Robust fuzzy control for robot manipulators

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Abstract: A robust fuzzy control is developed for robot manipulators to guarantee both global stability and performance. Robot dynamics under consideration may include large nonlinear uncertainties, such as nonlinear load variations and unmodelled dynamics. Fuzzy sets are chosen based on performance requirements and stability regions of the control system. For each fuzzy set, a sub-control is designed, based on nonlinear robust control design using Lyapunov's direct method; this is blended with others into a final fuzzy control. The resulting control provides not only robust and global stability, but also more accurate control performance than fuzzy controls obtained from constant sub-controls. The proposed design is applied to a robot trajectory control problem and compared with a standard nonlinear robust controller. The simulation results show that the proposed control is effective and yields superior tracking performance.

1 Introduction

Dynamics of robot manipulators are highly nonlinear and may contain uncertain elements such as friction. Many efforts have been made in developing control schemes to achieve the precise tracking control of robot manipulators [1–3]. Among available options, fuzzy control has a great potential since it is able to compensate for the uncertain nonlinear dynamics using the programming capability of human control behaviour. Many results have been published in the area of design and stability of fuzzy control systems [4–9]. However, one of the critical issues in fuzzy control design is, although designed in a heuristic manner, how to ensure global and robust stability of the system under control.

A robust fuzzy control design has been developed [5] for a class of nonlinear systems, and the fuzzy control is robustly and globally stabilising. The design assumes a general structure and needs no supervisory control. In this approach, a robust sub-control is designed first and fuzzified for each rule to guarantee closed-loop stability in each fuzzy set. Individual robust controls are then blended into the overall fuzzy controller.

In this paper, the idea of robust fuzzy control design and its associated Lyapunov technique [10] are applied to develop a robust fuzzy control for robotic manipulators. The resulting control is shown to guarantee global stability, and to yield better performance than a fixed robust control as the fuzzy controller is configured to be a refined robust control in which non-conservative bounding functions of uncertain dynamics may be available. Consequently, the proposed fuzzy controller can be configured to be a refined robust control so that better performance can be achieved.

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2 Problem formulation

Consider the dynamic equation of an n rigid-link robot manipulator:

$$M(q)\ddot{q} + N(q, \dot{q}) = T \tag{1}$$

where

$$N(q, \dot{q}) = V_m(q, \dot{q})\dot{q} + G(q) + F(\dot{q}) + T_d(q, \dot{q})$$

where $q \in \Re^n$ is a vector of joint angle variables; $M(q)$ is an $[n \times n]$ inertia matrix, which is symmetric and positive definite; $V_m(q, \dot{q})$, $G(q)$, and $F(\dot{q})$ are $[n \times 1]$ vectors representing the centripetal and Coriolis terms, gravity terms, and static and dynamic friction terms, respectively; $T_d(q, \dot{q})$ represents an additive bounded disturbance due to load variation and/or modelling error; and $T \in \Re^n$ is a control vector of torque by the joint actuators.

The following properties and assumptions [11] are introduced for the proposed design.

2.1 Robot dynamics

Inertia matrix $M(q)$ is symmetric and positive definite, and it is bounded from above and below as

$$\underline{m}I \le M(q) \le \overline{m}(q)I \tag{2}$$

for some positive constant $\underline{m}$ and function $\overline{m}(q)$. The centripetal/Coriolis term $V_m(q, \dot{q})$ is bounded as

$$\|V_m(q, \dot{q})\| \le a_1\|\dot{q}\| \tag{3}$$

The friction and gravity terms are bounded as

$$\|G(q) + F(\dot{q})\| \le a_2 + a_3\|\dot{q}\| \tag{4}$$

where a_i are known constants. □

2.2 Disturbance

$T_d(q, \dot{q}, t)$ is bounded by known function $\eta(q, \dot{q})$ as

$$\|T_d(q, \dot{q}, t)\| \le \eta(q, \dot{q}) \tag{5}$$

□

2.3 Trajectory

The desired trajectory $q_d \in \Re^n$ and its derivatives $\dot{q}_d$ and $\ddot{q}_d$ are bounded by constants as

$$\|\ddot{q}_d\| \leq c_1 \text{ and } \|\dot{q}_d\| \leq c_2 \tag{6}$$

for constants c_1 and c_2.

