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Lyapunov recursive design of robust adaptive tracking control with L_2-gain performance for electrically-driven robot manipulators

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This paper develops a new Lyapunov recursive design for the tracking control problem of rigid-link electrically-driven robot manipulators with uncertainty by taking a tracking performance into account. The tracking performance is evaluated by L_2-gain from a torque level disturbance signal to a penalty signal for the tracking error between outputs of the manipulator and desired trajectories. The novelty of our approach is in the strategy to construct such a Lyapunov function recursively that ensures not only stability of a tracking error system but also an L_2-gain constraint, which provides a closed-form solution for non-linear H_∞ control problem without using a Hamilton–Jacobi inequality. Two controllers, i.e. robust and robust adaptive control laws with L_2-gain performance, are designed such that the closed-loop error system is globally stable in the sense of uniform ultimate bounded stability with the L_2-gain less than any given small level. Experimental works are carried out for a two-link electrically-driven manipulator. Experimental results show an enhanced tracking performance of the proposed control scheme.

1. Introduction

The tracking control problem of rigid-link robot manipulators has attracted the attention of robot control engineers. Many different approaches to this problem have been proposed such as adaptive control schemes (e.g. Slotine *et al.* 1987, Ortega 1989), and robust control schemes (e.g. Slotine 1985, Spong *et al.* 1987). Recently, H_∞ control theory tends to be applied to this problem (e.g. Chen *et al.* 1994, Ishii *et al.* 1997). However, all these control schemes have been developed for second-order non-linear differential equations used to represent rigid-link robot manipulator dynamics, in which the torques or forces acting on arm joints are inputs to the system. In other words, the actuator dynamics are excluded from the robot model. However, as addressed in Goor (1982), the actuator dynamics play an important role in robot control problems, especially in the cases of high-velocity movements and highly varying loads. Several researchers also have pointed out that the detrimental effects of neglected actuator dynamics degrade a tracking performance of the robot. Tarn *et al.* (1991) demonstrated by experimental works that better performance could be achieved when the actuator dynamics were considered during the design of the robot controller. Therefore, it is believed that additional progress can be made by including the effects of actuator dynamics in the controller synthesis.

Recently, the tracking control problem of robotic systems including actuator dynamics has been studied. The inclusion of actuators into system dynamics complicates stability analysis and controller synthesis. The reason for this is that the dynamics of robotic systems including the actuators are described by third-order non-linear differential equations. In early literature (e.g. Guez 1983, Beekmann and Lee 1988), non-linear feedback linearization and decoupling was studied. However, it should be noted that the design procedure in these control schemes is based on full knowledge of the dynamics of robotic systems. If there are uncertainties in the system dynamics, use of a designed controller based on an inaccurate system model will degrade a tracking performance and may incur instability. To deal with the uncertainties in the dynamics of robotic systems, various control methods have been developed, including robust control schemes (e.g. Dawson *et al.* 1992), adaptive control schemes (e.g. Yuan 1995), and hybrid schemes (e.g. Su and Stepanenko 1995, 1997). Most recently, along the same lines, Burg *et al.* (1996) and Su *et al.* (1998) proposed a partial state-feedback controller to avoid the velocity measurements. However, it should be mentioned that the main attention of these control schemes is focused on only stability analysis and evaluation for the tracking performance is not considered in control design.

In this paper, we develop a new non-linear feedback control design to the tracking control of rigid-link electrically-driven robot manipulators with uncertainty that incorporates manipulator dynamics as well as actuator dynamics, taking a tracking performance into consideration. In modelling of the electrically-driven robot

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manipulators, different choices of link actuators provide vastly different electrical dynamics. In this study, we consider the effects of permanent magnet brushed dc motor dynamics in the overall robot dynamic model. In practice, most of the literature (e.g. Tarn *et al.* 1991, Dawson *et al.* 1992, Mahmoud 1993, Su and Stepanenko 1995, 1997, 1998) have adopted this electrical dynamic model.

On the other hand, although many researchers applied H_∞ control theory to the tracking control problem for the second-order rigid-link robot dynamic model (see, e.g. Fujita *et al.* 1992, Hashimoto *et al.* 1992, Chen *et al.* 1994), H_∞ control approach for robotic systems including actuator dynamics has not been studied enough at this point. Tomei (1999) proposed an excellent adaptive control algorithm which guaranteed arbitrary transient performance as well as arbitrary disturbance attenuation for second-order robot manipulators with unknown and time-varying parameters, which are subject to bounded disturbances. It takes account of time-varying parameter variations in the control synthesis. However, a modelling error such as unmodelled dynamics is not taken into account in the theoretical development. The modelling error is not easily dealt with in the approach pursued by Tomei (1999). Since a modelling error is inevitable in actual systems, a control scheme is needed that guarantees robustness in the presence of the modelling error. Besides, the validation of the control scheme is shown in only a simulation example applied to a simple single-link robot arm. In Ishii *et al.* (1997), we developed a robust model following a control scheme with L_2-gain performance for a second-order robot dynamic model, in which robot dynamics were linearized by feedback linearization and a reference model was introduced to give a desired trajectory. However, feedback linearization erased the Hamiltonian structure of the robot dynamic model, which failed to use several physical properties of the mathematical manipulator model in control design. Besides, introduction of the reference model restricted the kinds of trajectories that the manipulator must follow.

In this paper, we improve these drawbacks, and cast an L_2-gain synthesis to the robotic systems including actuators. We use no linearization and no reference model in control design, which allows us to exploit some useful physical properties of manipulators to facilitate controller synthesis. Furthermore, unknown torque level disturbance is introduced in robot dynamics, and disturbance attenuation is considered to accomplish high tracking performance. Then, the tracking performance is evaluated by L_2-gain from the disturbance signal to the penalty signal for the tracking error between the outputs of the manipulator and the desired trajectories. A voltage level state feedback control law is designed recursively based on a Lyapunov recursive design (Khalil 1996, Sepulchre *et al.* 1997) such that the tracking error system is globally stable in the sense of uniform ultimate bounded stability with the L_2-gain less than any given small level. The proposed controller does not require the joint acceleration feedback, which only requires measurement of link position, link velocity and electrical current. The contributions of this paper are described as follows.

First, the proposed control scheme enlarged the previous methods of Dawson *et al.* (1992), Yuan (1995), and Su and Stepanenko (1995, 1997) by incorporating the criterion of tracking performance given by L_2-gain constraint in controller synthesis. The novelty is in the strategy to construct such a Lyapunov function recursively that ensures not only stability of the tracking error system but also satisfies the dissipation inequality ensuring L_2-gain performance. However, it should be noted that our scheme is not merely a simple extention of the aforementioned existing approaches to the L_2-gain synthesis, since the control design problem that combines both stability and L_2-gain performance is not trivial. Besides, the proposed scheme was extended so as to deal with uncertain payloads without knowledge of parameters in manipulator dynamics.

Second, it is well known that a control design to achieve L_2-gain performance for a non-linear affine system, also called non-linear H_∞ control synthesis, often reduces to the problem of solving a Hamilton–Jacobi inequality. In general, the inequality is a matrix partial differential inequality and hence very difficult to solve. We proposed a recursive design approach to obtain a closed-form solution for the non-linear H_∞ control problem without using a Hamilton–Jacobi inequality.

Finally, the control scheme was applied to a two-link horizontal robot manipulator driven by dc motors. The effectiveness of the proposed control scheme was verified by experimental works. As a matter of fact, in the literature developed for the tracking control problem of robotic systems including actuator dynamics, their results are mainly supported by simulation works. Only a few papers (e.g. Tarn *et al.* 1991) present the experimental results on the actual robot.

The paper is organized as follows. In §2, robot dynamics including actuators is expressed in the form of two cascaded loops, and the objective of control design is stated. Then, a robust tracking control law with L_2-gain performance is derived in §3. In §4, the result in §3 is extended to a robust adaptive tracking control scheme in which the manipulator parameters are estimated by a parameter adaptation law. Experimental results are presented in §5. Finally, conclusions are given in §6. Although our scheme depends on full state-feedback, the measurements of velocity can be obtained without difficulty in experiments, and the experimental results show an enhanced tracking performance. A preliminary version of this paper appears in Ishii *et al.* (1999 a,b).

2. Problem statement

Consider an n-link manipulator with each joint driven by a dedicated, armature-controlled dc motor. In general, it is considered that there exist several kinds of uncertainties in actual robot systems described as follows.

(1) Parameter perturbation induced by load variation.
(2) Modelling error such as unmodelled dynamics of friction term and so on, which can be expressed as a function of joint angle and its derivative.
(3) Disturbance such as measurement noise.

Taking these uncertainties into account, the proper dynamics of the robotic system should be given by

$$M(\theta)\ddot{\theta} + C(\theta,\dot{\theta})\dot{\theta} + D(\dot{\theta}) + G(\theta) + \Delta f(\theta,\dot{\theta}) + \boldsymbol{w} = K_N \boldsymbol{I} \tag{1}$$

$$L\dot{\boldsymbol{I}} + R\boldsymbol{I} + K_e\dot{\theta} = \boldsymbol{v} \tag{2}$$

where $\theta \in \boldsymbol{R}^n$ is the vector of joint angle, $\boldsymbol{I} \in \boldsymbol{R}^n$ is the vector of armature currents and $\boldsymbol{v} \in \boldsymbol{R}^n$ is the vector of armature voltages in each joint actuator. $M(\theta) \in \boldsymbol{R}^{n\times n}$ is the inertia matrix, $C(\theta,\dot{\theta})\dot{\theta} \in \boldsymbol{R}^n$ represents the centripetal and Coriolis forces, $D(\dot{\theta}) \in \boldsymbol{R}^n$ represents the static and dynamic friction terms, $G(\theta) \in \boldsymbol{R}^n$ denotes the gravitational force, $\Delta f(\theta,\dot{\theta}) \in \boldsymbol{R}^n$ is an unknown non-linear function representing modelling error and parameter peturbation, and $\boldsymbol{w} \in \boldsymbol{R}^n$ denotes unknown torque level disturbance. $L \in \boldsymbol{R}^{n\times n}$ is the actuator inductance matrix, $R \in \boldsymbol{R}^{n\times n}$ is the actuator resistance matrix, $K_e \in \boldsymbol{R}^{n\times n}$ is the voltage constant matrix of motor back-electromotive forces, and $K_N \in \boldsymbol{R}^{n\times n}$ is the matrix that characterizes the electromechanical conversion from electrical current and mechanical torque. Matrices L, R, K_e and K_N are positive definite, constant and diagonal.

The statement of the control objective is based on the following two assumuptions.

Assumption 1: *Let $\theta_d \in \boldsymbol{R}^n$ denote an achievable desired trajectory for joint angles of the robot. While desired trajectory θ_d is normally generated through off-line trajectory planning, it is necessary for guaranteed asymptotic tracking that θ_d has the following properties:*

- *θ_d is smooth in the sense that its third-order time derivative is bounded by a constant for all time.*
- *If $\boldsymbol{w} = 0$ and if $\theta(t_0) = \theta_d(t_0)$, there exists an uncertainty-independent, smooth voltage control $\boldsymbol{v}$ (or simply a torque $\tau = K_N \boldsymbol{I}$) under which tracking of θ_d by θ is achieved asymptotically.*

The combination of the two properties in Assumption 1 ensures trackability of θ_d under a bounded control. To facilitate the introduction of the second assumption, let us define the so-called vector of tracking errors, that is

$$\bar{\boldsymbol{e}} = [\boldsymbol{e}^{\mathrm{T}} \quad \dot{\boldsymbol{e}}^{\mathrm{T}}]^{\mathrm{T}}$$

where $\boldsymbol{e} := \theta - \theta_d$.

