number of approaches has been devised for robot hand/eye calibration. These approaches can be divided into two categories. In the first category, the problem is solved by a two-stage linear method [1]–[5]. More specifically, the unknown rotation of the camera frame with respect to the hand frame is solved in the first stage, using a number of relative robot hand motions and the induced relative camera motions. The obtained rotation together with the measured quantities is used to solve for the unknown position of the camera frame with respect to the robot hand frame. In the second category, a nonlinear least squares procedure is applied to solve simultaneously for both the rotation and position of the camera frame with respect to the robot hand frame [6].

While the two-stage linear solution method is computationally more efficient, it has two drawbacks. First, rotation estimation errors propagate to position estimation errors; therefore, the accuracy performance of the method is not optimal. Second, this type of approach requires the complete knowledge of relative robot hand motion $\mathbf{A}$ and relative camera motion $\mathbf{B}$.

The problems encountered by the two-stage method can be avoided using a one-stage iterative algorithm [6]. Not only is the one-stage algorithm less sensitive to noise, it can handle the case in which the camera orientation information is not available.

The key component of this algorithm is an Identification Jacobian relating measurement residuals to pose error parameters of the unknown hand/eye transformation. The Jacobian matrix given in [6] is, however, not well-structured in the sense that the dimension of the Jacobian matrix is unnecessarily large. For instance, the Jacobian is a 24×6 matrix when two pose measurements are used. Due to the complexity of the Jacobian, conditions for the observability of the error parameters, defined in terms of the rank of the Jacobian matrix, are difficult to derive.

A more compact Identification Jacobian is presented in this paper. The derivation is straightforward and simple, owing to an alternative mathematical formulation of the hand/eye calibration problem, which is equivalent to the one given in Equation (1). The new Jacobian has a much lower dimension. The Jacobian is a 12×6 matrix when two pose measurements are used. Observability conditions of the pose error parameters in the unknown hand/eye transformation are also easy to prove using this new Jacobian.

Important issues such as the comparison study of one-stage and two-stage algorithms and the assessment of using only position measurements of the camera motion $\mathbf{B}$ have been investigated in [6]. Therefore, this paper will concentrate on the formulation of the calibration problem, the derivation of the Identification Jacobian, and the discussion of observability conditions of the unknown pose error parameters.

II. AN ALTERNATIVE MATHEMATICAL FORMULATION OF THE HAND/EYE CALIBRATION PROBLEM

A frequently used notation is defined first. Let $\Omega(v)$ denote a skew symmetric matrix which corresponds to $\mathbf{v} \equiv [v_x, v_y, v_z]^T$,

$$\Omega(v) \equiv \begin{bmatrix} 0 & -v_z & v_y \\ v_z & 0 & -v_x \\ -v_y & v_x & 0 \end{bmatrix} \quad (2)$$

Now let us concentrate on the alternative mathematical formulation. Eq. (1) can be broken into the following equations

$$R_A R_X = R_X R_B \quad (3a)$$

and

$$R_A \mathbf{p}_X + \mathbf{p}_A = R_X \mathbf{p}_B + \mathbf{p}_X \quad (3b)$$

where R_X and $\mathbf{p}_X$ are the 3×3 rotational and 3×1 position parts of $\mathbf{X}$, respectively. Similar definitions are used for R_A , R_B , $\mathbf{p}_A$ and $\mathbf{p}_B$.

R_A and R_B must have the same angle of rotation by Lemma 4 of [1]. Let $R_A = \text{Rot}(\mathbf{k}_A, \theta)$, $R_B = \text{Rot}(\mathbf{k}_B, \theta)$, and $R_X = \text{Rot}(\mathbf{k}_X, \omega)$. Define

$$z \equiv \tan(\omega/2) \mathbf{k}_X \quad (4)$$

Lemma 1 If $\theta \neq 0$, $R_A R_X = R_X R_B$ is equivalent to the following equation

$$\Omega(\mathbf{k}_A + \mathbf{k}_B)z = \mathbf{k}_B - \mathbf{k}_A \quad (5)$$

Proof: Refer to [5].

