



# Multi machine power system excitation control design via theories of feedback linearization control and nonlinear robust control

DEQIANG GAN<sup>†</sup>, ZHIHUA QU<sup>†</sup> and HONGZHI CAI<sup>†</sup>

*The dynamics of a large-scale power system are both nonlinear and interconnected. The equilibrium of such a system is typically unknown and uncertain, and the controllers within are also subject to physical limitations. In this paper, a new application of nonlinear robust control is presented for power system control design. It is assumed that the controllers are designed as a part of generator excitation system design. First, a customized exact feedback linearization scheme is developed for the power system under investigation. This new linearization scheme allows one to transform the power system with a single-axis system model into a linear uncertain system with an unknown equilibrium. Based on the latest development of nonlinear robust control theory, a novel control design is then applied to stabilize the resulting linearized uncertain system. Finally, a nonlinear decentralized excitation control is obtained by the inverse transformation. Compared with existing control schemes, the proposed control is free from such common deficiencies of power system nonlinear controllers as network dependence and equilibrium dependence. Detailed stability analysis and engineering judgment in the control design are provided. The results of simulation studies are presented.*

## 1. Introduction

The main objective of installing controllers in power systems is to achieve desired stability and security at a reasonable cost. Controllers designed using modern state-space theory have been serving power systems for decades, and conventional single-input single-output controllers designed using classical control theory were put into use even earlier. In recent years there has been an increasing interest on applying advanced control designs in power engineering area. A number of novel control design methodologies, for example adaptive control,  $H_\infty$  control and  $\mu$  synthesis, nonlinear control, feedback linearization, fuzzy logic control and neural control, have been reported. The goal of these studies is to achieve stability and performance robustness. Conventional and state-space controllers are not designed in a way to guarantee the desired level of robustness. The focus of this paper is on nonlinear con-

trol design methodologies (Hill *et al.*, 1993, Falker and Hecker 1995, Rajkumar and Mohler 1995).

Feedback linearization control (FBLC) theory has been applied to power system control designs, including turbine valve control design, excitation control design, high voltage direct current (HVDC) power control design and static voltage condenser (SVC) control design (Lu and Sun 1989, 1993, Sun and Lu 1996). The results of Chapman and Ilic (1992), Chapman *et al.* (1993), King *et al.* (1994) and Allen *et al.* (1996) uncovered several critical issues in FBLC-based control design and extended the FBLC to reduced-order practical system and to the investigation of torsion dynamics. FBLC has also been used in the design of a controller for synchronous generator connected to an infinite bus (Mielczarski and Zajaczkowski 1994). The so-called direct feedback linearization (DFBL) theory was introduced especially for power system control design (Gao *et al.* 1992; Wang *et al.* 1993, 1994 a) as DFBL is much more understandable to the power engineering audience. However, the application of DFBL is limited only to single-input single-output systems. Power system stabilizer design through FBLC has been reported by several researchers (Cao *et al.* 1994, Nambu and Ohsawa 1996). The effect of unknown

---

Received 26 August 1998. Revised 12 April 1999. Accepted 22 April 1999.

<sup>†</sup>Department of Electrical and Computer Engineering, University of Central Florida, Orlando, FL 32816, USA. Tel: 1 (407) 823-5976; Fax: 1 (407) 823-5835. e-mail: qu@pegasus.cc.ucf.edu

interconnection was studied recently, and an adaptive FBLC control law was proposed by Jain *et al.* (1994). The FBLC design of a simple ac–dc power system was examined by Kaprielian *et al.* 1990). Other nonlinear control design methods include, for example, predictive control (Rajkumar and Mohler 1994), variable structure control (Wang *et al.* 1994 b), and nonlinear control proposed by the present authors and co-workers (Qu *et al.* 1992, Jiang *et al.* 1993, 1997, Cai *et al.* 1996). This later method originated from the Lyapunov–based robust control theory for general uncertain nonlinear systems (Qu 1995, 1998).

Although many developments have been proposed in the area of nonlinear control designs for power system, there are still several issues worth investigating further.

- (1) As indicated by Chapman *et al.* (1993), implementation of an excitor control based on FBLC theory requires that the equilibrium point of power system is known and fixed. This has not yet been thoroughly studied.
- (2) An ideal excitation system should be independent on the topology of power network. The controllers designed using FBLC, however, require the availability of information of power systems topology.
- (3) Most of the existing nonlinear controllers do not guarantee that terminal voltages are within their permissible ranges.
- (4) The output of a real-world controller is always limited because of physical limitations. The effects of such limits on the dynamics of controlled power systems should be studied.
- (5) The controller should be as simple as possible, which has three aspects. The feedback signals required by the controller must be measurable; the controller must be physically implementable (not just in theory but also in practice); the control must be capable of compensating for the unknowns and it does not require sophisticated tuning of design parameters.

The main thrust of this paper is to resolve some of the above-mentioned problems in nonlinear excitation control design. Based on the latest development in two nonlinear control design methodologies exact FBLC and Lyapunov-based robust control, an improved exact feedback linearization scheme is proposed in the paper to transform the dynamics of a nonlinear power system into an *uncertain linear system with an unknown equilibrium*. A novel control design is then presented to stabilize the uncertain linear system, and a nonlinear decentralized excitation control is obtained by inverse transformation. The proposed control, which is composed of two additive parts (a linear optimal state-

space feedback and a saturation-type nonlinear robust control), has several advantages. Firstly, it does not require knowledge about the system equilibrium as most other control designs require. Instead, it needs only an approximate estimate of system equilibrium. Secondly, the control is decentralized and partially independent of network topology. Thirdly, loss of bus voltage can be alleviated by judicious choices of design parameters, and physical limits of controllers can be respected. Simulation results of a test power system are provided.