Since the control objective is trajectory tracking, the tracking errors are defined as

$$e = q^d - q, \quad \dot{e} = \dot{q}^d - \dot{q} \tag{7}$$

where measurements of q and $\dot{q}$ are required in the subsequent control design. The state of the tracking error system is then chosen to be $x = [e^T \ \dot{e}^T]$.

To design a trajectory tracking control, rewrite eqn. 1 in terms of the tracking error given by eqn. 7 and formulate the state-space equation

$$\dot{x} = \bar{A}x + BM(q)^{-1}(\Delta A - T) \tag{8}$$

where

$$A = \begin{bmatrix} 0 & I_n \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ I_n \end{bmatrix},$$

$$\bar{A} = A - BR^{-1}B^TP,$$

$$\Delta A = M(q)(R^{-1}B^TPx + \ddot{q}^d) + N(q, \dot{q}) \tag{9}$$

I_n is the identity matrix, and matrix P is the positive definite solution of the Riccati equation: for any given pair of matrices $Q, R > 0$

$$A^TP + PA - PBR^{-1}B^TP + Q = 0 \tag{10}$$

Equivalently, if we have a positive definite matrix in the form of

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix} \tag{11}$$

where P_{ij} are sub-matrix blocks in P, and $P_{12} = P_{21}$. If matrices Q and R are set to be

$$Q = \begin{bmatrix} I_n & 0 \\ 0 & I_n \end{bmatrix}, \quad R = \begin{bmatrix} r_1 I_n & 0 \\ 0 & r_2 I_n \end{bmatrix}. \tag{12}$$

For positive constants r_1 and r_2, the positive definite solution for matrix P is given by

$$P_{ij} = \alpha_{ij} I_n \quad \text{for} \quad i, j = 1, 2 \tag{13}$$

where $\alpha_{i,j}$ are as follows:

$$\alpha_{11} = \sqrt{1 + 2r_2^{-\frac{1}{2}}}, \quad \alpha_{12} = \alpha_{21} = r_2^{-\frac{1}{2}}, \quad \alpha_{22} = r_2^{-\frac{1}{2}}\sqrt{1 + 2r_2^{-\frac{1}{2}}} \tag{14}$$

It is obvious that the bounding function for uncertainty ΔA can be obtained as

$$\begin{aligned} \|\Delta A\| \leq\ & \overline{m}(q)\|R^{-1}\| \cdot \|B^TPx\| + \overline{m}(q)\|\ddot{q}^d\| \\ & + \|V_m(q, \dot{q})\dot{q} + G(q) + F(\dot{q}) + T_d(q, \dot{q})\| \\ \triangleq\ & \rho_s(x) \end{aligned} \tag{15}$$

where $\|\cdot\|$ denotes the Euclidean norm.

3 Robust Fuzzy Control

The proposed control is in the form of

$$T = u_f = \frac{\sum_{i=1}^{l} \mu_{M_i}(z) \cdot u_i(x)}{\sum_{i=1}^{l} \mu_{M_i}(z)} \tag{16}$$

where l is the number of sub-fuzzy sets chosen; u_i is the individual control in the ith sub-fuzzy set; M_i is the membership function to be chosen later; $\mu_{M_i}(x)$ is the degree of membership function M_i; and z is the auxiliary state defined by

$$z = B^TPx = \begin{bmatrix} P_{12} & P_{22} \end{bmatrix} x = P_{12}e + P_{22}\dot{e} \tag{17}$$

From eqn. 13. $P_{22}^{-1}P_{12}$ is a positive definite matrix. The idea behind choosing auxiliary state z is that, since $P_{22}^{-1}P_{12}$ is a positive definite, then

$$\limsup_{t\to\infty,\ t\geq\tau} \|z(\tau)\| \leq \epsilon \quad \text{implies} \quad \limsup_{t\to\infty,\ t\geq\tau} \|e\| \leq \|P_{12}^{-1}\|\epsilon \tag{18}$$

Our objective is to design a fuzzy control of the form of eqn. 16, which guarantees stability and performance for the system in eqn. 3. The proposed scheme is based on the standard nonlinear robust control [12, 13] and standard fuzzy control design.