Assumption 2: *The unknown non-linear function $\Delta f(\theta,\dot{\theta}) = \Delta f(\boldsymbol{e} + \theta_d,\ \dot{\boldsymbol{e}} + \dot{\theta}_d)$ and torque level disturbance $\boldsymbol{w}$ are bounded in norm as*

$$\|\Delta f(\theta,\dot{\theta})\| \le \rho_{\Delta f}(\boldsymbol{e},\dot{\boldsymbol{e}}) \tag{3}$$

$$\|\boldsymbol{w}\| \le \bar{w} \tag{4}$$

where $\rho_{\Delta f}(\boldsymbol{e},\dot{\boldsymbol{e}})$ and $\bar{w}$ are a non-negative, known function and a constant, respectively.

It will be shown in the next section that dynamics of a rigid-body robot are at most quadrate. Therefore, it can be assumed without loss of any generality that

$$\rho_{\Delta f}(\boldsymbol{e},\dot{\boldsymbol{e}}) = \zeta_2\|\bar{\boldsymbol{e}}\|^2 + \zeta_1\|\bar{\boldsymbol{e}}\| + \zeta_0 \tag{5}$$

where constants ζ_2, ζ_1 and ζ_0 can be easily found using information about the upper bounds on parameters and on the desired trajectory (see Lemma A.2 in Dawson *et al.* 1990).

The objective of control design is to achieve trajectory tracking in the presence of non-linear uncertainty $\Delta f(\theta,\dot{\theta})$ and disturbance $\boldsymbol{w}$, which is restated mathematically as follows.

Design problem: Under Assumptions 1 and 2, find a voltage level state feedback control law such that the closed-loop system satisfies the following design specifications.

($S1$) *Stability*: If $\boldsymbol{w} \ne 0$, the closed-loop error system is globally stable in the sense of uniform ultimate bounded stability. If $\boldsymbol{w} = 0$, the vector of tracking errors, $\bar{\boldsymbol{e}}(t)$, converges to zero asymptotically.

($S2$) *L_2-gain performance*: Under zero initial conditions ($\boldsymbol{e}(0) = 0$ and $\dot{\boldsymbol{e}}(0) = 0$), integral inequality $\|\boldsymbol{z}\|_T \le \gamma\|\boldsymbol{w}\|_T + \varepsilon_0$ holds for all $T \ge 0$ and for all $\boldsymbol{w} \in \boldsymbol{L}_2(0,T)$, where

$$\|\cdot\|_T = \left\{\int_0^T \|\cdot\|^2 \,\mathrm{d}t\right\}^{1/2}$$

$\boldsymbol{L}_2(0,T)$ is the space consisting of all functions whose $\|\cdot\|_T$ norm is finite, $\boldsymbol{z}$ is the vector defined by

$$z = W\bar{e} \quad \text{with} \quad W = \text{diag}\{q_1 I, q_2 I\}$$

to evaluate the tracking performance, q_1 and q_2 are positive weighting coefficients chosen to specify the level of penalty on tracking errors, γ is a positive constant specifying the attenuation level and ε_0 is a sufficiently small positive constant.

In what follows, we shall develop such a control for trajectory tracking in robots.

3. Derivation of the robust control law

Traditionally, the robotic control literature (see Slotine 1988) had emphasized the use of the manipulator's physical properties to facilitate the stability analysis. Therefore, we note the following useful properties of manipulator model.

Property 1. Inertia: *The inertia matrix $M(\boldsymbol{\theta})$ defined in* (1) *is positive definite symmetric and is uniformly bounded as a function of $\boldsymbol{\theta}$. That is, there exist positive constants $\lambda_{\min}$ and $\lambda_{\max}$ which satisfy*

$$\lambda_{\min} I \le M(\boldsymbol{\theta}) \le \lambda_{\max} I, \qquad \forall \boldsymbol{\theta} \tag{6}$$

Property 2. Skew symmetry: *For a proper definition of $C(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$, the matrix $\dot{M}(\boldsymbol{\theta}) - 2C(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$ is skew symmetric. That is, for any $\boldsymbol{x} \in \boldsymbol{R}^n$*

$$\boldsymbol{x}^{\mathrm{T}}\{\dot{M}(\boldsymbol{\theta}) - 2C(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})\}\boldsymbol{x} = 0 \tag{7}$$

holds.

The dynamic model (1) and (2) consists of two cascade loops. Unlike the dynamic model of manipulators, assuming the joint torque can be commanded directly, the torque term $K_N \boldsymbol{I}$ in (1) cannot be synthesized directly. Instead, it is an output of the actuator dynamics. In accordance with the backstepping control strategy described in Dawson *et al.* (1992) and Su and Stepanenko (1995), the design procedure is organized in the following two steps. First, the armature currents vector $\boldsymbol{I}$ is regarded as a control variable for subsystem (1) and an embedded control input $\boldsymbol{I}_d$ is designed so that the tracking objective may be achieved. Then, voltage level control input $\boldsymbol{v}$ is designed such that $\boldsymbol{I}$ tracks $\boldsymbol{I}_d$. To this end, we rewrite (1) as

$$M(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + C(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})\dot{\boldsymbol{\theta}} + D(\dot{\boldsymbol{\theta}}) + G(\boldsymbol{\theta}) + \Delta f(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) + \boldsymbol{w} = K_N \boldsymbol{I}_d + K_N \boldsymbol{\eta} \tag{8}$$

where $\boldsymbol{\eta} := \boldsymbol{I} - \boldsymbol{I}_d$ represent a current level perturbation to the rigid-link dynamics. The subsystem (8) can be viewed as a rigid model system with an input disturbance $K_N \boldsymbol{\eta}$, controlled by torque level input $K_N \boldsymbol{I}_d$. In this paper, (8) is called the manipulator loop and (2) is called the actuator loop.

Consider the following desired $\boldsymbol{I}_d$ given by

$$\boldsymbol{I}_d = K_N^{-1}\{M(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}}_d + C(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})\dot{\boldsymbol{\theta}}_d + D(\dot{\boldsymbol{\theta}}) + G(\boldsymbol{\theta}) + \boldsymbol{u}_1\} \tag{9}$$

where $\boldsymbol{u}_1$ is an auxiliary control input, which is determined later.

Now, consider the following change of coordinate (which is known as the filtered tracking error (see Slotine 1988))

$$\boldsymbol{y} = \dot{\boldsymbol{e}} + \alpha_1 \boldsymbol{e} \tag{10}$$

where $\alpha_1 > 0$ is a control gain that will appear in the controller to be designed.

Then, the tracking error dynamics of the manipulator loop are given by

$$\dot{\boldsymbol{e}} = \boldsymbol{y} - \alpha_1 \boldsymbol{e} \tag{11}$$

$$\begin{aligned} M(\boldsymbol{\theta})\dot{\boldsymbol{y}} = {} & (\alpha_1 M(\boldsymbol{\theta}) - C(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}))\dot{\boldsymbol{e}} - \Delta f(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) \\ & + K_N \boldsymbol{\eta} + \boldsymbol{u}_1 - \boldsymbol{w} \end{aligned} \tag{12}$$

A control law is derived by pursuing the following three steps in turn.

Step 1. **Filtered tracking error**: Define a positive definite function $V_0(\boldsymbol{e})$ by

$$V_0(\boldsymbol{e}) = \tfrac{1}{2}\boldsymbol{e}^{\mathrm{T}}\boldsymbol{e} \tag{13}$$

The time derivative of $V_0(\boldsymbol{e})$ along the trajectory of (11) is given by

$$\begin{aligned} \dot{V}_0(\boldsymbol{e}) &= \boldsymbol{e}^{\mathrm{T}}\dot{\boldsymbol{e}} \\ &= \boldsymbol{e}^{\mathrm{T}}(-\alpha_1 \boldsymbol{e} + \boldsymbol{y}) \\ &= -\alpha_1 \|\boldsymbol{e}\|^2 + \boldsymbol{e}^{\mathrm{T}}\boldsymbol{y} \end{aligned} \tag{14}$$

Step 2. **Error dynamics for the manipulator loop**: Let a positive definite function $V_1(\boldsymbol{e}, \boldsymbol{y}, \boldsymbol{\theta})$ be

$$V_1(\boldsymbol{e}, \boldsymbol{y}, \boldsymbol{\theta}) = \frac{\alpha_1 + q_1^2 + \alpha_1^2 q_2^2}{\alpha_1} V_0(\boldsymbol{e}) + \tfrac{1}{2}\boldsymbol{y}^{\mathrm{T}} M(\boldsymbol{\theta})\boldsymbol{y} \tag{15}$$

Note that the positive definiteness of the function $V_1(\boldsymbol{e}, \boldsymbol{y}, \boldsymbol{\theta})$ is ensured from Property 1.

Using (14) the time derivative of $V_1(\boldsymbol{e}, \boldsymbol{y}, \boldsymbol{\theta})$ along the trajectory of (11) and (12) is given by

$$\begin{aligned} \dot{V}_1(\boldsymbol{e}, \boldsymbol{y}, \boldsymbol{\theta}) = {} & \frac{\alpha_1 + q_1^2 + \alpha_1^2 q_2^2}{\alpha_1} \dot{V}_0(\boldsymbol{e}) + \boldsymbol{y}^{\mathrm{T}} M(\boldsymbol{\theta})\dot{\boldsymbol{y}} \\ & + \tfrac{1}{2}\boldsymbol{y}^{\mathrm{T}}\dot{M}(\boldsymbol{\theta})\boldsymbol{y} \\ = {} & -(\alpha_1 + q_1^2 + \alpha_1^2 q_2^2)\|\boldsymbol{e}\|^2 + \frac{\alpha_1 + q_1^2 + \alpha_1^2 q_2^2}{\alpha_1}\boldsymbol{e}^{\mathrm{T}}\boldsymbol{y} \\ & + \boldsymbol{y}^{\mathrm{T}}\{(\alpha_1 M(\boldsymbol{\theta}) - C(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}))\dot{\boldsymbol{e}} - \Delta f(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) \\ & + K_N \boldsymbol{\eta} + \boldsymbol{u}_1 - \boldsymbol{w}\} + \tfrac{1}{2}\boldsymbol{y}^{\mathrm{T}}\dot{M}(\boldsymbol{\theta})\boldsymbol{y} \end{aligned}$$

Using Property 2

$$\dot{V}_1(\boldsymbol{e},\boldsymbol{y},\boldsymbol{\theta}) = -(\alpha_1 + q_1^2 + \alpha_1^2 q_2^2)\|\boldsymbol{e}\|^2 + \boldsymbol{y}^{\mathrm{T}} K_N \boldsymbol{\eta} + \boldsymbol{y}^{\mathrm{T}}\Big\{\alpha_1 M(\boldsymbol{\theta})\dot{\boldsymbol{e}} + \alpha_1 C(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})\boldsymbol{e} - \Delta f(\boldsymbol{\theta},\dot{\boldsymbol{\theta}}) + \frac{\alpha_1 + q_1^2 + \alpha_1^2 q_2^2}{\alpha^1}\boldsymbol{e} + \boldsymbol{u}_1 - \boldsymbol{w}\Big\} \tag{16}$$