## 2. De-composed feedback linearization for power systems with the single-axis model

Let us at first review the exact linearized model of a multimachine power system. Under several standard assumptions, the state equation of *i*th generator can be written by (Sauer and Pai 1991)

$$\dot{\delta}_i = \omega_i, \quad (1)$$

$$\dot{\omega}_i = \frac{1}{M_i} [P_{mi} - P_{ei} - D_i \omega_i], \quad (2)$$

$$\dot{E}'_{qi} = \frac{1}{T'_{doi}} [E_{fdi} - E'_{qi} + (X'_{di} - X_{di}) I_{di}], \quad (3)$$

$$P_{ei} = E'_{qi} I_{qi} + (X_{qi} - X'_{di}) I_{di} I_{qi} \quad (4)$$

$$I_{di} = \sum_{k=1}^{NG} E'_{qk} (G_{ik} \cos \delta_{ik} - B_{ik} \sin \delta_{ik}) \quad (5)$$

and

$$I_{qi} = \sum_{k=1}^{NG} E'_{qk} (G_{ik} \sin \delta_{ik} + B_{ik} \cos \delta_{ik}), \quad (6)$$

where  $\delta_{ik} = \delta_i - \delta_k$ . The output of each machine is the rotor angle  $\delta$ ; the vector of state variables of the power system can be chosen to be

$$z = [\delta_1, \omega_1, \dot{\omega}_1, \dots, \delta_n, \omega_n, \dot{\omega}_n]. \quad (7)$$

This new state vector allows one to transform the system into the canonical form given by

$$\dot{z}_{1i} = z_{2i}, \quad (8)$$

$$\dot{z}_{2i} = z_{3i}, \quad (9)$$

$$\dot{z}_{3i} = v_i, \quad (10)$$

where

$$v_i = -\frac{1}{M_i} \left\{ \sum_{j=1}^{NG} \frac{\partial P_{ei}}{\partial E'_{qj}} \frac{1}{T'_{doj}} [-E'_{qj} + (X'_{dj} - X_{dj}) I_{dj}] \right. \\ \left. + \sum_{j=1}^{NG} \frac{\partial P_{ei}}{\partial \delta_j} \omega_j + D_i \dot{\omega}_i \right\} - \frac{1}{M_i} \sum_{j=1}^{NG} \frac{\partial P_{ei}}{\partial E'_{qj}} \frac{1}{T'_{doj}} E_{fdj} \quad (11)$$

is the intermediate control input of the so-called feedback linearized system, that is the system consists of (8)–(10).

Based on modern control theory of linear systems, the optimal state feedback law of form

$$v_i = -k_1(z_{1i} - z_{1i}^0) - k_2(z_{2i} - z_{2i}^0) - k_3(z_{3i} - z_{3i}^0) \quad (12)$$

can be used to stabilize the linearized system around the equilibrium  $z^0$ . The controls  $v_i$  drive the output dynamics of the original systems, and the physical controls  $E_{fdi}$  can be obtained through the mapping in (11). This design process is currently of interest in the literature.

A main difficulty in the above control design using FBLC is that the equilibrium  $z^0$  is generally unknown. In reality, approximate settings of  $(z_{1i}^0, z_{2i}^0, z_{3i}^0)$  have to be used. Another obstacle is that the control law obtained above is centralized; consequently both global feedback signals and network topology must be available. To develop a decentralized topology-independent control law, the designer has to separate the dynamics of the system. In the existing results, an approximation has been used to discard the coupling dynamics.

Our idea to deal with the two problems mentioned above is to decompose the nonlinear terms into locally measurable and locally unmeasurable parts such that the measurable part is exactly compensated for by a local control, while the unmeasurable part is compensated for by an additional robust control input. It is the same robust control that enables the designer to remove the requirement of exact knowledge of  $z^0$ . The decomposition procedure will be described in detail here, while the control design will be discussed in the next section.

Let us begin with rewriting  $\dot{z}_{3i}$  as

$$\dot{z}_{3i} = a_i(\mathbf{x}) + b_i(\mathbf{x})E_{fdi} \quad (13)$$

where,

$$\begin{aligned} a_i(\mathbf{x}) = & -\frac{1}{M_i} \left\{ \sum_{j=1}^{NG} \frac{\partial P_{ei}}{\partial E'_{qj}} \frac{1}{T'_{doj}} [-E'_{qj} + (X'_{dj} - X_{dj})I_{dj}] \right. \\ & \left. + \sum_{j=1}^{NG} \frac{\partial P_{ei}}{\partial \delta_j} \omega_j + D_i \dot{\omega}_i \right\} \\ & - \frac{1}{M_i} \sum_{j=1, j \neq i}^{NG} \frac{\partial P_{ei}}{\partial E'_{qj}} \frac{1}{T'_{doj}} E_{fdj} \end{aligned} \quad (14)$$

$$b_i(\mathbf{x}) = -\frac{1}{M_i} \frac{\partial P_{ei}}{\partial E'_{qi}} \frac{1}{T'_{doi}} \quad (15)$$

and  $\mathbf{x} = [\delta_1, \omega_1, E'_{q1}, \dots, \delta_n, \omega_n, E'_{qn}]^T$  denotes the vector of the state variable before feedback linearization transformation. Note that both  $a_i(\mathbf{x})$  and  $b_i(\mathbf{x})$  are not totally known at the locally  $i$ -th power station.

Next, decompose  $a_i(\mathbf{x})$  and  $b_i(\mathbf{x})$  into measurable and non-measurable parts respectively. Let us first rewrite  $a_i(\mathbf{x})$  as

$$\begin{aligned} a_i(\mathbf{x}) = & -\frac{1}{M_i} \left\{ \frac{\partial P_{ei}}{\partial E'_{qi}} \frac{1}{T'_{doi}} [-E'_{qi} + (X'_{di} - X_{di})I_{di}] \right. \\ & \left. + \frac{\partial P_{ei}}{\partial \delta_i} \omega_i + D_i \dot{\omega}_i \right\} \\ & - \frac{1}{M_i} \left\{ \sum_{j=1, j \neq i}^{NG} \frac{\partial P_{ei}}{\partial E'_{qj}} \dot{E}'_{qj} + \sum_{j=1, j \neq i}^{NG} \frac{\partial P_{ei}}{\partial \delta_j} \omega_j \right\}. \end{aligned} \quad (16)$$