Lemma 2 If $\theta \neq 0$, $R_A \mathbf{p}_X + \mathbf{p}_A = R_X \mathbf{p}_B + \mathbf{p}_X$ is equivalent to the following equation

$$\Omega(\mathbf{u} + \mathbf{p}_B)z = \mathbf{p}_B - \mathbf{u} \quad (6a)$$

Or

$$-\Omega(z)(\mathbf{u} + \mathbf{p}_B) = \mathbf{p}_B - \mathbf{u} \quad (6b)$$

where $\mathbf{u} \equiv (R_A - I)\mathbf{p}_X + \mathbf{p}_A$.

Proof: Lemma 2 can be proven in a similar way of proving lemma 1. Let $\mathbf{x} \equiv (x, x)$ be a quaternion corresponding to the rotation matrix R_X , where x is a scalar and $\mathbf{x}$ is a 3×1 vector [7]. Then

$$R_X \mathbf{p}_B = \mathbf{u} \quad (7)$$

However according to [7],

$$R_X \mathbf{p}_B = \mathbf{x} \otimes \mathbf{p}_B \otimes \mathbf{x}^* \quad (8)$$

where the superscript $*$ denotes the Hamiltonian conjugate of the quaternion, and $\otimes$ denotes a quaternion product operation. Thus

$$\mathbf{u} = \mathbf{x} \otimes \mathbf{p}_B \otimes \mathbf{x}^* \quad (9)$$

where $\mathbf{p}_B$ and $\mathbf{u}$ have been treated as pure quaternions [7]. Rearranging (9),

$$\mathbf{x} \otimes \mathbf{p}_B = \mathbf{u} \otimes \mathbf{x} \quad (10)$$

Expanding (10) provides [7], [8]

$$(\mathbf{p}_B \cdot \mathbf{x}, \mathbf{x} \mathbf{p}_B + \mathbf{x} \times \mathbf{p}_B) = (\mathbf{u} \cdot \mathbf{x}, \mathbf{x} \mathbf{u} + \mathbf{u} \times \mathbf{x}) \quad (11)$$

where $\cdot$ denotes a dot product operation and $\times$ a cross product operation. Eq. (11) is equivalent to

$$(\mathbf{p}_B + \mathbf{u}) \times \mathbf{x} = \mathbf{x}(\mathbf{p}_B - \mathbf{u}) \quad (12)$$

since the scalar part $(\mathbf{p}_B - \mathbf{u}) \cdot \mathbf{x} = 0$ is implied by (12). One obtains (6a) by using $z \equiv \mathbf{x}/x$, and

$$\mathbf{x} = \sin(\omega/2) \mathbf{k}_X \quad (13c)$$

$$x = \cos(\omega/2) \quad (13d)$$

Eq. (6b) is just another form of (6a), in which the unknown position vector $\mathbf{p}_X$ appears linearly. This completes the proof of the lemma.

It is well known that a minimum of two sets of (5) and (6) are needed for a unique solution of z and $\mathbf{p}_X$. After z is computed, $\mathbf{k}_X$ and ω can be obtained from (4). If $\|z\| \neq 0$,

$$\mathbf{k}_X = z/\|z\| \quad (14a)$$

$$\omega = 2a \tan(z_{\max}/k_{X,\max}) \quad (14b)$$

where $z_{\max}$ and $k_{X,\max}$ are the elements having the maximum absolute values among the three components of z and $\mathbf{k}_X$, respectively.

If $\|z\| = 0$, from (4), ω is zero and $\mathbf{k}_X$ can be arbitrarily chosen as $[0, 0, 1]^T$.

Eqs. (5) and (6) provide an alternative formulation of the hand/eye calibration problem. Although this formulation is derived using quaternions, the final results do not use quaternion notations.