3.1 Robust fuzzy control design

The procedure of designing fuzzy control consists of four steps.

Step 1: selections of fuzzy sets and membership functions
As one of many possible choices, subsets F_i in the state space can be chosen as follows. For $i = 1, \ldots, l-1$, $F_i \triangleq \{x{:}x \in \Re^{2n}$, and x is either on, inside, or close to the hyper-ball defined by $\|z\| = d_i\}$. $d_l = 0$, and $d_i > d_j$ for $i > j$ is a finite sequence of positive increasing numbers chosen by the designer to reflect which stability regions and performance are desired or can be achieved.

$F_l \triangleq \{x{:}x \in \Re^{2n}$, and x is on the outside and not close to the hyper-balls defined by $\|z\| = d_{l-1}\}$. It is then obvious that $\cup_{i=1}^{l} F_i = \Re^{2n}$ as long as close to and not close to are complementary statements. From many possible choices [9], select a membership function $M_i(x)$ to make sets F_i fuzzy. The only requirement on membership function is that the degree of membership function $\mu_{M_i}(z)$ is between zero and one.

Step 2: selections of Lyapunov function and bounding function
The Lyapunov function is chosen to be

$$V(x) = \frac{1}{2}x^TPx \tag{19}$$

where matrix P is given by eqn. 13, and bounding function $\rho_s(x)$ is given by eqn. 15.

Step 3: selection of individual fuzzy control
For $i = 1, \ldots, l$, design individual control $u_i(x)$ according to the fuzzy rule.

Rule i
if $x \in F_i$, then control is given by $u = u_i(x)$.

Choice of individual control $u_i(x)$ is not unique, but it must satisfy the following three conditions.

(1) If $x \in F_i \cap F_j$ for some i and j, the signs of control vectors $\boldsymbol{u}_i$ and $\boldsymbol{u}_j$ satisfy the property that

$$\operatorname{sign}(\boldsymbol{u}_i) = \operatorname{sign}(\boldsymbol{u}_j) = \operatorname{sign}(\boldsymbol{u}_r) \tag{20}$$

where sign(·) is the generalisation of scalar sign function to vector case. (Although other forms of control can be chosen, sign condition (eqn. 20) implies that all individual controllers have the same direction of driving the state towards the origin for all values of z (no matter to which F_i they belong), along a ray originating from the origin in the z plane. This choice is the simplest way to achieve stabilisation.)

$\boldsymbol{u}_r$ is the robust control defined by

$$\boldsymbol{u}_r = -\overline{m}\frac{[\rho_s(\boldsymbol{x})]^2}{\rho_s(\boldsymbol{x})\|z\| + \epsilon\varphi(t)} z \tag{21}$$

In control (eqn. 21), $\epsilon > 0$ is a design constant, and $\varphi(t) > 0$ is a uniformly continuous L_1 or L_∞ time function.