In the above equation,  $\partial P_{ei}/\partial E'_{qi}$  (the exact expressions of  $\partial P_{ei}/\partial E'_{qi}$  are given in appendix A) can be decomposed into two additive parts as

$$\frac{\partial P_{ei}}{\partial E'_{qi}} = \frac{\partial \hat{P}_{ei}}{\partial E'_{qi}} + \frac{\partial \Delta P_{ei}}{\partial E'_{qi}} \quad (17)$$

where

$$\frac{\partial \hat{P}_{ei}}{\partial E'_{qi}} = I_{qi} + (E'_{qi} + \Delta X I_{di}) \hat{B}_{ii} + \Delta X I_{qi} \hat{G}_{ii} \quad (18)$$

$$\frac{\partial \Delta P_{ei}}{\partial E'_{qi}} = (E'_{qi} + \Delta X I_{di}) \Delta B_{ii} + \Delta X I_{qi} \Delta G_{ii} \quad (19)$$

and  $(\hat{B}_{ii}, \hat{G}_{ii})$  denote normal values of  $(B_{ii}, G_{ii})$  (which defines the nominal topology), while  $(\Delta B_{ii}, \Delta G_{ii})$  stands for the uncertain components of  $(B_{ii}, G_{ii})$ , and  $\Delta X = X_{qi} - X'_{di}$ . It follows that

$$a_i(\mathbf{x}) = a'_i(\mathbf{x}_i) + a''_i(\mathbf{x}) \quad (20)$$

where

$$\begin{aligned} a'_i(\mathbf{x}_i) = & -\frac{1}{M_i} \left\{ \frac{\partial \hat{P}_{ei}}{\partial E'_{qi}} \frac{1}{T'_{doi}} [-E'_{qi} + (X'_{di} - X_{di})I_{di}] \right. \\ & \left. + \frac{\partial P_{ei}}{\partial \delta_i} \omega_i + D_i \dot{\omega}_i \right\} \end{aligned} \quad (21)$$

and

$$\begin{aligned} a''_i(\mathbf{x}) = & -\frac{1}{M_i} \left\{ \frac{\partial \Delta P_{ei}}{\partial E'_{qi}} \frac{1}{T'_{doi}} [-E'_{qi} + (X'_{di} - X_{di})I_{di}] \right. \\ & \left. + \sum_{j=1, j \neq i}^{NG} \left( \frac{\partial P_{ei}}{\partial E'_{qj}} \dot{E}'_{qj} + \frac{\partial P_{ei}}{\partial \delta_j} \omega_j \right) \right\}. \end{aligned} \quad (22)$$

Decomposition of (21) and (22) is made so that the terms in (21) require only the physical information available locally and (22) contains the variables not available locally.

The decomposition of  $b_i(\mathbf{x})$  is straightforward, that is

$$b_i(\mathbf{x}) = -\frac{1}{M_i} \frac{1}{T'_{doi}} \frac{\partial P_{ei}}{\partial E'_{qi}} = b'_i + b''_i, \quad (23)$$

where

$$b'_i = -\frac{1}{M_i T'_{doi}} \frac{\partial \hat{P}_{ei}}{\partial E'_{qi}}, \quad b''_i = -\frac{1}{M_i T'_{doi}} \frac{\partial \Delta P_{ei}}{\partial E'_{qi}} \quad (24)$$

In summary, the decomposed form of  $\dot{z}_{3i}$  is

$$\dot{z}_{3i} = a'_i(\mathbf{x}_i) + b'_i(\mathbf{x}_i)E_{fdi} + a''_i(\mathbf{x}) + b''_i(\mathbf{x})E_{fdi} \quad (25)$$

where  $\mathbf{x}_i = [\delta_i, \omega_i, \dot{\omega}_i]$  is the substate vector locally measurable, and so is  $z_i$ . Furthermore, the feedback linearized system can now be mapped into the following form:

$$\dot{z}_{1i} = z_{2i}, \quad (26)$$

$$\dot{z}_{2i} = z_{3i}, \quad (27)$$

$$\dot{z}_{3i} = w_i + d_i. \quad (28)$$

where

$$w_i = a'_i(\mathbf{x}_i) + b'_i(\mathbf{x}_i)E_{fdi} \quad (29)$$

is the intermediate control input that is local and decentralized, and

$$d_i = a''_i(\mathbf{x}) + b''_i(\mathbf{x})E_{fdi} \quad (30)$$

is the lumped nonlinear uncertainty that is not measured locally.

In the next section, we shall design the nonlinear control  $w_i$  to stabilize the above uncertain partially linearized system based on optimal control theory and nonlinear robust control theory (Qu 1995, 1998). Once  $w_i$  has been designed, the physical control  $E_{fdi}$  can be solved from (29).

### 3. The proposed control law and stability property

Control design for the partially linearized system (26)–(28) is more straightforward than that for the original nonlinear system (1)–(6). Note that the system equilibrium of the partially linearized system is still unknown. Therefore, the proposed controller is set to have two additive parts  $w'_i$  and  $w''_i$ , that is

$$w_i = w'_i + w''_i. \quad (31)$$

The first part  $w'_i$  is a state feedback control that stabilizes the ‘normal’ linearized system (system (26)–(28) with uncertainty  $d_i = 0$ ). It is easy to see that

$$w'_i = -k_{1i}(z_{1i} - z_{1i}^d) - k_{2i}(z_{2i} - z_{2i}^d) - k_{3i}(z_{3i} - z_{3i}^d) \quad (32)$$

is sufficient, where  $k_{1i}, k_{2i}$  and  $k_{3i}$  are gains that can easily be selected using modern control theory, and  $z_{1i}^d, z_{2i}^d$  and  $z_{3i}^d$  represent desired equilibrium values for the  $i$ th machine. Obviously,  $z_{2i}^d$  and  $z_{3i}^d$  should be set to zero, while  $z_{1i}^d$  should be chosen to be any desired value (which may be equal or close to the pre-fault equilibrium). The introduction of  $z_i^d$  generates a control even though the post-fault system equilibrium is often unknown.

Let us define the following state variables:

$$y_{1i} = z_{1i} - z_{1i}^d, \quad (33)$$

$$y_{2i} = z_{2i} - z_{2i}^d, \quad (34)$$

$$y_{3i} = z_{3i} - z_{3i}^d. \quad (35)$$

It follows that

$$\dot{y}_{1i} = y_{2i}, \quad (36)$$

$$\dot{y}_{2i} = y_{3i}, \quad (37)$$

$$\dot{y}_{3i} = -k_{1i}y_{1i} - k_{2i}y_{2i} - k_{3i}y_{3i} + d_i + w''_i. \quad (38)$$

Let us write the above equations in compact form as

$$\dot{\mathbf{y}}_i = \mathbf{A}_i \mathbf{y}_i + \mathbf{B}_i (d_i + w''_i), \quad (39)$$

where  $\mathbf{y}_i = (y_{1i}, y_{2i}, y_{3i})$  and

$$\mathbf{A}_i = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -k_{1i} & -k_{2i} & -k_{3i} \end{bmatrix}, \quad \mathbf{B}_i = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}. \quad (40)$$

Note that

$$d_i \leq \xi_i \quad (41)$$

where  $\xi_i$  (whose expression is given in appendix A) is a constant.