III. IDENTIFICATION JACOBIAN

The Identification Jacobian is a Jacobian matrix relating measurement residuals to pose parameter errors of the unknown transformation matrix $\mathbf{X}$. It plays a central role in the identification of $\mathbf{X}$.

Let

$$f(z) = \Omega(\mathbf{k}_A + \mathbf{k}_B)z - \mathbf{k}_B + \mathbf{k}_A = 0 \quad (14a)$$

$$g(z, \mathbf{p}_X) = \Omega(\mathbf{u} + \mathbf{p}_B)z - \mathbf{p}_B + \mathbf{u} = 0 \quad (14b)$$

where $\mathbf{u} \equiv (R_A - I)\mathbf{p}_X + \mathbf{p}_A$.

The problem is to solve z and $\mathbf{p}_X$ from the above equations, which is nonlinear if z and $\mathbf{p}_X$ are to be determined simultaneously.

Assume for the moment that the initial solution z^0 and $\mathbf{p}_X^0$ deviated by a small amount from z^* and $\mathbf{p}_X^*$, the optimal solution. To a first order approximation,

$$f(z^0 + dz) \approx f(z^0) + \frac{\partial f}{\partial z} dz \quad (15a)$$

$$g(z^0 + dz, \mathbf{p}_x^0 + d\mathbf{p}_x) \approx g(z^0, \mathbf{p}_x^0) + \frac{\partial g}{\partial z} dz + \frac{\partial g}{\partial \mathbf{p}_x} d\mathbf{p}_x \quad (15b)$$

where

$$\frac{\partial f}{\partial z} = \Omega(\mathbf{k}_A + \mathbf{k}_B) \quad (16a)$$

$$\frac{\partial g}{\partial z} = \Omega(\mathbf{p}_B + \mathbf{p}_A + (R_A - I)\mathbf{p}_x^0) \quad (16b)$$

$$\frac{\partial g}{\partial \mathbf{p}_x} = (I - \Omega(z^0))(R_A - I) \quad (16c)$$

Note that (6b) has been used for the derivation of (16c). Eq. (15) can be rewritten into

$$\begin{bmatrix} \frac{\partial f}{\partial z} & \mathbf{0} \\ \frac{\partial g}{\partial z} & \frac{\partial g}{\partial \mathbf{p}_x} \end{bmatrix} \begin{bmatrix} dz \\ d\mathbf{p}_x \end{bmatrix} = - \begin{bmatrix} f(z^0) \\ g(z^0, \mathbf{p}_x^0) \end{bmatrix} \quad (17)$$

It can be easily shown that the coefficient matrix in (17) is singular, therefore at least two sets of the equations need to be used to solve for dz and $d\mathbf{p}_X$. Let

$$\rho \equiv \begin{bmatrix} z \\ \mathbf{p}_x \end{bmatrix} \quad (18a)$$

$$\mathbf{h} \equiv - \begin{bmatrix} f_1(z) \\ f_2(z) \\ g_1(z, \mathbf{p}_x) \\ g_2(z, \mathbf{p}_x) \end{bmatrix} \quad (18b)$$

$$\mathbf{J} \equiv \begin{bmatrix} \frac{\partial f_1}{\partial z} & \mathbf{0} \\ \frac{\partial f_2}{\partial z} & \mathbf{0} \\ \frac{\partial g_1}{\partial z} & \frac{\partial g_1}{\partial \mathbf{p}_x} \\ \frac{\partial g_2}{\partial z} & \frac{\partial g_2}{\partial \mathbf{p}_x} \end{bmatrix} \quad (18c)$$

where the subscripts "1" and "2" in (18) and later in Section IV denote the quantities evaluated using the first and second measurements, and the superscript "0" is dropped. In (18), ρ is a pose vector of $\mathbf{X}$, $\mathbf{h}$ is a measurement residual vector, and $\mathbf{J}$ is an Identification Jacobian for hand/eye calibration. The dimensions of ρ , $\mathbf{h}$, and $\mathbf{J}$ are respectively 6×1 , 12×1 , and 12×6 . Further, let $d\rho$ be a pose error vector of $\mathbf{X}$. Using these notations, one has