(2) Fuzzy control $\boldsymbol{u}_i$ must have the property that

$$z^T \boldsymbol{u}_i \geq z^T \boldsymbol{u}_r \tag{22}$$

(3) Fuzzy control $\boldsymbol{u}_i$ must satisfy the inequality that, for all $\boldsymbol{x}$

$$\frac{\sum\limits_{i \in N} \mu_{M_i}(z) \cdot |z^T \boldsymbol{u}_i(\boldsymbol{x})|}{\sum\limits_{i=1}^{l} \mu_{M_i}(z)} \geq |z^T \boldsymbol{u}_r(\boldsymbol{x})| \tag{23}$$

In this paper, fuzzy control $\boldsymbol{u}_i(\boldsymbol{x})$ is selected to be

$$\boldsymbol{u}_i = -\overline{m}\frac{k_i}{\rho_s(\boldsymbol{x})|z| + \epsilon\varphi(t)} z, \quad i = 1, 2, \ldots, l-1$$

$$\boldsymbol{u}_l = \boldsymbol{u}_r, \tag{24}$$

where

$$k_i = \sup_{0 \leq |z| \leq d_{i+1}} [\rho_s(\boldsymbol{x})]^2 \tag{25}$$

Step 4: selection of fuzzy control law

The overall fuzzy control $\boldsymbol{u}_f$ is found by blending the individual controls $\boldsymbol{u}_i$ according to the standard fuzzifying formula:

$$\boldsymbol{u}_f = \frac{\sum\limits_{i=1}^{l} \mu_{M_i}(z) \cdot \boldsymbol{u}_i(\boldsymbol{x})}{\sum\limits_{i=1}^{l} \mu_{M_i}(z)}$$

$$= -\overline{m}\frac{1}{\rho_s(\boldsymbol{x})|z| + \epsilon\varphi(t)} \frac{\left[\sum\limits_{i=1}^{l-1} k_i \mu_{M_i}(z)\right] + \mu_{M_l}(z)[\rho_s(\boldsymbol{x})]^2}{\sum\limits_{i=1}^{l} \mu_{M_i}(z)} z \tag{26}$$

The above design procedure guarantees both performance and robustness. Control in eqn. 24 u_l can be interpreted as a nonlinear supervisory control which ensures global stability. For $i = 1, \ldots, l-1$ controls u_i provide accurate control without over-estimating the control gain, and the overall fuzzy control (eqn. 25) executes similarly as a variable structure controller.

3.2 System stability

The proposed fuzzy control is based on the existing nonlinear robust control design; therefore, stability analysis under fuzzy control is performed in parallel to that under the nonlinear robust control design.

Lemma 1: System in eqn. 8 is globally and asymptotically stable, or uniformly ultimately bounded under nonlinear robust control (eqn. 21), i.e. $\boldsymbol{T} = \boldsymbol{u}_r$.

Proof: to show that system in eqn. 8 is stable under robust control (eqn. 21), note that the time derivative of the Lyapunov function (eqn. 19) is

$$\begin{aligned}
\dot{V} &= -\frac{1}{2}\boldsymbol{x}^T\boldsymbol{Q}\boldsymbol{x} + \boldsymbol{x}^T\boldsymbol{PBM}(\boldsymbol{q})^{-1}(\Delta A - \boldsymbol{u}_r) \\
&= -\frac{1}{2}\boldsymbol{x}^T\boldsymbol{Q}\boldsymbol{x} + [z^T\boldsymbol{M}(\boldsymbol{q})^{-1}\Delta A - z^T\boldsymbol{M}(\boldsymbol{q})^{-1}\boldsymbol{u}_r] \\
&\leq -\frac{\lambda_{min}(\boldsymbol{Q})}{\lambda_{max}(\boldsymbol{P})}V + \frac{1}{\underline{m}}\|z\| \cdot \rho_s(\boldsymbol{x}) - z^T\boldsymbol{M}(\boldsymbol{q})^{-1}\boldsymbol{u}_r \\
&\leq -\frac{\lambda_{min}(\boldsymbol{Q})}{\lambda_{max}(\boldsymbol{P})}V + \epsilon\varphi(t)
\end{aligned} \tag{27}$$

As shown previously [13], you can solve the above differential inequality to show that V and $\|\boldsymbol{x}\|$ converge exponentially to zero or to a uniform ultimate bound. □

It has also been shown [13] that the system is exponentially convergent in the large and that transient excursions can be estimated. In fact, transient response can be adjusted by proper choices of design parameter ϵ and function $\varphi(t)$ [3]. Based on Lemma 1, the following stability result can be easily concluded.