The second part of the control input,  $w''_i$ , is to compensate for the uncertainty  $d_i$ . The following saturation-type controller is used:

$$w''_i = -\xi_i^2 \frac{\mathbf{B}_i^T \mathbf{P}_i \mathbf{y}_i}{\|\mathbf{B}_i^T \mathbf{P}_i \mathbf{y}_i\| \xi_i + \varepsilon_i}. \quad (42)$$

The above control design is driven by stability analysis of the closed system under control  $w_i = w'_i + w''_i$ . Choose the Lyapunov function candidate as

$$V = \frac{1}{2} \mathbf{y}^T \mathbf{P} \mathbf{y} = \sum_{i=1}^n V_i = \sum_{i=1}^n \frac{1}{2} \mathbf{y}_i^T \mathbf{P}_i \mathbf{y}_i \quad (43)$$

where  $\mathbf{P} = \text{diag} \{ \mathbf{P}_1, \dots, \mathbf{P}_n \}$ . It follows that

$$\begin{aligned} \dot{V} &= \sum [\frac{1}{2} \mathbf{y}_i^T (\mathbf{P}_i \mathbf{A}_i + \mathbf{A}_i^T \mathbf{P}_i) \mathbf{y}_i + \mathbf{y}_i^T \mathbf{P}_i \mathbf{B}_i (d_i + w''_i)] \\ &= \sum [-\mathbf{y}_i^T \mathbf{Q}_i \mathbf{y}_i + \mathbf{y}_i^T \mathbf{P}_i \mathbf{B}_i (d_i + w''_i)], \end{aligned} \quad (44)$$

where  $\mathbf{Q}_i = -\frac{1}{2}(\mathbf{P}_i \mathbf{A}_i + \mathbf{A}_i^T \mathbf{P}_i)$  is positive definite. It should be noted that  $\mathbf{Q}_i > 0$  is guaranteed by proper choices of control gains  $k_{1i}, k_{2i}$  and  $k_{3i}$ . Furthermore, the combined term  $\mathbf{y}_i^T \mathbf{P}_i \mathbf{B}_i (d_i + w''_i)$  is no larger than a constant, which can be seen as

$$\begin{aligned}
\mathbf{y}_i^T \mathbf{P}_i \mathbf{B}_i (d_i + w_i'') &= \mathbf{y}_i^T \mathbf{P}_i \mathbf{B}_i \left( d_i - \xi_i^2 \frac{\mathbf{B}_i^T \mathbf{P}_i \mathbf{y}_i}{\|\mathbf{B}_i^T \mathbf{P}_i \mathbf{y}_i\| \xi_i + \varepsilon_i} \right) \\
&= \frac{\mathbf{y}_i^T \mathbf{P}_i \mathbf{B}_i d_i \|\mathbf{B}_i^T \mathbf{P}_i \mathbf{y}_i\| \xi_i + \mathbf{y}_i^T \mathbf{P}_i \mathbf{B}_i}{\|\mathbf{B}_i^T \mathbf{P}_i \mathbf{y}_i\| \xi_i + \varepsilon_i} \\
&\quad \times d \varepsilon_i - \mathbf{y}_i^T \mathbf{P}_i \mathbf{B}_i \xi_i^2 \mathbf{B}_i^T \mathbf{P}_i \mathbf{y}_i \\
&\leq \frac{\mathbf{y}_i^T \mathbf{P}_i \mathbf{B}_i d_i \varepsilon_i}{\|\mathbf{B}_i^T \mathbf{P}_i \mathbf{y}_i\| \xi_i + \varepsilon_i} \\
&\leq \varepsilon_i. \tag{45}
\end{aligned}$$

It follows that

$$\begin{aligned}
\dot{V} &\leq - \sum_{i=1}^n \mathbf{y}_i^T \mathbf{Q}_i \mathbf{y}_i + \sum_{i=1}^n \varepsilon_i \\
&= - \sum_{i=1}^n \mathbf{y}_i^T \mathbf{Q}_i \mathbf{y}_i + \eta \\
&< -\gamma V + \eta, \tag{46}
\end{aligned}$$

where  $\gamma = \lambda_{\min}(\mathbf{Q})/\lambda_{\max}(\mathbf{P})$ .

Therefore, we are able to conclude that the closed-loop system is *practically stable* under the proposed control (Qu 1998). Note that the physical excitation control  $E_{fdi}$  is given by

$$E_{fdi} = \frac{w_i' + w_i'' - a_i'(\mathbf{x}_i)}{b_i'(\mathbf{x}_i)}, \tag{47}$$

which is decentralized and independent to network change.

#### 4. Simulation studies

The New England ten machine 39-bus system (Pai 1989) is used to carry out simulation studies to illustrate the effectiveness of the control. The transient response under IEEE type I excitors are also provided here to make a comparison. Several disturbances have been applied to the testing power system. Throughout the simulation studies, it is assumed that fault clearing time is five circles. A disturbance represented by line 26–25 is that a 3- $\phi$  ground fault occurs at bus 26, and is later cleared by the removal of line 26–25. Similarly, the meaning of disturbance 26 is self-explanatory.

The parameters of IEEE type I excitors are carefully tuned such that the system behaves almost optimally under disturbance 26–25 and 25. The proposed excitor output limits  $\varepsilon$  and  $\xi$  are listed in table 1. Excessively large values of bounds  $\xi_i$ , if chosen, may cause instability or performance degradation because they tend to excite un-modelled dynamics and to saturate the input channels. For most applications, 20–30% variation or uncertainty is typical, and robust control will do well. In case no information on bounds is available, an adaptive version of the proposed robust control can

**Table 1. Parameter ranges in the proposed controller.**

Machine number.	Excitor Output limits	$\varepsilon$	$\xi$
1	−2.0/2.0	0.01–0.1	4.0–9.0
2	−2.0/2.0	0.01–0.1	4.0–9.0
3	−2.0/2.0	0.01–0.1	4.0–9.0
4	−2.0/2.0	0.01–0.1	4.0–9.0
5	−4.0/4.0	0.01–0.1	4.0–9.0
6	−2.0/2.0	0.01–0.1	4.0–9.0
7	−2.0/2.0	0.01–0.1	4.0–9.0
8	−2.0/2.0	0.01–0.1	4.0–9.0
9	−2.2/2.2	0.01–0.1	4.0–9.0

be easily introduced and implemented. Optimal choice of matrix  $\mathbf{Q}_i$  is the identity matrix, the resulting matrix  $\mathbf{P}_i$  is obtained (by MATLAB) as

$$\mathbf{P}_i = \begin{bmatrix} 2.1345 & 1.7660 & 0.5000 \\ 1.7660 & 3.2116 & 0.9895 \\ 0.5000 & 0.9895 & 0.6960 \end{bmatrix}.$$

Optimal control theory is used to choose the gains in control  $w_i'$ ; thus we have  $k_{1i} = 1.0$ ,  $k_{2i} = 2.29$  and;  $k_{3i} = 2.14$ .