Now, we add and subtract the terms $\gamma^2\|\boldsymbol{w}\|^2$ and $\|\boldsymbol{z}\|^2$ to the right-hand side of (16), and complete the square. Then

$$\begin{aligned}\dot{V}_1(\boldsymbol{e},\boldsymbol{y},\boldsymbol{\theta}) &= -(\alpha_1 + q_1^2 + \alpha_1^2 q_2^2)\|\boldsymbol{e}\|^2 - \boldsymbol{y}^{\mathrm{T}}\Delta f(\boldsymbol{\theta},\dot{\boldsymbol{\theta}}) \\ &\quad + \boldsymbol{y}^{\mathrm{T}}\Big\{\alpha_1 M(\boldsymbol{\theta})\dot{\boldsymbol{e}} + \alpha_1 C(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})\boldsymbol{e} + \frac{\alpha_1 + q_1^2 + \alpha_1^2 q_2^2}{\alpha_1}\boldsymbol{e} + \boldsymbol{u}_1\Big\} \\ &\quad - \left\|\frac{1}{2\gamma}\boldsymbol{y} + \gamma\boldsymbol{w}\right\|^2 + \gamma^2\|\boldsymbol{w}\|^2 + \frac{1}{4\gamma^2}\boldsymbol{y}^{\mathrm{T}}\boldsymbol{y} - \|\boldsymbol{z}\|^2 \\ &\quad + q_1^2\|\boldsymbol{e}\|^2 + q_2^2\|\dot{\boldsymbol{e}}\|^2 + \boldsymbol{y}^{\mathrm{T}} K_N \boldsymbol{\eta} \\ &\leq -\alpha_1\|\boldsymbol{e}\|^2 + \gamma^2\|\boldsymbol{w}\|^2 - \|\boldsymbol{z}\|^2 + \boldsymbol{y}^{\mathrm{T}} K_N \boldsymbol{\eta} \\ &\quad + \|\boldsymbol{y}\|\cdot\|\Delta f(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})\| \\ &\quad + \boldsymbol{y}^{\mathrm{T}}\Big\{\alpha_1 M(\boldsymbol{\theta})\dot{\boldsymbol{e}} + \alpha_1 C(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})\boldsymbol{e} + \frac{\alpha_1 + q_1^2 + \alpha_1^2 q_2^2}{\alpha_1}\boldsymbol{e} \\ &\quad + \left(\frac{1}{4\gamma^2} + q_2^2\right)\boldsymbol{y} - 2\alpha_1 q_2^2\boldsymbol{e} + \boldsymbol{u}_1\Big\}\end{aligned} \tag{17}$$

Determine $\boldsymbol{u}_1$ as follows so as to make the last term in (17) negative

$$\begin{aligned}\boldsymbol{u}_1 &= -\alpha_1 M(\boldsymbol{\theta})\dot{\boldsymbol{e}} - \alpha_1 C(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})\boldsymbol{e} - \frac{\alpha_1 + q_1^2 + \alpha_1^2 q_2^2}{\alpha_1}\boldsymbol{e} \\ &\quad - \left(\frac{1}{4\gamma^2} + q_2^2\right)\boldsymbol{y} + 2\alpha_1 q_2^2\boldsymbol{e} - \alpha_2 M(\boldsymbol{\theta})\boldsymbol{y} \\ &\quad - \frac{\boldsymbol{y}\rho_{\Delta f}^2}{\|\boldsymbol{y}\|\rho_{\Delta f} + \varepsilon_1 \mathrm{e}^{-\beta_1 t}}\end{aligned} \tag{18}$$

where α_2 is an arbitrary positive constant chosen by the designer which plays an important role in stability analysis, ε_1 is a sufficiently small positive scalar control gain, and β_1 is an adequate positive constant. Thus, we have

$$\begin{aligned}\dot{V}_1(\boldsymbol{e},\boldsymbol{y},\boldsymbol{\theta}) &\leq -\alpha_1\|\boldsymbol{e}\|^2 - \alpha_2\|\boldsymbol{y}\|^2_{M(\boldsymbol{\theta})} + \gamma^2\|\boldsymbol{w}\|^2 - \|\boldsymbol{z}\|^2 \\ &\quad + \boldsymbol{y}^{\mathrm{T}} K_N \boldsymbol{\eta} + \|\boldsymbol{y}\|\rho_{\Delta f} - \frac{\|\boldsymbol{y}\|^2\rho_{\Delta f}^2}{\|\boldsymbol{y}\|\rho_{\Delta f} + \varepsilon_1 \mathrm{e}^{-\beta_1 t}} \\ &\leq -\alpha_1\|\boldsymbol{e}\|^2 - \alpha_2\|\boldsymbol{y}\|^2_{M(\boldsymbol{\theta})} + \gamma^2\|\boldsymbol{w}\|^2 - \|\boldsymbol{z}\|^2 \\ &\quad + \boldsymbol{y}^{\mathrm{T}} K_N \boldsymbol{\eta} + \varepsilon_1 \mathrm{e}^{-\beta_1 t}\end{aligned} \tag{19}$$

where $\|\boldsymbol{y}\|^2_{M(\boldsymbol{\theta})} = \boldsymbol{y}_{\mathrm{T}} M(\boldsymbol{\theta})\boldsymbol{y}$.

Step 3. **Error dynamics for the actuator loop:** We now design a control law at the voltage input $\boldsymbol{v}$ which forces $\boldsymbol{\eta}$ to zero. However, the Lyapunov recursive design requires us to calculate

$$\dot{\boldsymbol{I}}_d = K_N^{-1}\{\dot{M}(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}}_d + M(\boldsymbol{\theta})\boldsymbol{\theta}_d^{(3)} + \dot{C}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})\dot{\boldsymbol{\theta}}_d + C(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})\ddot{\boldsymbol{\theta}}_d + \dot{D}(\dot{\boldsymbol{\theta}}) + \dot{G}(\boldsymbol{\theta}) + \dot{\boldsymbol{u}}_1\} \tag{20}$$

Clearly, $\dot{\boldsymbol{I}}_d$ requires the measurement of $\ddot{\boldsymbol{\theta}}$. In order to avoid the intensive computation of $\dot{\boldsymbol{I}}_d$, we consider using a bound of $\dot{\boldsymbol{I}}_d$, rather than $\dot{\boldsymbol{I}}_d$ itself. It has been shown that $\|\dot{\boldsymbol{I}}_d\|$ can be written in terms of combinations of constants and functions of $\boldsymbol{\theta}$, $\dot{\boldsymbol{\theta}}$, $\boldsymbol{I}$ (see, e.g. Dawson *et al.* 1992). This implies that it is possible to generate a bounding function $\rho_{I_d}(\boldsymbol{e},\dot{\boldsymbol{e}},\boldsymbol{\eta})$ such that

$$\|\dot{\boldsymbol{I}}_d\| \leq \rho_{I_d}(\boldsymbol{e},\dot{\boldsymbol{e}},\boldsymbol{\eta}) \tag{21}$$

As shown in (5), the following form of $\rho_{I_d}(\boldsymbol{e},\dot{\boldsymbol{e}},\boldsymbol{\eta})$ is assumed

$$\rho_{I_d}(\boldsymbol{e},\dot{\boldsymbol{e}},\boldsymbol{\eta}) = \xi_4\|\bar{\boldsymbol{e}}\|^3 + \xi_3\|\bar{\boldsymbol{e}}\|^2 + \xi_2\|\bar{e}\| + \xi_1\|\boldsymbol{\eta}\| + \xi_0 \tag{22}$$

where constants ξ_4, ξ_3, ξ_2, ξ_1, and ξ_0 can be found using information about the upper bounds on parameters and on the desired trajectory (see also Lemma A.2 in Dawson *et al.* 1990).

Hence, we simply apply the following voltage control input $\boldsymbol{v}$ given by

$$\boldsymbol{v} = R\boldsymbol{I}_d + K_e\dot{\boldsymbol{\theta}}_d + \boldsymbol{u}_2 \tag{23}$$

where $\boldsymbol{u}_2$ is an auxiliary control input, which is determined later. Then, the error dynamics of the actuator loop is given by

$$L\dot{\boldsymbol{I}} + R\eta + K_e\dot{\boldsymbol{e}} = \boldsymbol{u}_2 \tag{24}$$

We subtract a 'fictitious' term $L\dot{\boldsymbol{I}}_d$ from both sides of (24). Then, the error dynamics (24) is rewritten as

$$L\dot{\eta} = -R\eta - K_e\dot{\boldsymbol{e}} - L\dot{\boldsymbol{I}}_d + \boldsymbol{u}_2 \tag{25}$$

Let a Lyapunov function candidate be

$$V_2(\boldsymbol{e},\boldsymbol{y},\boldsymbol{\theta},\eta) = V_1(\boldsymbol{e},\boldsymbol{y},\boldsymbol{\theta}) + \tfrac{1}{2}\boldsymbol{\eta}^{\mathrm{T}} L\boldsymbol{\eta} \tag{26}$$

Using (19) the time derivative of $V_2(\boldsymbol{e},\boldsymbol{y},\boldsymbol{\theta},\boldsymbol{\eta})$ along the trajectory of (11), (12) and (25) is given by

$$\begin{aligned}\dot{V}_2(\boldsymbol{e},\boldsymbol{y},\boldsymbol{\theta},\boldsymbol{\eta}) &= \dot{V}_1(\boldsymbol{e},\boldsymbol{y},\boldsymbol{\theta}) + \boldsymbol{\eta}^{\mathrm{T}} L\dot{\boldsymbol{\eta}} \\ &\le -\alpha_1\|\boldsymbol{e}\|^2 - \alpha_2\|\boldsymbol{y}\|^2_{M(\boldsymbol{\theta})} + \gamma^2\|\boldsymbol{w}\|^2 - \|\boldsymbol{z}\|^2 \\ &\quad + \boldsymbol{y}^{\mathrm{T}} K_N\boldsymbol{\eta} + \varepsilon_1 \mathrm{e}^{-\beta_1 t} \\ &\quad + \boldsymbol{\eta}^{\mathrm{T}}\{-R\boldsymbol{\eta} - K_e\dot{\boldsymbol{e}} - L\dot{\boldsymbol{I}}_d + \boldsymbol{u}_2\} \\ &\le -\alpha_1\|\boldsymbol{e}\|^2 - \alpha_2\|\boldsymbol{y}\|^2_{M(\boldsymbol{\theta})} + \gamma^2\|\boldsymbol{w}\|^2 - \|\boldsymbol{z}\|^2 \\ &\quad + \varepsilon_1 \mathrm{e}^{-\beta_1 t} + \|\boldsymbol{\eta}^{\mathrm{T}} L\|\cdot\|\dot{\boldsymbol{I}}_d\| \\ &\quad + \boldsymbol{\eta}^{\mathrm{T}}\{-R\boldsymbol{\eta} - K_e\dot{\boldsymbol{e}} + K_N^{\mathrm{T}}\boldsymbol{y} + \boldsymbol{u}_2\} \end{aligned} \tag{27}$$