Theorem 1: Under fuzzy control (eqn. 16), the system in eqn. 8 is globally and asymptotically stable.

Proof: The proposed control is designed to satisfy the following inequality:

$$z^T \boldsymbol{u}_f \leq z^T \boldsymbol{u}_r \tag{28}$$

The above inequality is guaranteed by eqns. 20, 22 and 23. It follows from the proof of the Lemma 1 that global stability can be concluded.

4 Simulation

A two degree-of-freedom robot manipulator is used in simulation to evaluate the proposed control scheme. The dynamic equations of the two rigid-link manipulator can be found elsewhere [11], and its parameters are set to be $m_1 = m_2 = 1.0\,\text{kg}$ and $l_1 = l_2 = 1.0\,\text{m}$.

In the simulation, initial conditions are given as $q_1(0) = q_2(0) = 0.0174\,\text{rad}$ (1 degree), $\dot{q}_1(0) = \dot{q}_2(0) = 0\,\text{rad/s}$, and the desired trajectory is given by $q_1^d(t) = q_2^d(t) = 1.0 - \cos(t)$. The friction and disturbance terms are assumed to be

$$\boldsymbol{T}_d = \begin{bmatrix} 5\cos(5t) \\ 5\cos(5t) \end{bmatrix}\text{N-m} \quad \text{and} \quad \boldsymbol{F}(\dot{\boldsymbol{q}}) = 0.5\operatorname{sign}(\dot{\boldsymbol{q}}) \tag{29}$$

where sign denotes the vector sign function.

For the robust fuzzy control, bounds and bounding functions for the system are set to be $\underline{m} = 0.5$, $\overline{m} = 9.0$ and $\rho_s(\boldsymbol{x}) = 250 + 50(|\boldsymbol{x}| + |\boldsymbol{x}|^2)$.

It follows from the Riccati equation that, given $\boldsymbol{Q} = \boldsymbol{R} = \boldsymbol{I}$, where $\boldsymbol{I}$ is the identity matrix,

$$\boldsymbol{P} = \begin{bmatrix} \sqrt{3}\boldsymbol{I}_2 & \boldsymbol{I}_2 \\ \boldsymbol{I}_2 & \sqrt{3}\boldsymbol{I}_2 \end{bmatrix} \quad \text{and} \quad z \triangleq [\boldsymbol{I}_2 \sqrt{3}\boldsymbol{I}_2]\boldsymbol{x} \tag{30}$$

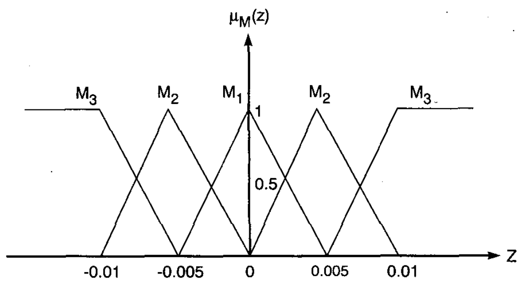

Fig. 1 *Symmetric triangle membership function*

Subsets of the state space F_i, $1 = 1,2,3$, are defined as follows: $F_1 \triangleq \{x{:}x \in \Re^{2n}$ and x is either at or close to the origin$\}$ $F_{2>} \triangleq \{x{:}x \in \Re^{2n}$ is either on, inside, or close to the hyper-ball defined by $\|z\| = 0.005$ $\}$ and $F_3 \triangleq \{x{:}x \in \Re^{2n}$ and x is on the outside and not close to the hyper-ball defined by $\|z\| = 0.005\}$.

The triangle membership function given in Fig. 1 is chosen, and it has the property that, for all z,

$$\mu_{M_1}(z) + \mu_{M_2}(z) + \mu_{M_3}(z) = 1 \tag{31}$$

Fig. 1 shows that any value of z does not belong to more than two fuzzy sets of $\boldsymbol{F}_i$.