Test results will be summarized in sections 4.1–4.4. In all the simulations, machine 10 is not equipped with any control since it is an equivalent machine of a large power system, and this machine is used as the reference machine.

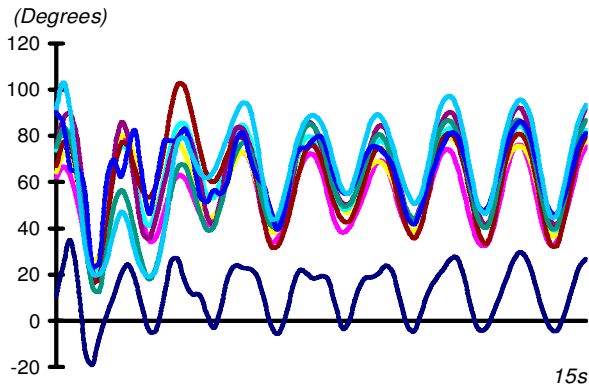
##### 4.1. Effects in stabilizing sustained oscillations

A sustained oscillation in the testing power system with IEEE type I excitors has been observed after the occurrence of disturbance 25 (figure 1(a)). The oscillation is diminished when the IEEE type I excitor is replaced with the new excitation control (figure 1(b)). Figure 1(c). shows the outcomes under different settings of control parameters ( $\varepsilon_i = 0.11$ ;  $\xi_i = 9.0$ ).

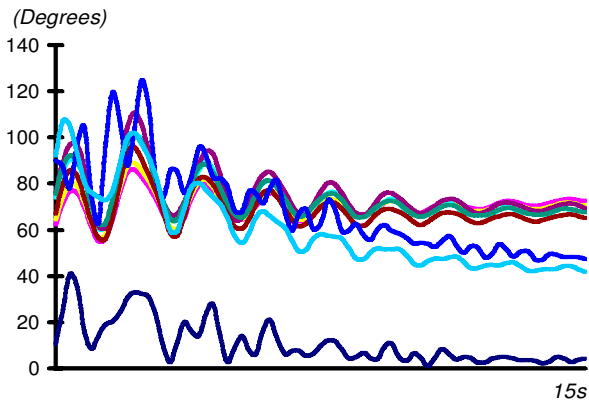
Similar results have been obtained when the following disturbances are applied to the system respectively: disturbances 1, 2, 9, 19, 22, 23, 24, 26, 27, 39. However, the new controller failed to stabilize disturbances 28 and 29 as the critical clearing times were past, although IEEE type I excitor failed as well.

##### 4.2. Impact on transient stability

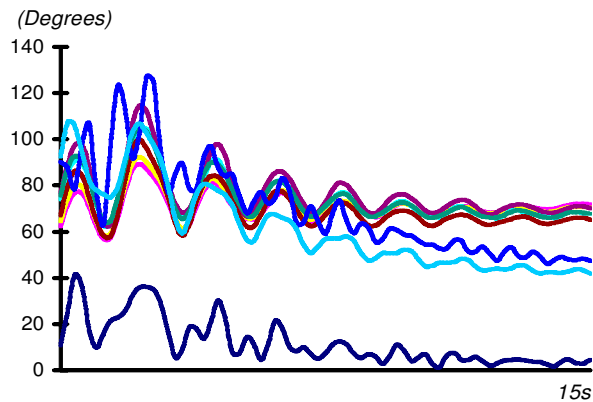
A transient instability in the test power system with IEEE type I excitors has been observed after the occurrence of disturbance 26–25 (figure 2(a)). The instability is eliminated by use of the proposed control (Figure 2(b)). Similar results were obtained under different control parameters ( $\varepsilon_i = 0.11$ ;  $\xi_i = 9.0$ ). These



(a)



(b)

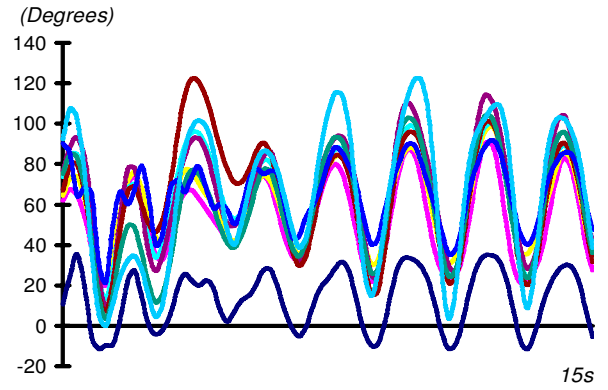


(c)

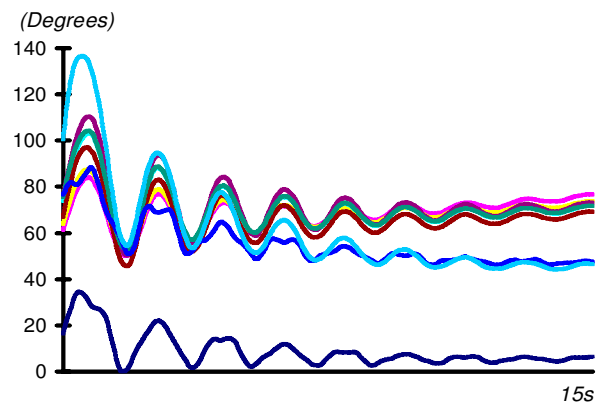
**Figure 1.** Machine relative angles caused by disturbance 25: (a) IEEE type I excitors are equipped at machines 1 to 9; (b) FBLC controllers are equipped at machines 1 to 9;  $\varepsilon_i = 0.01$  and  $\xi_i = 5.0$ ; (c) FBLC controllers are equipped at machines 1 to 9;  $\varepsilon_i = 0.11$  and  $\xi_i = 9.0$ .

results, however, are not included here because of space limitation.

Similar results have also been obtained under disturbances 1–2, 1–39, 24–16, 25–26.