$$\mathbf{J}d\rho = \mathbf{h} \quad (19)$$

A nonlinear least squares procedure for the problem at hand is then as follows. From a given initial guess ρ^0 , the sequence $\rho^0, \rho^1, \dots, \rho^k, \dots$ is generated by [9]

$$\rho^{k+1} = \rho^k + d\rho^k \quad (20)$$

where $d\rho^k$ is solved from (19). That is

$$d\rho^k = \mathbf{J}^+(\rho^k)\mathbf{h}(\rho^k) \quad (21)$$

where $\mathbf{J}^+$ is the pseudo inverse of $\mathbf{J}$. More robust updating rules such as that provided by the Levenberg-Marquardt algorithm [10] may be used to replace the updating rule given in (21).

It is well-known that for the algorithm to converge, initial conditions of the algorithm have to be in a convex set of the optimal point. In hand-eye calibration, good initial conditions can be obtained either by roughly measuring the hand/eye transformation using a gauger, or by a noniterative procedure such as those discussed in [1]–[5].

IV. OBSERVABILITY ISSUES

The observability of the pose error vector $d\rho$ can be investigated through the study of the Identification Jacobian [11]. The observability of kinematic error parameters is defined in terms of the Identification Jacobian in robot calibration literature. It is said that if the Identification Jacobian is full rank, the error parameters are observable. The next theorem reveals the observability conditions of $d\rho$.

Theorem 1: Assume that Equation (19) is used for the estimation of pose error parameter vector $d\rho$. $d\rho$ is observable if

- 1) *The rotation axes of R_{A1} and R_{A2} are neither parallel nor antiparallel to one another,*
- 2) *The rotation angles of R_{A1} and R_{A2} are both neither zero nor π , and*
- 3) *The rotation angle of R_X is not π .*

Proof: From the definition of observability, $d\rho$ is observable if the Jacobian matrix is full rank. The Jacobian matrix in this case is

$$\mathbf{J} \equiv \begin{bmatrix} \Omega(\mathbf{k}_{A1} + \mathbf{k}_{B1}) & \mathbf{0} \\ \Omega(\mathbf{k}_{A2} + \mathbf{k}_{B2}) & \mathbf{0} \\ \Omega(\mathbf{p}_{B1} + \mathbf{u}_1) & (I - \Omega(z))(R_{A1} - I) \\ \Omega(\mathbf{p}_{B2} + \mathbf{u}_2) & (I - \Omega(z))(R_{A2} - I) \end{bmatrix} \quad (22)$$

$\mathbf{J}$ is full rank if its two diagonal block matrices are full rank. It has been shown in [5] that an equivalent condition for the upper left block diagonal matrix to be full rank is

$$((I + R_X)\mathbf{k}_{A1}) \times ((I + R_X)\mathbf{k}_{A2}) \neq \mathbf{0} \quad (23)$$

The eigenvalues of $I + R_X$ are 2 , $1 + \cos \omega + j \sin \omega$, and $1 + \cos \omega - j \sin \omega$, (a similar observation is given in [12]). Therefore if $\omega \neq \pi$, (23) is equivalent to

$$\mathbf{k}_{A1} \times \mathbf{k}_{A2} \neq \mathbf{0} \quad (24)$$

which is the first condition. The lower right diagonal matrix can be written as

$$\begin{bmatrix} (I - \Omega(z))(R_{A1} - I) \\ (I - \Omega(z))(R_{A2} - I) \end{bmatrix} = \begin{bmatrix} I - \Omega(z) & \mathbf{0} \\ \mathbf{0} & I - \Omega(z) \end{bmatrix} \times \begin{bmatrix} R_{A1} - I \\ R_{A2} - I \end{bmatrix} \quad (25)$$