According to eqn. 24, individual controls $\boldsymbol{u}_i(x)$ for $i = 1,2,3$ are

$$\begin{aligned} \boldsymbol{u}_1 &= -\overline{m}\frac{k_1}{\rho_s(\boldsymbol{x})|z| + \epsilon\varphi(t)}z, \\ \boldsymbol{u}_2 &= -\overline{m}\frac{k_2}{\rho_s(\boldsymbol{x})|z| + \epsilon\varphi(t)}z \\ \boldsymbol{u}_3 &= \boldsymbol{u}_r \end{aligned} \tag{32}$$

The design constant and design function are chosen to be

$$\epsilon = 25 \quad \varphi(t) = \begin{cases} e^{-0.5t} & 0 \le t \le 10 \\ e^{-5} & t > 10 \end{cases} \tag{33}$$

It follows from eqn. 25 that $k_1 = 62{,}764$ and $k_2 = 62{,}626$.

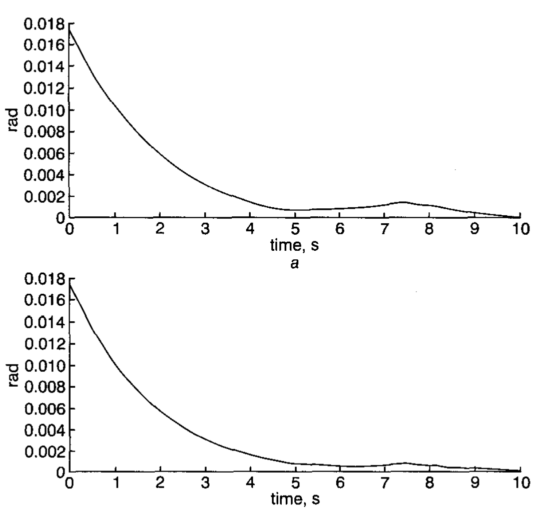

Fig. 2 *Position errors (a) e_1 and (b) e_2 under robust control*

According to eqn. 26, the overall fuzzy control is then designed as

$$\boldsymbol{u}_f = -\overline{m}\frac{1}{\rho_s(\boldsymbol{x})|z| + \epsilon\varphi(t)} \cdot \frac{[k_1\mu M_1(\boldsymbol{x}) + k_2\mu M_2(\boldsymbol{x})] + \mu M_3(z)\rho_s^2(\boldsymbol{x})}{\sum_{i=1}^{3}\mu M_i(z)}z \tag{34}$$

Robust control (eqn. 21) and robust fuzzy control (eqn. 34) are implemented for comparison. The simulation results are shown in Figs. 2 and 3. It is obvious from the results that the proposed robust fuzzy control system is comparable to the nonlinear robust control for the robot tracking. Fig. 4 shows the joint errors under robust control (eqn 21) minus the corresponding errors under robust fuzzy control (eqn. 34). This comparison shows that the proposed fuzzy control results in slightly better tracking performance than the nonlinear robust control for $t \geq 6.8$ [s].