Determine $\boldsymbol{u}_2$ as follows so as to make the last term in (27) negative

$$\boldsymbol{u}_2 = R\boldsymbol{\eta} + K_e\dot{\boldsymbol{e}} - K_N^{\mathrm{T}}\boldsymbol{y} - \alpha_3 L\boldsymbol{\eta} - \frac{LL^{\mathrm{T}}\boldsymbol{\eta}\rho^2_{I_d}}{\|\boldsymbol{\eta}^{\mathrm{T}} L\|\rho_{I_d} + \varepsilon_2 \mathrm{e}^{-\beta_2 t}} \tag{28}$$

where α_3 is an arbitrary positive constant chosen by the designer which plays an important role in stability analysis, ε_2 is a sufficiently small positive scalar control gain and β_2 is an adequate positive constant. Thus, we have

$$\begin{aligned}\dot{V}_2(\boldsymbol{e},\boldsymbol{y},\boldsymbol{\theta},\boldsymbol{\eta}) &\le -\alpha_1\|\boldsymbol{e}\|^2 - \alpha_2\|\boldsymbol{y}\|^2_{M(\boldsymbol{\theta})} + \gamma^2\|\boldsymbol{w}\|^2 - \|\boldsymbol{z}\|^2 \\ &\quad + \varepsilon_1 \mathrm{e}^{-\beta_1 t} - \alpha_3\|\boldsymbol{\eta}\|^2_L + \|\boldsymbol{\eta}^{\mathrm{T}} L\|\rho_{I_d} \\ &\quad - \frac{\|\boldsymbol{\eta}^{\mathrm{T}} L\|^2\rho^2_{I_d}}{\|\boldsymbol{\eta}^{\mathrm{T}} L\|\rho_{I_d} + \varepsilon_2 \mathrm{e}^{-\beta_2 t}} \\ &\le -\alpha_1\|\boldsymbol{e}\|^2 - \alpha_2\|\boldsymbol{y}\|^2_{M(\boldsymbol{\theta})} - \alpha_3\|\boldsymbol{\eta}\|^2_L \\ &\quad + \gamma^2\|\boldsymbol{w}\|^2 - \|\boldsymbol{z}\|^2 + \varepsilon_1 \mathrm{e}^{-\beta_1 t} + \varepsilon_2 \mathrm{e}^{-\beta_2 t} \end{aligned} \tag{29}$$

where $\|\boldsymbol{\eta}\|^2_L = \boldsymbol{\eta}^{\mathrm{T}} L\boldsymbol{\eta}$.

Then, (29) is rewritten as

$$\begin{aligned}\dot{V}_2(\boldsymbol{e},\boldsymbol{y},\boldsymbol{\theta},\boldsymbol{\eta}) &\le -2\alpha V_2(\boldsymbol{e},\boldsymbol{y},\boldsymbol{\theta},\boldsymbol{\eta}) \\ &\quad + \gamma^2\|\boldsymbol{w}\|^2 - \|\boldsymbol{z}\|^2 + 2\varepsilon \mathrm{e}^{-\beta t}\end{aligned} \tag{30}$$

where

$$\alpha = \min\left\{\frac{\alpha_1^2}{\alpha_1 + q_1^2 + \alpha_1^2 q_2^2}, \alpha_2, \alpha_3\right\}$$

$$\varepsilon = \max\{\varepsilon_1, \varepsilon_2\}$$

$$\beta = \min\{\beta_1, \beta_2\}$$

Now, we show that specifications ($S1$) and ($S2$) hold. When $\boldsymbol{w} \neq 0$, from the boundedness of $\boldsymbol{w}$ in Assumption 2, along the same line in Dawson *et al.* (1992) we can conclude that the closed-loop error system is globally stable in the sense of uniform ultimate bounded stability. Moreover, take $\boldsymbol{w} = 0$, from (30) we have

$$\dot{V}_2(\boldsymbol{e},\boldsymbol{y},\boldsymbol{\theta},\boldsymbol{\eta}) \le -2\alpha V_2(\boldsymbol{e},\boldsymbol{y},\boldsymbol{\theta},\boldsymbol{\eta}) + 2\varepsilon \mathrm{e}^{-\beta t}, \qquad \forall t \ge 0 \tag{31}$$

Equation (31) implies that $V_2(\boldsymbol{e},\boldsymbol{y},\boldsymbol{\theta},\boldsymbol{\eta})$ tends to zero as t tends to infinity. This means that the tracking errors $\boldsymbol{e}$, $\dot{\boldsymbol{e}}$ and $\boldsymbol{\eta}$ tend to zero as t tends to infinity when $\boldsymbol{w} = 0$.

Next, we show the L_2-gain condition. For any $T \ge 0$, integrating (30) from 0 to T with zero initial conditions ($\boldsymbol{e}(0) = 0$, $\dot{\boldsymbol{e}}(0) = 0$ and $\boldsymbol{\eta}(0) = 0$) yields

$$\int_0^{\mathrm{T}} \|\boldsymbol{z}\|^2\,\mathrm{d}\tau \le \gamma^2\int_0^{\mathrm{T}} \|\boldsymbol{w}\|^2\,\mathrm{d}\tau + 2\varepsilon\int_0^{\mathrm{T}} \mathrm{e}^{-\beta t}\mathrm{d}\tau$$
$$\forall \boldsymbol{w} \in \boldsymbol{L}_2(0, T) \tag{32}$$

This implies $\|\boldsymbol{z}\|_T \le \gamma\|\boldsymbol{w}\|_T + \varepsilon_0$, $\forall \boldsymbol{w} \in \boldsymbol{L}_2(0, T)$. It should be noted that the time-independent decaying exponential terms $\mathrm{e}^{-\beta_1 t}$ and $\mathrm{e}^{-\beta_2 t}$, which appear in the denominator of the control inputs (18) and (28) respectively, allow a constant ε_0 being sufficiently small.

From the above observations, we have the following result.

Theorem 1: *The voltage level robust tracking control law that accomplishes control objectives* ($S1$) *and* ($S2$) *is given by*

$$\begin{aligned}\boldsymbol{v} &= RK_N^{-1}\{M(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}}_d + C(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})\dot{\boldsymbol{\theta}}_d + D(\dot{\boldsymbol{\theta}}) + G(\boldsymbol{\theta}) + \boldsymbol{u}_1\} \\ &\quad + K_e\dot{\boldsymbol{\theta}}_d + \boldsymbol{u}_2\end{aligned} \tag{33}$$

where control inputs $\boldsymbol{u}_1$ *and* $\boldsymbol{u}_2$ *are given by* (18) *and* (28) *respectively.*

Remark 1: In this paper, a recursive design approach is proposed to obtain a closed-form solution for non-linear H_∞ control problem without using a Hamilton–Jacobi inequality. This is possible for two reasons. First, the Lyapunov direct method can be used to generate and manipulate Hamilton–Jacobi inequality (Qu 1998). Second, a simpler L_2-gain performance is pursued in this paper. In general, an L_2-gain performance is characterized by inequality $\|\boldsymbol{z}\|_T \le \gamma\|\boldsymbol{w}\|_T + \varepsilon_0$, where $\boldsymbol{z} = h(\boldsymbol{x}, \boldsymbol{v})$ is a so-called penalty function, $\boldsymbol{x}$ is a state, and $\boldsymbol{v}$ is a control input. The L_2 performance in this paper is defined by the choice of $\boldsymbol{z} = W\bar{\boldsymbol{e}}$. If function $h(\cdot)$ depends on control input $\boldsymbol{v}$, a closed-form controller can only be obtained by numerically solving the corresponding Hamilton–Jacobi inequality. In this sense, removing $\boldsymbol{v}$ from penalty function $h(\cdot)$ is a good choice in order to design our controller in closed-form. This choice of $\boldsymbol{z}$, independent of control input $\boldsymbol{v}$, was motivated from almost the disturbance decoupling problem (e.g. Isidori 1996, Mario *et al.* 1994).

Remark 2: In this formulation of the penalty function $\boldsymbol{z}$, theoretically we can select infinitely smaller γ. However, such a small choice of γ allows excessive large control input. Since saturation for actuators is inevitable in practical problems, trade-off is required between choice of smaller γ and practical tracking performance. The proper value of γ will only be found by trial-and-error through the experiments.

4. Derivation of the robust adaptive control law

In this section, the result in the previous section is extended to a robust adaptive control scheme so as to estimate the parameters in the manipulator dynamics. It should be noted that the third property of the manipulator model is as follows.

Property 3. Linearity: *The matrices* $M(\boldsymbol{\theta})$, $C(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})$, $D(\dot{\boldsymbol{\theta}})$ *and* $G(\boldsymbol{\theta})$ *can be expressed as*

$$M(\boldsymbol{\theta})\boldsymbol{\phi} + C(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})\psi + D(\dot{\boldsymbol{\theta}}) + G(\boldsymbol{\theta}) = \Phi(\boldsymbol{\theta},\dot{\boldsymbol{\theta}},\boldsymbol{\phi},\psi)\boldsymbol{a} \tag{34}$$

where $\Phi(\boldsymbol{\theta},\dot{\boldsymbol{\theta}},\boldsymbol{\phi},\psi) \in \boldsymbol{R}^{n\times m}$ *is the regressor matrix independent of the unknown parameters and* $\boldsymbol{a} \in \boldsymbol{R}^m$ *is the vector of unknown parameters.*

To derive a control law, we add more two assumptions.

Assumption 3: *The upper bound of* $M(\boldsymbol{\theta})$ *is known, i.e.* $\lambda_{\max}$ *satisfying* (6) *is known.*

Assumption 3 requires the knowledge of the upper bounds of the inertia matrix. However, this is not a strict assumption, since it can be found easily by the information of the upper bounds on parameters.

Assumption 4: *The unknown parameters are only in the manipulator dynamics, parameters in actuator dynamics are known.*

Remark 3: Su and Stepanenko (1995, 1997, 1998) considered not only an uncertainty of parameters in manipulator dynamics but also an uncertainty of parameters in actuator dynamics in control design. It is also possible to discuss the uncertainty of actuator parameters in this paper by establishing the bounds of parameter fluctuation as in the same way given in Su and Stepanenko (1995). However, we omit to include the parameter fluctuation in the actuator dynamics, since in comparison with manipulator parameters, actuator parameters are not liable to variation.

The control objective is the same as that in § 2. Hence, the design problem is stated as follows: Under Assumptions 1–4, find a parameter update law and a voltage level state feedback control law such that the closed-loop system satisfies the design specifications ($S1$) and ($S2$). The procedure for derivation of the control law is also almost in the same lines as § 3. Therefore, we just outline the procedure. We rewrite the manipulator dynamics (1) as (8), and define the change of coordinate (10). Besides, we introduce the variable

$$\dot{\boldsymbol{\theta}}_r = \dot{\boldsymbol{\theta}}_d - \alpha_1 \boldsymbol{e} \tag{35}$$

Then, (10) is rewritten as

$$\boldsymbol{y} = \dot{\boldsymbol{\theta}} - \dot{\boldsymbol{\theta}}_r \tag{36}$$

Now consider the following desired $\boldsymbol{I}_d$ based on the adaptive tracking control law given in Slotine *et al.* (1987)

$$\boldsymbol{I}_d = K_N^{-1}\{\Phi(\boldsymbol{\theta},\dot{\boldsymbol{\theta}},\ddot{\boldsymbol{\theta}}_r,\dot{\boldsymbol{\theta}}_r)\hat{\boldsymbol{a}} + \boldsymbol{u}_1\} \tag{37}$$

where $\hat{\boldsymbol{a}}$ represents the estimation of the unknown parameter vector $\boldsymbol{a}$.