(a)



(b)

**Figure 2.** Machine relative angles caused by disturbance 26–25: (a) IEEE type I excitors are equipped at machines 1 to 9; (b) new controllers are equipped at machines 1 to 9;  $\varepsilon_i = 0.01$  and  $\xi_i = 5.0$ .

#### 4.3. Voltage distribution and excitation output

The generator terminal voltage distribution and excitation output in a typical transient process (corresponding to figure 2(b)) are illustrated in figures 3 and 4 respectively. It can be seen in figure 3 that no severe over-voltage is observed in figure 3 (this is because the excitation outputs are carefully chosen). From figure 4, we see that the controls  $E_{fd}$  of all generators are at their upper limits. This can be explained by the intuition that a high excitation output raises the generator terminal voltage, weakens the effect of disturbance and thus contributes positively to stabilizing the disturbance in a power system.

#### 4.4. Effects of changed working conditions

Simulations of the dynamics caused by disturbance 26–25 have been done when the system is operated under the following different active power generation patterns:

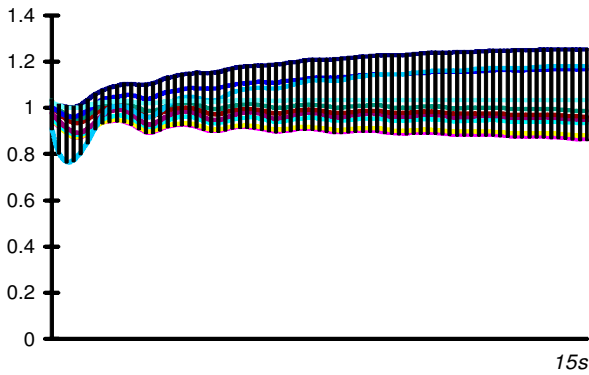


Figure 3. Terminal voltages of all generators after disturbance 26–25. New controllers are equipped at machines 1 to 9;  $\varepsilon_i = 0.01$  and  $\xi_i = 5.0$ .

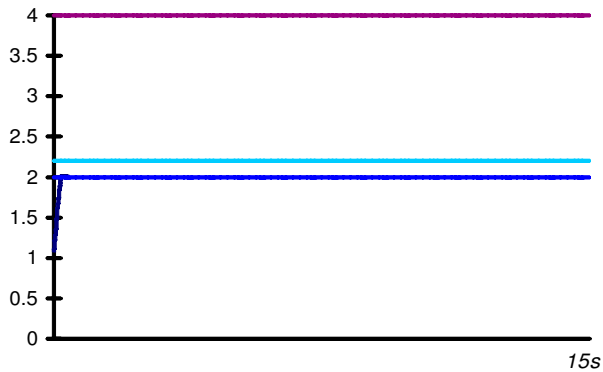


Figure 4. Outputs of all exciters after disturbance 26–25. New controllers are equipped at machines 1 to 9;  $\varepsilon_i = 0.01$  and  $\xi_i = 5.0$ .

machine 1, 2.5; machine 8: 5.4; others, unchanged;  
 machine 1, 2.5; machine 8: 5.8; others, unchanged;  
 machine 1, 2.5; machine 8: 6.4; others, unchanged;  
 machine 1, 2.5; machine 8: 7.0; others, unchanged;  
 machine 1, 3.5; machine 8: 7.0; others, unchanged;  
 machine 1, 4.5; machine 8: 7.0; others, unchanged.

In the above scenarios, neither sustained oscillation nor transient instability has been observed. Figure 5 shows the machine relative angles under the last pattern.

## 5. Conclusions and Discussions

A new feedback linearization scheme and its associated control design method have been described in this paper. The control design has several advantages compared with existing control design methodologies. The effects of the control have been simulated under a variety of disturbances and at different operating conditions of the system. A comparison has also been done between the proposed control and IEEE type I exciters. Since a nonlinear optimal control can only be obtained

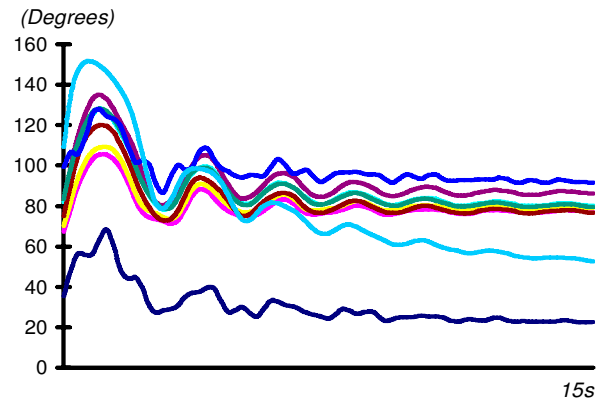


Figure 5. Machine relative angles caused by disturbance 26–25. New controllers are equipped at machines 1 to 9;  $\varepsilon_i = 0.01$  and  $\xi_i = 5.0$ . The system is under a more stressed condition.

through solving a two-point boundary value problem and under full knowledge of system dynamics, such a control is impractical for a power system. The proposed control, as a suboptimal control, can guarantee the local optimality of the linearized system and global stability. For a practical dynamic system, global stability is a necessary requirement, and local optimality represents an improved performance that is concerned most. From this point of view, FBLC-type controls should be quite favourable to power engineers. In summary, the following observations can be made from the study.

- (1) The proposed controller is capable of suppressing sustained oscillations as well as stabilizing transient instabilities caused by a variety of disturbances.
- (2) Improved stability robustness of power system under the proposed control can be obtained.
- (3) Increase in power transfer capability of the testing power system is possible upon the installation of the new control.
- (4) Loss of voltage can be alleviated if control parameters (in particular, the limits of  $E_{fd}$ ) are carefully chosen.

## Acknowledgments

The research reported in this paper was partly supported by Kentex Corporation, Redwood City, California, USA.