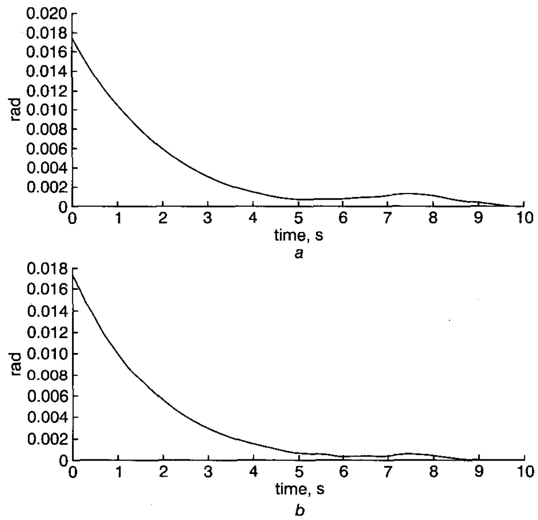

Fig. 3 *Position errors (a) e_1 and (b) e_2 under robust fuzzy control*

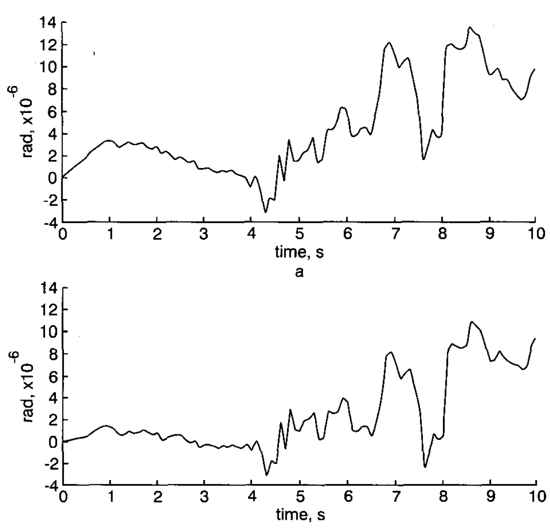

Fig. 4 *Comparison of tracking performance, $e_{robust} - e_{fuzzy}$*

5 Conclusions

The proposed robust fuzzy control guarantees not only desired performance, but also global stability and robustness for robot manipulators. The proposed design is to synthesise individual nonlinear robust controllers for each fuzzy rule and then to blend them into a fuzzy control. The design is to combine the advantages of both nonlinear robust control and fuzzy control. Simulation results have shown the effectiveness of the proposed scheme.

6 References

1 GE, S.: 'Advanced control techniques of robotic manipulators.'. Proc. Amer. Control Conf., 1998, pp. 2185–2199

2 HAM, C., and QU, Z.: 'A new nonlinear learning control for robotic manipulators', *Advanced Robotics*, 1996, **10**, (5), pp. 1–15

3 QU, Z., and DAWSON, D.: 'Robust tracking control of robot manipulators' (IEEE Press, New York, 1996)

4 BIRDWELL, J., and WANG, Y.: 'Lyapunov stability analysis of systems using the fuzzy-PID controller'. Proc. Amer. Control Conf., 1994, Baltimore, Maryland, pp. 966–970

5 HAM, C., QU, Z., and KALOUST, J.: 'Design of globally stabilizing robust fuzzy control for a class of nonlinear systems', *Int. J. Intell. Control Syst.*, 1996, **1**, (2), pp. 261–271

6 LANGARI, G., and TOMIZUKA, M.: 'Stability of fuzzy linguistic control systems.'. Proc. 29th IEEE Conf. on *Decision and Control*, 1990, Honolulu, Hawaii, pp. 2185–2190

7 MALKI, H., LI, H., and CHEN, G.: 'New design and stability analysis of fuzzy proportional-derivative control systems', *IEEE Trans.*, 1994, **FS-2**, pp. 245–254

8 TANAKA, K., and SANO, M.: 'Concept of stability margin for fuzzy systems and design of robust fuzzy controls'. Proc. 2nd IEEE *Fuzzy Systems* Conf., 1993, pp. 29–34

9 WANG, L.: 'A course in fuzzy systems and control' (Prentice Hall, New Jersey, 1997)

10 KHALIL, H.: 'Nonlinear systems' (Macmillan, New York, 1992)

11 CRAIG, J.: 'Adaptive control of mechanical manipulators' (Addison-Wesley, New York, 1988)

12 CORLESS, M., and LEITMANN, G.: 'Continuous State feedback guaranteeing uniform ultimate boundedness for uncertain dynamic systems', *IEEE Trans.*, 1981, **AC-26**, pp. 1139–1144

13 QU, Z.: 'Robust control of nonlinear uncertain systems' (John Wiley & Sons, New York, 1998)