Then, the tracking error dynamics of the manipulator loop are given by

$$\dot{\boldsymbol{e}} = \boldsymbol{y} - \alpha_1 \boldsymbol{e} \tag{38}$$

$$\begin{aligned} M(\boldsymbol{\theta})\dot{\boldsymbol{y}} = &-C(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})\boldsymbol{y} + \Phi(\boldsymbol{\theta},\dot{\boldsymbol{\theta}},\ddot{\boldsymbol{\theta}}_r,\dot{\boldsymbol{\theta}}_r)\tilde{\boldsymbol{a}} - \Delta f(\boldsymbol{\theta},\dot{\boldsymbol{\theta}}) \\ &+ K_N\boldsymbol{\eta} + \boldsymbol{u}_1 - \boldsymbol{w} \end{aligned} \tag{39}$$

where $\tilde{\boldsymbol{a}} = \hat{\boldsymbol{a}} - \boldsymbol{a}$ denotes the parameter estimation error and (39) is derived as follows. Substitute (37) to (8), and subtract the term $\Phi(\boldsymbol{\theta},\dot{\boldsymbol{\theta}},\ddot{\boldsymbol{\theta}}_r,\dot{\boldsymbol{\theta}}_r)\boldsymbol{a}$ from both sides, noting Property 3, we have

$$\begin{aligned} M(\boldsymbol{\theta})(\ddot{\boldsymbol{\theta}} - \ddot{\boldsymbol{\theta}}_r) + C(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})(\dot{\boldsymbol{\theta}} - \dot{\boldsymbol{\theta}}_r) + \Delta f(\boldsymbol{\theta},\dot{\boldsymbol{\theta}}) + \boldsymbol{w} \\ = \Phi(\boldsymbol{\theta},\dot{\boldsymbol{\theta}},\ddot{\boldsymbol{\theta}}_r,\dot{\boldsymbol{\theta}}_r)\tilde{\boldsymbol{a}} + \boldsymbol{u}_1 + K_N\boldsymbol{\eta} \end{aligned} \tag{40}$$

Thus, from (36), (39) is obtained.

However, in the case of robust adaptive controller design, $\boldsymbol{I}_d$ includes the estimated parameter vector $\hat{\boldsymbol{a}}$ as in (37). Therefore, the existence of such a bounding function of the form (22) that satisfies (21) is not guaranteed if boundedness of the estimated parameters is not assured. To guarantee the boundedness of $\hat{\boldsymbol{a}}$, we adopt a projection operator given in Su *et al.* (1998) in parameter update law.

Consider a set

$$\Pi = \{a | a_{i\min} \le a_i \le a_{i\max}, \quad i \in \{1, m\}\} \tag{41}$$

where $a_{i\min}$ and $a_{i\max}$ are real numbers denoting the lower bounds and the upper bounds of the ith element of $\hat{\boldsymbol{a}}$ respectively.

The projection operator is defined as

$$\{\mathrm{Proj}(\hat{\boldsymbol{a}}, -\Gamma\Phi^{\mathrm{T}}\boldsymbol{y})\}_i = \begin{cases} 0, & \text{if } \hat{a}_i = a_{i\max} \text{ and } \sigma_i(\Phi^{\mathrm{T}}\boldsymbol{y})_i < 0 \\ -\sigma_i(\Phi^{\mathrm{T}}\boldsymbol{y})_i, & \text{if } [a_{i\min} < \hat{a}_i < a_{i\max}] \\ & \text{or } [\hat{a}_i = a_{i\min} \text{ and } \sigma_i(\Phi^{\mathrm{T}}\boldsymbol{y})_i \geq 0] \\ & \text{or } [\hat{a}_i = a_{i\min} \text{ and } \sigma_i(\Phi^{\mathrm{T}}\boldsymbol{y})_i \leq 0] \\ 0, & \text{if } \hat{a}_i = a_{i\min} \text{ and } \sigma_i(\Phi^{\mathrm{T}}\boldsymbol{y})_i > 0 \end{cases} \tag{42}$$

where $\Gamma \in R^{m\times m}$ is arbitrary positive definite and diagonal matrix and σ_i is (i,i) element of Γ.

Then, the following properties hold.

(a) if $\hat{\boldsymbol{a}}(0) \in \Pi$, $\hat{\boldsymbol{a}}(t) \in \Pi$.

(b) $\|\mathrm{Proj}(\boldsymbol{p},\boldsymbol{q})\| \leq \|\boldsymbol{q}\|$.

(c) $(\boldsymbol{p}-\boldsymbol{p}^*)^{\mathrm{T}}\Lambda\,\mathrm{Proj}(\boldsymbol{p},\boldsymbol{q}) \leq (\boldsymbol{p}-\boldsymbol{p}^*)^{\mathrm{T}}\Lambda\boldsymbol{q}$, where Λ is a positive definite and diagonal matrix.

Now, we show the existence of the bounding function $\rho_{I_d}(\boldsymbol{e},\dot{\boldsymbol{e}},\boldsymbol{\eta})$. Determine parameter update law as

$$\dot{\hat{\boldsymbol{a}}} = \mathrm{Proj}(\hat{\boldsymbol{a}}, -\Gamma\Phi^{\mathrm{T}}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}},\ddot{\boldsymbol{\theta}}_r,\dot{\boldsymbol{\theta}}_r)\boldsymbol{y}), \qquad \hat{\boldsymbol{a}}(0) \in \Pi \tag{43}$$

Then, from (37) we have

$$\begin{aligned}\dot{\boldsymbol{I}}_d &= K_N^{-1}\{\dot{\Phi}\hat{\boldsymbol{a}} + \Phi\dot{\hat{\boldsymbol{a}}} + \dot{\boldsymbol{u}}_1\} \\ &= K_N^{-1}\{\dot{\Phi}\hat{\boldsymbol{a}} + \Phi\,\mathrm{Proj}(\hat{\boldsymbol{a}}, -\Gamma_d\Phi^{\mathrm{T}}\boldsymbol{y}) + \dot{\boldsymbol{u}}_1\}\end{aligned} \tag{44}$$

Using properties (a) and (b), the following inequality holds.

$$\begin{aligned}\|\dot{\boldsymbol{I}}_d\| &\leq \|K_N^{-1}\|\{\|\dot{\Phi}\|\|\hat{\boldsymbol{a}}\| + \|\Phi\|\|\,\mathrm{Proj}(\boldsymbol{a}, -\Gamma_d\Phi^{\mathrm{T}}\boldsymbol{y})\| + \|\dot{\boldsymbol{u}}_1\|\} \\ &\leq \|K_N^{-1}\|\{\bar{m}\|\dot{\Phi}\| + \|\Phi\|\|\Gamma_d\Phi^{\mathrm{T}}\boldsymbol{y}\| + \|\dot{\boldsymbol{u}}_1\|\} \\ &\leq \|K_N^{-1}\|\{\bar{m}\|\dot{\Phi}\| + \|\Gamma_d\|\|\Phi\|^2\|\boldsymbol{y}\| + \|\dot{\boldsymbol{u}}_1\|\}\end{aligned} \tag{45}$$

where $\bar{m}$ is a positive constant which satisfies $\|\hat{\boldsymbol{a}}\| \leq \bar{m}$.

Therefore, from the same argument in Step 3 of § 3, we can insist on the existence of such bounding function (22) that satisfies (21).

As in § 3, a control law is derived recursively by following the three steps.

Step 1. **Filtered tracking error:** Same as Step 1 in § 3.

Step 2. **Error dynamics for the manipulator loop:** Let a positive definite function $V_1(\boldsymbol{e},\boldsymbol{y},\boldsymbol{\theta},\tilde{\boldsymbol{a}})$ be

$$V_1(\boldsymbol{e},\boldsymbol{y},\boldsymbol{\theta},\tilde{\boldsymbol{a}}) = \frac{\alpha_1 + q_1^2 + \alpha_1^2 q_2^2}{\alpha_1} V_0(\boldsymbol{e}) + \tfrac{1}{2}\boldsymbol{y}^{\mathrm{T}} M(\boldsymbol{\theta})\boldsymbol{y} + \tfrac{1}{2}\tilde{\boldsymbol{a}}^{\mathrm{T}}\Gamma^{-1}\tilde{\boldsymbol{a}} \tag{46}$$

where $\Gamma \in \boldsymbol{R}^{m\times m}$ is arbitrary positive definite matrix.

Using the same technique as Step 2 in § 3, finally the time derivative of $V_1(\boldsymbol{e},\boldsymbol{y},\boldsymbol{\theta},\tilde{\boldsymbol{a}})$ along the trajectory of (11) and (12) is given by

$$\begin{aligned}\dot{V}_1(\boldsymbol{e},\boldsymbol{y},\boldsymbol{\theta},\tilde{\boldsymbol{a}}) \leq{}& -\alpha_1\|\boldsymbol{e}\|^2 + \gamma^2\|\boldsymbol{w}\|^2 - \|\boldsymbol{z}\|^2 \\ &+ \boldsymbol{y}^{\mathrm{T}}K_N\boldsymbol{\eta} + \|\boldsymbol{y}\|\cdot\|\Delta f(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})\| \\ &+ \boldsymbol{y}^{\mathrm{T}}\left\{\frac{\alpha_1 + q_1^2 + \alpha_1^2 q_2^2}{\alpha_1}\boldsymbol{e} \right. \\ &\left. + \left(\frac{1}{4\gamma^2} + q_2^2\right)\boldsymbol{y} - 2\alpha_1 q_2^2\boldsymbol{e} + \boldsymbol{u}_1\right\} \\ &+ \tilde{\boldsymbol{a}}^{\mathrm{T}}\{\Phi^{\mathrm{T}}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}},\ddot{\boldsymbol{\theta}}_r,\dot{\boldsymbol{\theta}}_r)\boldsymbol{y} + \Gamma^{-1}\dot{\tilde{\boldsymbol{a}}}\}\end{aligned} \tag{47}$$

From property (c) of the projection operator

$$\begin{aligned}&\tilde{\boldsymbol{a}}^{\mathrm{T}}\{\Phi^{\mathrm{T}}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}},\ddot{\boldsymbol{\theta}}_r,\dot{\boldsymbol{\theta}}_r)\boldsymbol{y} + \Gamma^{-1}\dot{\tilde{\boldsymbol{a}}}\} \\ &\quad = \tilde{\boldsymbol{a}}^{\mathrm{T}}\{\Phi^{\mathrm{T}}\boldsymbol{y} + \Gamma^{-1}\,\mathrm{Proj}(\hat{\boldsymbol{a}}, -\Gamma\Phi^{\mathrm{T}}\boldsymbol{y})\} \\ &\quad \leq \tilde{\boldsymbol{a}}^{\mathrm{T}}\Phi^{\mathrm{T}}\boldsymbol{y} - \tilde{\boldsymbol{a}}^{\mathrm{T}}\Gamma^{-1}\Gamma\Phi^{\mathrm{T}}\boldsymbol{y}\end{aligned} \tag{48}$$

Thus, (47) becomes

$$\begin{aligned}\dot{V}_1(\boldsymbol{e},\boldsymbol{y},\boldsymbol{\theta},\tilde{\boldsymbol{a}}) \leq{}& -\alpha_1\|\boldsymbol{e}\|^2 + \gamma^2\|\boldsymbol{w}\|^2 - \|\boldsymbol{z}\|^2 + \boldsymbol{y}^{\mathrm{T}}K_N\boldsymbol{\eta} \\ &+ \|\boldsymbol{y}\|\cdot\|\Delta f(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})\| \\ &+ \boldsymbol{y}^{\mathrm{T}}\left\{\frac{\alpha_1 + q_1^2 + \alpha_1^2 q_2^2}{\alpha_1}\boldsymbol{e} + \left(\frac{1}{4\gamma^2} + q_2^2\right)\boldsymbol{y} \right. \\ &\left. - 2\alpha_1 q_2^2\boldsymbol{e} + \boldsymbol{u}_1\right\}\end{aligned} \tag{49}$$