## Appendix A

In this appendix, formulae relevant to the main text are provided. The expression for  $v_i$  in (11) is obtained as follows:

$$\begin{aligned}
\ddot{\omega}_i &= -\frac{1}{M_i}(\dot{P}_{ei} + D_i\dot{\omega}_i) \\
&= -\frac{1}{M_i}\left(\sum_{j=1}^{NG}\frac{\partial P_{ei}}{\partial E'_{qj}}\dot{E}'_{qj} + \sum_{j=1}^{NG}\frac{\partial P_{ei}}{\partial \delta_j}\dot{\delta}_j + D_i\dot{\omega}_i\right) \\
&= -\frac{1}{M_i}\left\{\sum_{j=1}^{NG}\frac{\partial P_{ei}}{\partial E'_{qj}}\frac{1}{T'_{doj}}[E_{fdj} - E'_{qj} + (X'_{dj} - X_{dj})I_{dj}] \right. \\
&\quad \left. + \sum_{j=1}^{NG}\frac{\partial P_{ei}}{\partial \delta_j}\omega_j + D_i\dot{\omega}_i\right\} \\
&= -\frac{1}{M_i}\left\{\sum_{j=1}^{NG}\frac{\partial P_{ei}}{\partial E'_{qj}}\frac{1}{T'_{doj}}[-E'_{qj} + (X'_{dj} - X_{dj})I_{dj}] \right. \\
&\quad \left. + \sum_{j=1}^{NG}\frac{\partial P_{ei}}{\partial \delta_j}\omega_j + D_i\dot{\omega}_i\right\} - \frac{1}{M_i}\sum_{j=1}^{NG}\frac{\partial P_{ei}}{\partial E'_{qj}}\frac{1}{T'_{doj}}E_{fdj}.
\end{aligned}$$

The unknowns in the above expression are  $\partial P_{ei}/\partial E'_{qj}$  and  $\partial P_{ei}/\partial \delta_j$ . Letting  $\Delta X = X_{qi} - X'_{di}$ , we can obtain the expressions for  $\partial P_{ei}/\partial E'_{qj}$  and  $\partial P_{ei}/\partial \delta_j$  as

$$\begin{aligned}
\frac{\partial I_{di}}{\partial E'_{qj}} &= G_{ij}, & i = j, \\
\frac{\partial I_{di}}{\partial E'_{qj}} &= G_{ij} \cos \delta_{ij} - B_{ij} \sin \delta_{ij}, & i \neq j,
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial I_{qi}}{\partial E'_{qj}} &= B_{ij}, & i = j, \\
\frac{\partial I_{qi}}{\partial E'_{qj}} &= G_{ij} \sin \delta_{ij} + B_{ij} \cos \delta_{ij}, & i \neq j.
\end{aligned}$$

Similarly, we have,

$$\begin{aligned}
\frac{\partial I_{di}}{\partial \delta_j} &= \sum_{k=1}^{NG} E'_{qk}(-G_{ik} \sin \delta_{ik} - B_{ik} \cos \delta_{ik}) = -I_{qi}, & i = j, \\
\frac{\partial I_{di}}{\partial \delta_j} &= E'_{qj}(G_{ij} \sin \delta_{ij} + B_{ij} \cos \delta_{ij}), & i \neq j,
\end{aligned}$$

and,

$$\begin{aligned}
\frac{\partial I_{qi}}{\partial \delta_j} &= \sum_{k=1}^{NG} E'_{qk}(G_{ik} \cos \delta_{ik} - B_{ik} \sin \delta_{ik}) = I_{di}, & i = j \\
\frac{\partial I_{qi}}{\partial \delta_j} &= E'_{qj}(-G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij}). & i \neq j.
\end{aligned}$$

If  $i = j$ , we have

$$\begin{aligned}
\frac{\partial P_{ei}}{\partial E'_{qi}} &= I_{qi} + E'_{qi} \frac{\partial I_{qi}}{\partial E'_{qi}} + \Delta X I_{di} \frac{\partial I_{qi}}{\partial E'_{qi}} + \Delta X I_{qi} \frac{\partial I_{di}}{\partial E'_{qi}} \\
&= I_{qi} + (E'_{qi} + \Delta X I_{di})B_{ii} + \Delta X I_{qi}G_{ii}.
\end{aligned}$$

In the case when  $i \neq j$ , we have

$$\begin{aligned}
\frac{\partial P_{ei}}{\partial E'_{qj}} &= E'_{qi} \frac{\partial I_{qi}}{\partial E'_{qj}} + \Delta X I_{di} \frac{\partial I_{qi}}{\partial E'_{qj}} + \Delta X I_{qi} \frac{\partial I_{di}}{\partial E'_{qj}} \\
&= [(E'_{qi} + \Delta X I_{di})B_{ij} + \Delta X I_{qi}G_{ij}] \cos \delta_{ij} \\
&\quad + (E'_{qi} + \Delta X I_{di})G_{ij} - \Delta X I_{qi}B_{ij} \sin \delta_{ij}.
\end{aligned}$$

Finally, it follows that

$$\begin{aligned}
\frac{\partial P_{ei}}{\partial \delta_i} &= E'_{qi} \frac{\partial I_{qi}}{\partial \delta_i} + \Delta X I_{di} \frac{\partial I_{qi}}{\partial \delta_i} + \Delta X I_{qi} \frac{\partial I_{di}}{\partial \delta_i} \\
&= I_{qi} + (E'_{qi} + \Delta X I_{di})I_{di} - \Delta X I_{qi}^2
\end{aligned}$$

and that, if  $i \neq j$ ,

$$\begin{aligned}
\frac{\partial P_{ei}}{\partial \delta_j} &= E'_{qi} \frac{\partial I_{qi}}{\partial \delta_j} + \Delta X I_{di} \frac{\partial I_{qi}}{\partial \delta_j} + \Delta X I_{qi} \frac{\partial I_{di}}{\partial \delta_j} \\
&= [-(E'_{qi} + \Delta X I_{di})E'_{qj}G_{ij} + \Delta X I_{qi}E'_{qj}B_{ij}] \cos \delta_{ij} \\
&\quad + [(E'_{qi} + \Delta X I_{di})E'_{qj}B_{ij} + \Delta X I_{qi}E'_{qj}G_{ij}] \sin \delta_{ij}.
\end{aligned}$$

Now we can proceed with deriving a bounding function of uncertainty  $d_i$ . It follows that

$$\begin{aligned}
d_i &\leq |a_i''(\mathbf{x}) + b_i''E_{fdi}| \\
&= \left| -\frac{1}{M_i} \left[ \frac{\partial \Delta P_{ei}}{\partial E'_{qi}} \frac{1}{T'_{doi}} \dot{E}'_{qi} \right. \right. \\
&\quad \left. \left. + \sum_{j=1, j \neq i}^{NG} \left( \frac{\partial P_{ei}}{\partial E'_{qj}} \frac{1}{T'_{doj}} \dot{E}'_{qj} + \frac{\partial P_{ei}}{\partial \delta_j} \omega_j \right) \right] \right| \\
&\leq \xi_i,
\end{aligned}$$

in which  $|a_i''(\mathbf{x})|$  is assumed to be bounded as  $E_{fd}$  is bounded in practice. The specific value of  $\xi_i$  is not important; the stability analysis is independent of  $\xi_i$ . If the value is derived analytically, it may be too large to be used in control design. In our simulation studies,  $\xi_i$  in table 1 was chosen based on engineering judgment.