By using the symbolic reduction package MACSYMA [13], it was shown that the determinant of $I - \Omega(z)$ is $(z_3^2 + 1)$, where z_3 is the third element of z . Thus $I - \Omega(z)$ is full rank. According to [1],

$$\text{Rank} \left(\begin{bmatrix} R_{A1} - I \\ R_{A2} - I \end{bmatrix} \right) = 3$$

if the first two conditions are satisfied. Consequently,

$$\text{Rank} \left(\begin{bmatrix} (I - \Omega(Z))(R_{A1} - I) \\ (I - \Omega(z))(R_{A2} - I) \end{bmatrix} \right) = 3$$

if the first two conditions are satisfied. This completes the proof of the theorem.

Remarks:

- 1) Theorem 1 provides sufficient conditions. In practice, even if the conditions are violated, the error parameter vector may still be observable.
- 2) More than two measurements may be used to reduce the effect of measurement noise.

In [6], it is suggested that using only the position measurements of B , one may still be able to obtain the unknown hand/eye transformation. In such a case, (14b) becomes the only governing equation, and the Identification Jacobian using two measurements becomes

$$J \equiv \begin{bmatrix} \Omega(p_{b1} + u_1) & (I - \Omega(z))(R_{A1} - I) \\ \Omega(p_{b2} + u_2) & (I - \Omega(z))(R_{A2} - I) \end{bmatrix} \quad (26)$$

which is a 6×6 matrix. Using (7), the above equation can be rewritten as

$$J = \begin{bmatrix} \Omega((R_X + I)p_{b1}) & (I - \Omega(z))(R_{A1} - I) \\ \Omega((R_X + I)p_{b2}) & (I - \Omega(z))(R_{A2} - I) \end{bmatrix} \quad (27)$$

Theorem 2: Assume that the Jacobian in (22) is replaced by (27). That is, only the position vectors of B_i , $i = 1, 2$, are used for the estimation of pose error parameter vector $d\rho$. $d\rho$ is observable only if

- 1) The rotation axes of R_{A1} and R_{A2} are neither parallel nor antiparallel to one another,
- 2) The rotation angles of R_{A1} and R_{A2} are both nonzero, and
- 3) p_{B1} and p_{B2} are nonzero and neither parallel nor antiparallel.

Proof: The Jacobian given in (27) is full rank only if its columns are linearly independent. According to [1] and the analysis of Theorem 1, the second three columns are linearly independent only if the first two conditions are satisfied. Similar to the proof of Theorem 1, an equivalent condition for the first three columns to be linearly independent is

$$((I + R_X)p_{B1}) \times ((I + R_X)p_{B2}) \neq 0 \quad (28)$$

Therefore if $\Omega \neq \pi$, Equation (28) is equivalent to

$$p_{B1} \times p_{B2} \neq 0 \quad (29)$$

which is the third condition. This completes the proof of the theorem.

Remarks:

- 1) This set of necessary conditions is the same as the given in [6].
- 2) Again, more than two measurements may be used to reduce measurement noise effect.

V. CONCLUSION

A new Identification Jacobian for hand/eye calibration is presented in this correspondence, which is based on an alternative mathematical formulation of the hand/eye calibration problem. The Jacobian matrix is much more compact than the one provided in [6]. Moreover, observability conditions of the pose error parameters of the unknown hand/eye transformation are derived based on the new Identification Jacobian. It is straightforward to apply this Jacobian matrix to robot hand/eye calibration. An observability measure was defined in [14] using all singular values of the Identification Jacobian for robot calibration. Studies were also performed which demonstrated that a relatively low number of well-selected measurements can produce superior calibration results, compared with identification based on a set of large number of randomly selected measurement configurations [14]. A future work is to extend the technique to robot hand/eye calibration using the proposed Identification Jacobian.

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