Determine control input $\boldsymbol{u}_1$ as follows so as to make the last term in (49) negative

$$\begin{aligned}\boldsymbol{u}_1 ={}& -\frac{\alpha_1 + q_1^2 + \alpha_1^2 q_2^2}{\alpha_1}\boldsymbol{e} - \left(\frac{1}{4\gamma^2} + q_2^2\right)\boldsymbol{y} + 2\alpha_1 q_2^2\boldsymbol{e} \\ &- \alpha_2\cdot\lambda_{\max}\boldsymbol{y} - \frac{\boldsymbol{y}\rho_{\Delta f}^2}{\|\boldsymbol{y}\|\rho_{\Delta f} + \varepsilon_1\,\mathrm{e}^{-\beta_1 t}}\end{aligned} \tag{50}$$

Thus, we have

$$\begin{aligned}\dot{V}_1(\boldsymbol{e},\boldsymbol{y},\boldsymbol{\theta},\tilde{\boldsymbol{a}}) \leq{}& -\alpha_1\|\boldsymbol{e}\|^2 - \alpha_2\|\boldsymbol{y}\|^2_{M(\boldsymbol{\theta})} + \gamma^2\|\boldsymbol{w}\|^2 - \|\boldsymbol{z}\|^2 \\ &+ \boldsymbol{y}^{\mathrm{T}}K_N\boldsymbol{\eta} + \varepsilon_1\,\mathrm{e}^{-\beta_1 t}\end{aligned} \tag{51}$$

Step 3. **Error dynamics for the actuator loop:** Here, we also use the bounding function $\rho_{I_d}(\boldsymbol{e},\dot{\boldsymbol{e}},\boldsymbol{\eta})$ satisfying (21) instead of $\dot{\boldsymbol{I}}_d$ itself. By applying the voltage control input (23), we have the error dynamics of the actuator loop (25).

Let a Lyapunov function candidate be

$$V_2(\boldsymbol{e},\boldsymbol{y},\boldsymbol{\theta},\tilde{\boldsymbol{a}},\boldsymbol{\eta}) = V_1(\boldsymbol{e},\boldsymbol{y},\boldsymbol{\theta},\tilde{\boldsymbol{a}}) + \tfrac{1}{2}\boldsymbol{\eta}^{\mathrm{T}}L\boldsymbol{\eta} \tag{52}$$

Consider the time derivative of $V_2(\boldsymbol{e},\boldsymbol{y},\boldsymbol{\theta},\tilde{\boldsymbol{a}},\boldsymbol{\eta})$ along the trajectory of (11), (12) and (25) as Step 3 in § 3, and determine control input $\boldsymbol{u}_2$ as

$$\boldsymbol{u}_2 = R\boldsymbol{\eta} + K_e\dot{\boldsymbol{e}} - K_N^{\mathrm{T}}\boldsymbol{y} - \alpha_3 L\boldsymbol{\eta} - \frac{LL^{\mathrm{T}}\boldsymbol{\eta}\rho_{I_d}^2}{\|\boldsymbol{\eta}^{\mathrm{T}}L\|\rho_{I_d} + \varepsilon_2\,\mathrm{e}^{-\beta_2 t}} \tag{53}$$

Thus, we have

$$\dot{V}_2(\boldsymbol{e},\boldsymbol{y},\boldsymbol{\theta},\tilde{\boldsymbol{a}},\boldsymbol{\eta}) \le -\alpha_1\|\boldsymbol{e}\|^2 - \alpha_2\|\boldsymbol{y}\|_{M(\theta)}^2 - \alpha_3\|\boldsymbol{\eta}\|_L^2 + \gamma^2\|\boldsymbol{w}\|^2 - \|\boldsymbol{z}\|^2 + \varepsilon_1\,\mathrm{e}^{-\beta_1 t} + \varepsilon_2\,\mathrm{e}^{-\beta_2 t} \tag{54}$$

The validation of the specifications $(S1)$ and $(S2)$ follows the same argument developed in § 3. Finally, we have the following result.

Theorem 2: *The voltage level robust adaptive tracking control law that accomplishes control objectives* $(S1)$ *and* $(S2)$ *is given by*

$$\boldsymbol{v} = RK_N^{-1}\{\Phi(\boldsymbol{\theta},\dot{\boldsymbol{\theta}},\ddot{\boldsymbol{\theta}}_r,\dot{\boldsymbol{\theta}}_r)\hat{\boldsymbol{a}} + \boldsymbol{u}_1\}K_e\dot{\boldsymbol{\theta}}_d + \boldsymbol{u}_2 \tag{55}$$

where control inputs $\boldsymbol{u}_1$ *and* $\boldsymbol{u}_2$ *are given by* (50) *and* (53) *respectively, and parameter update law is given by* (43).

Remark 4: Theorem 2 ensures the global uniform ultimate bounded stability of the link tracking errors $\boldsymbol{e}$ and $\dot{\boldsymbol{e}}$, and the boundedness of the estimated parameters. However, convergence of the estimated parameters to the real values is not guaranteed.

5. Experimental example

In order to evaluate the tracking performance of the derived controllers, robust tracking control law with L_2-gain performance (hereafter denoted as RBL controller) and robust adaptive tracking control law with L_2-gain performance combined with the parameter estimator (hereafter denoted as RAL controller), experimental works are carried out for a two-link horizontal robot manipulator driven by dc motors, shown in figure 1.

The dynamics of the system are described by (1) and (2) with

$$M(\boldsymbol{\theta}) = \begin{bmatrix} j_1 + j_2 + 2j_3\cos\theta_2 & j_2 + j_3\cos\theta_2 \\ j_2 + j_3\cos\theta_2 & j_2 \end{bmatrix}$$

$$C(\boldsymbol{\theta},\dot{\boldsymbol{\theta}}) = j_3\sin\theta_2 \begin{bmatrix} -\dot{\theta}_2 & -\dot{\theta}_1 - \dot{\theta}_2 \\ \dot{\theta}_1 & 0 \end{bmatrix}$$

$$D(\dot{\boldsymbol{\theta}}) = \begin{bmatrix} d_1\dot{\theta}_1 \\ d_2\dot{\theta}_2 \end{bmatrix} \qquad G(\boldsymbol{\theta}) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \boldsymbol{\theta} = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$$

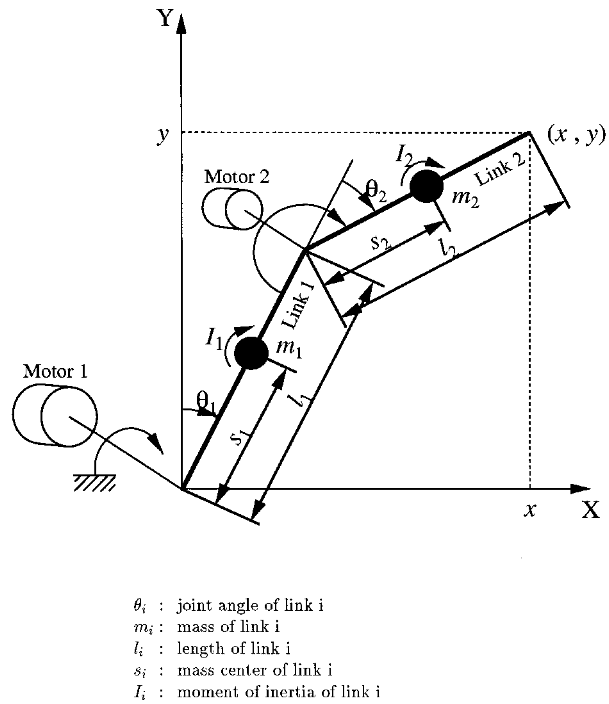


Figure 1. Two-link horizontal robot manipulator driven by dc motors.

$$\boldsymbol{I} = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \qquad \boldsymbol{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$K_N = \begin{bmatrix} K_{N1} & 0 \\ 0 & K_{N2} \end{bmatrix} \qquad L = \begin{bmatrix} L_1 & 0 \\ 0 & L_2 \end{bmatrix}$$

$$R = \begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix} \qquad K_e = \begin{bmatrix} K_{e1} & 0 \\ 0 & K_{e2} \end{bmatrix}$$

where $j_1 = m_1 s_1^2 + m_2 l_1^2 + I_1$, $j_2 = m_2 s_2^2 + I_2$, $j_3 = m_2 l_1 s_2$, d_1 and d_2 are parameters. For the implementation of RAL controller, the regressor matrix $\Phi(\boldsymbol{\theta},\dot{\boldsymbol{\theta}},\boldsymbol{\phi},\psi)$ and the unknown parameter vector $\boldsymbol{a}$ in (34) are expressed as

$$\Phi(\boldsymbol{\theta},\dot{\boldsymbol{\theta}},\boldsymbol{\phi},\psi) = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} & \sigma_{14} & \sigma_{15} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} & \sigma_{24} & \sigma_{25} \end{bmatrix} \tag{56}$$

$$\boldsymbol{a} = [j_1 \;\; j_2 \;\; j_3 \;\; d_1 \;\; d_2]^{\mathrm{T}} \tag{57}$$

where

$$\sigma_{11} = \phi_1 \qquad \sigma_{12} = \phi_1 + \phi_2$$

$$\sigma_{13} = (2\phi_1 + \phi_2)\cos\theta_2 - (\dot{\theta}_2\psi_1 + \dot{\theta}_1\psi_2 + \dot{\theta}_2\psi_2)\sin\theta_2$$

$$\sigma_{14} = \dot{\theta}_1 \qquad \sigma_{15} = 0 \qquad \sigma_{21} = 0 \qquad \sigma_{22} = \phi_1 + \phi_2$$

$$\sigma_{23} = \phi_1\cos\theta_2 + \dot{\theta}_1\psi_1\sin\theta_2 \qquad \sigma_{24} = 0 \qquad \sigma_{25} = \dot{\theta}_2$$

Link 1 Mass	m_1	1.8970	kg
Link 2 Mass	m_2	0.253	kg
Link 1 Length	l_1	0.32	m
Link 2 Length	l_2	0.23	m
Motor 1 Inductance	L_1	0.016	H
Motor 2 Inductance	L_2	0.003	H
Motor 1 Resistance	R_1	7.5	Ω
Motor 2 Resistance	R_2	8.9	Ω
Motor 1 Back EMF constant	K_{e1}	0.4994	Vs/rad
Motor 2 Back EMF constant	K_{e2}	0.0859	Vs/rad
Motor 1 Torque constant	K_{N1}	0.5017	Nm/A
Motor 2 Torque constant	K_{N2}	0.0854	Nm/A

Table 1. Kinematic parameters of the manipulator and actuator.

The kinematic parameters of the manipulator and actuator are shown in table 1.