## References

- ALLEN, E. H., CHAPMAN, J. W., and ILIC, M. D., 1996, Effects of torsional dynamics on nonlinear generator control. *IEEE Transactions on Control Systems Technology*, **4**, 125–140.
- CAI, H., QU, Z., and DORSEY, J. F., 1996, Robust decentralized excitation control for large scale power systems. *Proceedings of the IFAC World Congress*, Vol. 0, San Francisco, California, USA, June 1996, 217–222.
- CAI, H., QU, Z., GAN, D., DORSEY, J. F., 1997, A robust power systems stabilizer design based on Lyapunov's approach. *Proceedings of the 1997 American Control Conference*, Albuquerque, New Mexico, USA, June 1997.
- CAO, Y., JIANG, L., CHENG, S., CHEN, D., MALIK, O. P., and HOPE, G. S., 1994, A nonlinear variable structure stabilizer for power system stability. *IEEE Transactions On Energy Conversion*, **9**, 489–495.



- CHAPMAN, J. W., ILIC, M. D., and KING, C. A., 1993, Stabilizing a multi-machine power system via decentralized feedback linearizing excitation Control. *IEEE Transactions on Power Systems*, **8**, 830–838.
- CHAPMAN, J. W., and ILIC, M. D., 1992, Some robustness results for feedback linearizing control of generator. *Proceedings of the 31st IEEE Conference on Decision and Control*, Tucson, Arizona, USA, 1992 (New York: IEEE), pp. 1123–1128.
- FALKNER, C. M., and HECK, B. S., 1995, A comparison of nonlinear controllers on a single generator–infinite bus power system. *Proceedings of the American Control Conference*, Seattle, Washington, USA, June 1995.
- GAO, L., CHEN, L., FAN, Y., and MA, H., 1992, A nonlinear control design for power systems. *Automatica*, **28**, 975–979.
- HILL, D. J., HISKENS, I. A., and WANG, Y., 1993, Robust, adaptive or nonlinear control for modern power systems, *Proceedings of the 32nd IEEE Conference on Decision and Control*, San Antonio, Texas, USA, December 1993 (New York: IEEE).
- JAIN, S., KHORRAMI, F., and FARDANESH, B., 1994, Adaptive nonlinear excitation control of power systems with unknown interconnections. *IEEE Transactions On Control Systems Technology*, **2**, 436–446.
- JIANG, H., CAI, H., DORSEY, J. F., and QU, Z., 1997, Towards a globally-robust decentralized control for large scale power systems. *IEEE Transactions on Control Systems Technology*, **5**, 309–319.
- JIANG, H., DORSEY, J. F., QU, Z., and HABETLER, T., 1993, Global robust adaptive control of a power system: primary control on the electrical side. *Proceedings of the IFAC World Congress*, Vol. 7, Sydney, Australia, July 1993, pp. 525–528.
- KAPRIELIAN, S., CLEMENTS, K., and TURI, J., 1990, Feedback stabilization for an ac/dc power system model. *Proceedings of the 29th IEEE Conference on Decision and Control*, Honolulu, Hawaii, USA, December 1990 (New York: IEEE) pp. 3367–3372.
- KING, C. A., CHAPMAN, J. W., and ILIC, M. D., 1994, Feedback linearizing excitation control on a full-state power system model. *IEEE Transactions on Power Systems*, **9**, 1102–1109.
- LU, Q., and SUN, Y., 1989, Nonlinear stabilizing control of multi-machine systems. *IEEE Transactions on Power Systems*, **4**, 236–241; 1993, *Nonlinear Control of Power Systems* (Beijing: Science Publishing Company).
- MIELCZARSKI, W., and ZAJACZKOWSKI, A. M., 1994, Nonlinear field voltage control of a synchronous generator using feedback linearization. *Automatica*, **30**, 1625–1630.
- NAMBU, M., and OHSAWA, Y., 1996, Development of an advanced power system stabilizer using a strict linearization approach. *IEEE Transactions on Power Systems*, **11**, 813–818.
- PAI, M. A., 1989, *Energy Function Analysis for Power System Stability* (Boston, Massachusetts: Kluwer).
- QU, Z., 1998, Robust control of nonlinear uncertain systems without generalized matching conditions. *IEEE Transactions on Automatic Control*, **40**, 1453–1460; 1998, *Robust Control of Nonlinear Uncertain Systems* (New York: Wiley–Interscience).
- QU, Z., DORSEY, J. F., BOND, J., and MCCALLEY, J. D., 1992, Application of robust control to sustained oscillations in power systems. *IEEE Transactions on Circuits and Systems*, **39**, 470–476.
- RAJKUMAR, V., and MOHLER, R. R., 1994, Nonlinear predictive control for the damping of multi-machine power system transients using FACTS devices. *Proceedings of the 33rd IEEE Conference on Decision and Control*, Lake Buena Vista, Florida, USA, December 1994 (New York: IEEE), pp. 4074–4079; 1995, Nonlinear control methods for power systems: a comparison. *IEEE Transactions on Control Systems Technology*, **3**.
- SAUER, P. W., and PAI, M. A., 1991, Modeling and simulation of multi-machine power system dynamics. *Control and Dynamic Systems*, Vol. 43, edited by C. T. Leondes, (New York: Academic Press).
- SUN, Y., and LU, Q., 1996, Decentralized excitation control by feedback linearization. *Proceedings of the IEEE 1996 Winter Meeting*, Baltimore, Maryland, USA, 1996 (New York: IEEE).
- WANG, Y., HILL, D. J., MIDDLETON, R. H., and GAO, L., 1993, Transient stability enhancement and voltage regulation of power systems. *IEEE Transactions on Power Systems*, **8**, 620–626.
- WANG, Y., HILL, D. J., MIDDLETON, R. H., and GAO, L., 1994 a, Transient stabilization of power systems with an adaptive control Law. *Automatic*, **30**, 1409–1413.
- WANG, Y., MITTELSTADT, W. A., and MARATUKULAM, D. J., 1994 b, Variable-structure braking-resistor control in a multi-machine power system. *IEEE Transactions On Power Systems*, **9**, 1557–1562.