In this experimental work, we consider the situation that the manipulator holds different weight of loads, and we set the non-linear function $\Delta f(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})$ as parameter perturbation due to the load variations. Let j_{ni} and d_{ni} represent nominal parameter values, and let real parameter values j_i and d_i be given by

$$j_i = j_{ni} + \Delta j_i \qquad (i = 1, 2, 3) \tag{58}$$

$$d_i = d_{ni} + \Delta d_i \qquad (i = 1, 2) \tag{59}$$

where Δj_i and Δd_i represent uncertain parameter perturbations. Then, $M(\boldsymbol{\theta})$, $C(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})$ and $D(\dot{\boldsymbol{\theta}})$ are written as

$$M(\boldsymbol{\theta}) = M_n(\boldsymbol{\theta}) + M_\Delta(\boldsymbol{\theta}) \tag{60}$$

$$C(\boldsymbol{\theta},\dot{\boldsymbol{\theta}}) = C_n(\boldsymbol{\theta},\dot{\boldsymbol{\theta}}) + C_\Delta(\boldsymbol{\theta},\dot{\boldsymbol{\theta}}) \tag{61}$$

$$D(\dot{\boldsymbol{\theta}}) = D_n(\dot{\boldsymbol{\theta}}) + D_\Delta(\dot{\boldsymbol{\theta}}) \tag{62}$$

where $M_n(\boldsymbol{\theta})$, $C_n(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})$ and $D_n(\dot{\boldsymbol{\theta}})$ represent nominal matrices of $M(\boldsymbol{\theta})$, $C(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})$ and $D(\dot{\boldsymbol{\theta}})$ respectively, which are composed of nominal parameter values, and $M_\Delta(\boldsymbol{\theta})$, $C_\Delta(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})$ and $D_\Delta(\dot{\boldsymbol{\theta}})$ represent non-linear functions with uncertain perturbations, which are composed of uncertain parameter perturbations. Let us represent the non-linear perturbation term $\Delta f(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})$ as

$$\Delta f(\boldsymbol{\theta},\dot{\boldsymbol{\theta}}) = M_\Delta(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + C_\Delta(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})\dot{\boldsymbol{\theta}} + D_\Delta(\dot{\boldsymbol{\theta}}) \tag{63}$$

It should be noted that $\Delta f(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})$ is occurred by the deviation from the real parameter values. This means that $\Delta f(\boldsymbol{\theta},\dot{\boldsymbol{\theta}}) = 0$, provided the dynamics of the robot have real parameters.

We will design controllers which provides robustness under the restriction that manipulator holds 0 g to 250 g load. Then, j_{ni}, d_{ni} and Δj_i, Δd_i are determined in the following way. We executed experiments for parameter identification $(j_i\, d_i)$ with two loads (0 g, 250 g). Let us represent the identified parameter values by

$j_1(0)$	0.1894	$j_1(250)$	0.2285
$j_2(0)$	0.0047	$j_2(250)$	0.0275
$j_3(0)$	0.0063	$j_3(250)$	0.0311
$d_1(0)$	0.0215	$d_1(250)$	0.0471
$d_2(0)$	0.0024	$d_2(250)$	0.0189

Table 2. Identified parameter values.

$$j_i(0),\, j_i(250) \qquad (i = 1, 2, 3)$$

$$d_i(0),\, d_i(250) \qquad (i = 1, 2)$$

They are shown in table 2. The identified parameter values $(j_i(0),\, d_i(0))$ are used as the nominal parameter values j_{ni} and d_{ni}, and $(j_i(250),\, d_i(250))$ are regarded as the real parameter values j_i and d_i. The parameter perturbations Δj_i and Δd_i can be obtained from (58) and (59).

We consider the desired joint trajectories

$$\left.\begin{aligned} \theta_{1d} &= \frac{\pi}{4}\sin\frac{4}{5}\pi t \\ \theta_{2d} &= -\frac{\pi}{4}\sin\frac{4}{5}\pi t \end{aligned}\right\} \tag{64}$$

In addition, additive disturbances are injected into the robot dynamics of the form

$$\boldsymbol{w} = \begin{bmatrix} 1.0\ \sin\frac{3}{2}\pi t \\ 0.1\ \sin\frac{3}{2}\pi t \end{bmatrix} \text{Nm} \tag{65}$$

Now, we must determine the bounding function $\rho_{\Delta f}(\boldsymbol{e},\dot{\boldsymbol{e}})$ given by (5) such that (3) is satisfied. The right-hand side of (63) includes $\ddot{\boldsymbol{\theta}}$. However, we evaluate $\Delta f(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})$ in the following way. Substitute the real parameters $(j_i(250),\, d_i(250))$ to (1), then $\Delta f(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})$ is annihilated due to the experimental setting of $\Delta f(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})$. Thus, the information of $\ddot{\boldsymbol{\theta}}$ is obtained from the inverse dynamics of (1) without $\Delta f(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})$. The coefficients ζ_2 and ζ_1 were estimated as $\zeta_2 = 1.0$ and $\zeta_1 = 10.0$ respectively by calculating the bounds of $\Delta f(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})$ using the inverse dynamics of (1) and (63) based on the information about the upper bounds on parameters and on the desired trajectory given by (64). As in the same way, the bounding function $\rho_{Id}(\boldsymbol{e},\dot{\boldsymbol{e}},\boldsymbol{\eta})$ given by (22) must be determined such that (21) is satisfied. The coefficients ξ_4, ξ_3, ξ_2 and ξ_1 were estimated as $\xi_4 = 25.0$, $\xi_3 = 15.0$, $\xi_2 = 10.0$ and $\xi_1 = 1.0$ respectively by calculation. In the estimation of coefficients ζ_i and ξ_i, the conservative estimation of the constant terms ζ_0 and ξ_0 is inevitable due to the norm expanding in the procedure to derive equations (5) and (22). This causes a designed controller to be too conservative and will degrade a tracking performance. Therefore, we set ζ_0 and ξ_0 as 0. Instead, the influence of omission of the constant terms ζ_0 and ξ_0 on the tracking performance is guaranteed from L_2-gain performance of disturbance attenuation. For the implementation of the RAL controller, $\lambda_{\max}$ was estimated as $\lambda_{\max} = 0.3295$ by calculating $\lambda(M(\boldsymbol{\theta}))$ for $0 \leq \theta_2 \leq 2\pi$.

All control design parameters were tuned by trial-and-error until the best link position tracking performance was achieved for the desired trajectories given by (64). They were determined as $q_1 = q_2 = 1.0$, $\alpha_1 = \alpha_2 = \alpha_3 = 10.0$, $\varepsilon_1 = \varepsilon_2 = 0.01$, and $\beta_1 = \beta_2 = 1.0$, in RAL controller the adaptation gain Γ was determined as $\Gamma = \mathrm{diag}\{0.01, 0.01\}$. We designed RBL and RAL controllers for $\gamma = 1.0$, $\gamma = 0.3$ and $\gamma = 0.15$ respectively. Experiments are carried out with the following three types of controllers for a purpose of comparison.

(a) A robust tracking control law proposed in Dawson *et al.* (1992) (hereafter denoted as RB controller).
(b) RBL controller.
(c) RAL controller.

The RB controller corresponds to a RBL controller in which the specified tracking performance level γ is chosen as $\gamma = \infty$. Note that the RB controller guarantees only the global uniform ultimate bounded stability of the tracking error system. However, the RBL and RAL controllers guarantee both the global uniform ultimate bounded stability of the tracking error system and L_2-gain performance.

In experiments, link position errors, link velocity errors, and motor current perturbations were initialized to zero. A standard backwards difference algorithm applied to the link position measurements and then passed through a low-pass filter was used to generate the link velocity signals. In the cases of RB and RBL controllers implementation, parameters in the state feedback control law (33) was based on the nominal parameters $(j_i(0),\ d_i(0))$, and in the case of RAL controller implementation, initial condition of the parameter update law (43) was set to $(j_i(0),\ d_i(0))$ and in the projection operator, $\boldsymbol{a}_{\min}$ was chosen as

$$\boldsymbol{a}_{\min} = [0.5j_1(250) \quad 0.15j_2(250) \quad 0.2j_3(250) \quad 0.4d_1(250) \quad 0.1d_2(250)]^{\mathrm{T}}$$

and $\boldsymbol{a}_{\max}$ was chosen as

$$\boldsymbol{a}_{\max} = [1.5j_1(250) \quad 1.85j_2(250) \quad 1.8j_3(250) \quad 1.6d_1(250) \quad 1.9d_2(250)]^{\mathrm{T}}$$

In practice, however, experiments were carried out under the condition that the manipulator had 250 g load. The sampling periods was 1 ms.

To see the influence of disturbance signal $\boldsymbol{w}$ on tracking errors in later experimental results clearly, we executed a fundamental experiment in the case of $\boldsymbol{w} = 0$ using the RBL controller for $\gamma = 0.15$ in advance. Experimental results are shown in figures 2 and 3. Figure 2 shows the position errors and velocity errors,

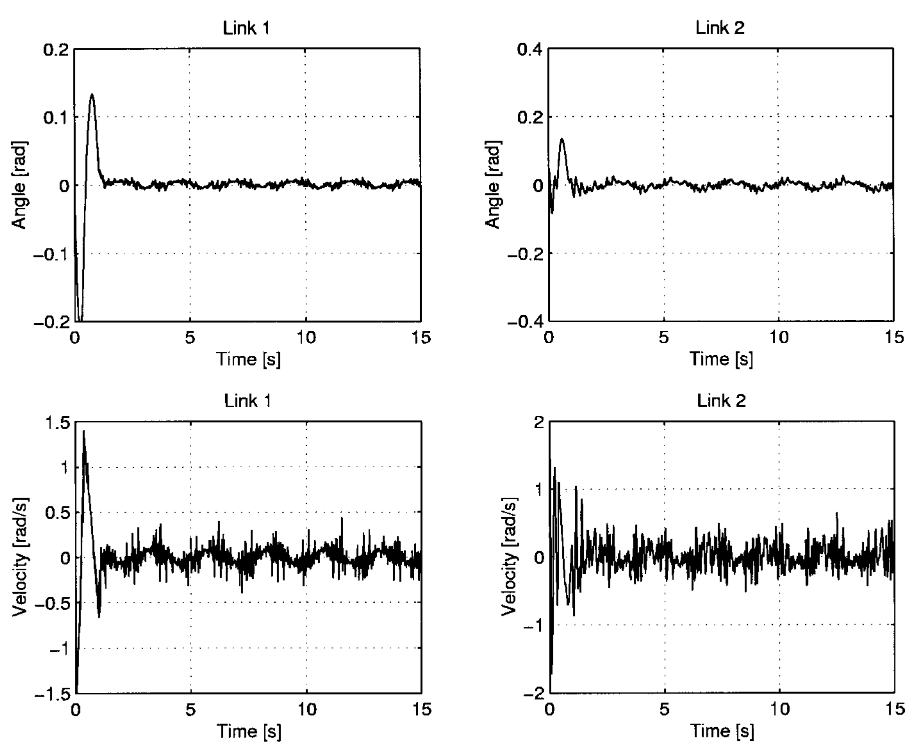


Figure 2. Position errors and velocity errors: $w = 0$.

and figure 3 shows the input voltages of each motor. As shown in figure 2, when $\boldsymbol{w} = 0$ the tracking errors for link position and link velocity are almost zero in spite of the presence of model inaccuracy. However, small oscillation can be seen in velocity errors. This seems to be caused by a mechanical reason that the derivative of the link position is obtained by a backwards difference algorithm applied to the link position measurements.

In the following experiments, disturbance $\boldsymbol{w}$ given by (65) was injected. Experimental results using the RB controller are shown in figures 4 and 5. Experimental results using the RBL controller for $\gamma = 0.15$ are shown in figures 6 and 7. To see the effect of the value of γ on the tracking performance, we executed experiments using the RBL controller for the different values of γ. Experimental results of the position for $\gamma = 0.3$ and $\gamma = 1.0$ are shown in figures 8 and 9. The same experimental study was implemented for the RAL controllers. Experimental results are shown in figures 10–14, where figure 12 shows the estimated parameter values for $\gamma = 0.15$.

Comparing the tracking performance of the RB controller (figure 4) and of the RBL controller (figure 6), the RB controller generates fairly large tracking

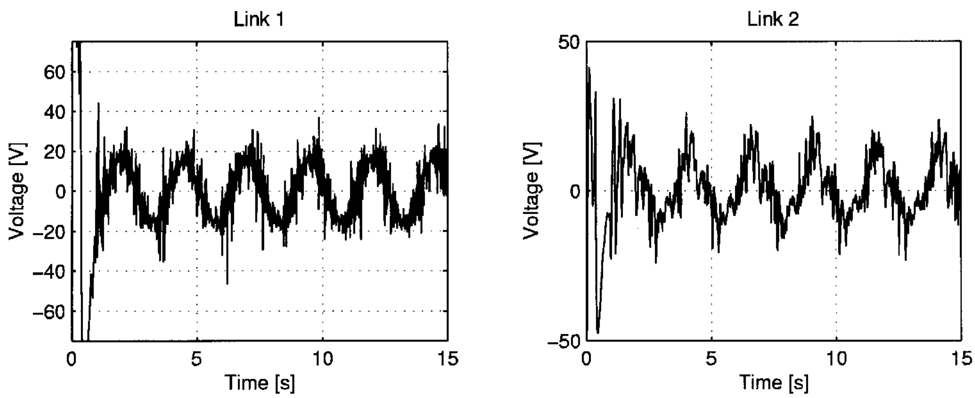


Figure 3. Input voltages: $w = 0$.

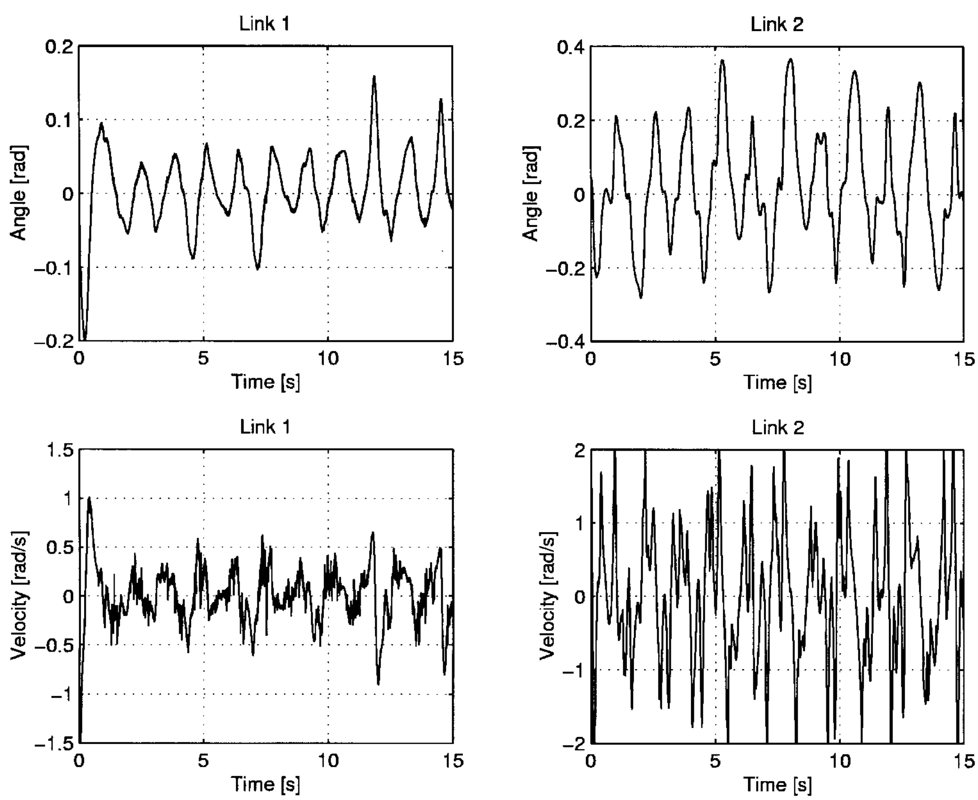


Figure 4. Position errors and velocity errors: RB controller.

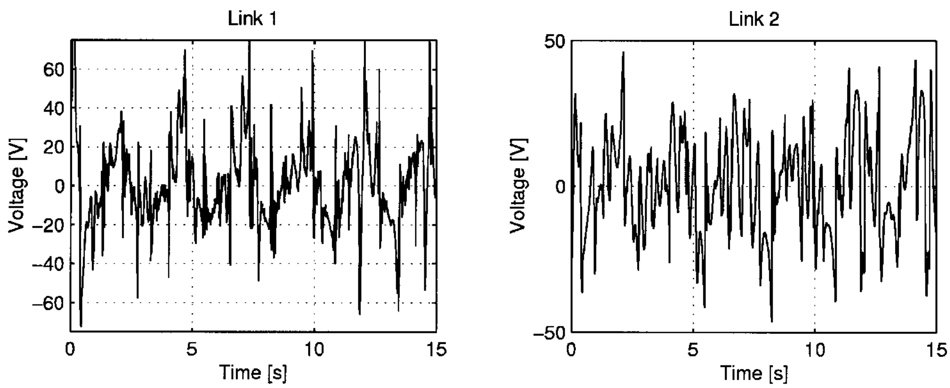


Figure 5. Input voltages: RB controller.

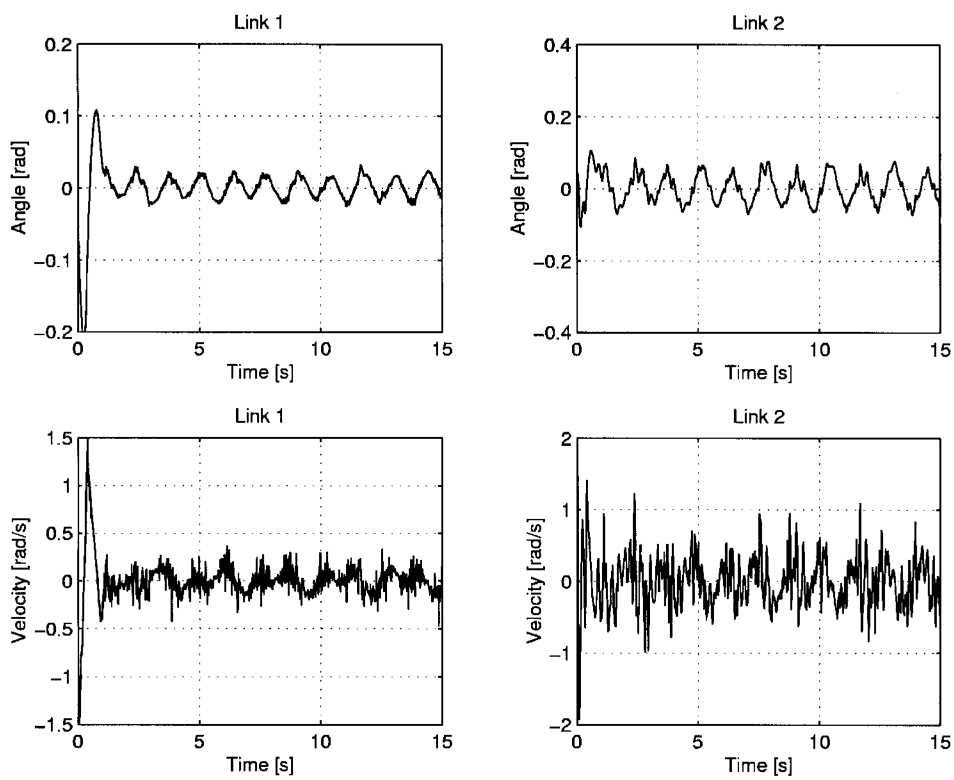


Figure 6. Position errors and velocity errors: RBL controller ($\gamma = 0.15$).

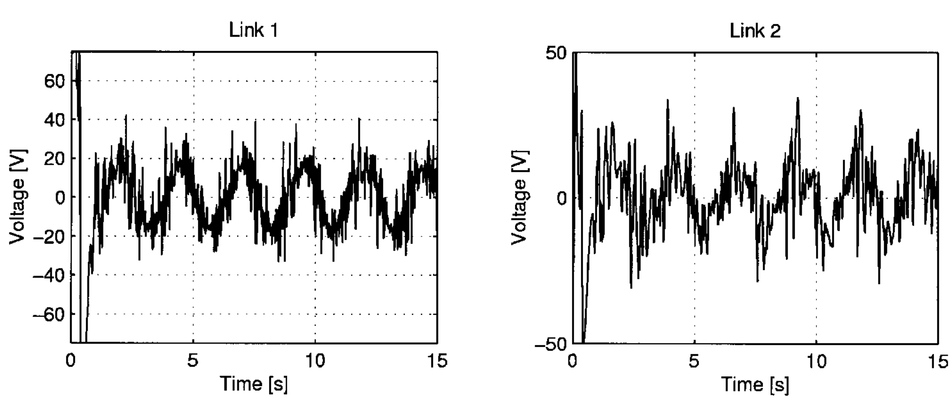


Figure 7. Input voltages: RBL controller ($\gamma = 0.15$).

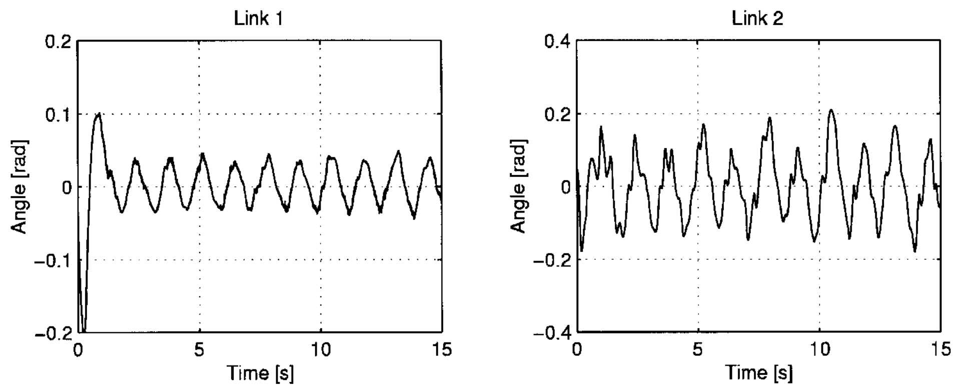


Figure 8. Position errors: RBL controller ($\gamma = 0.3$).

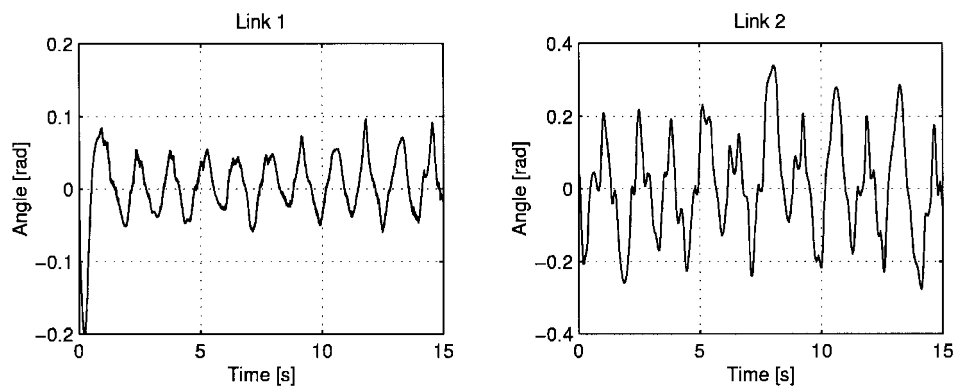


Figure 9. Position errors: RBL controller ($\gamma = 1.0$).

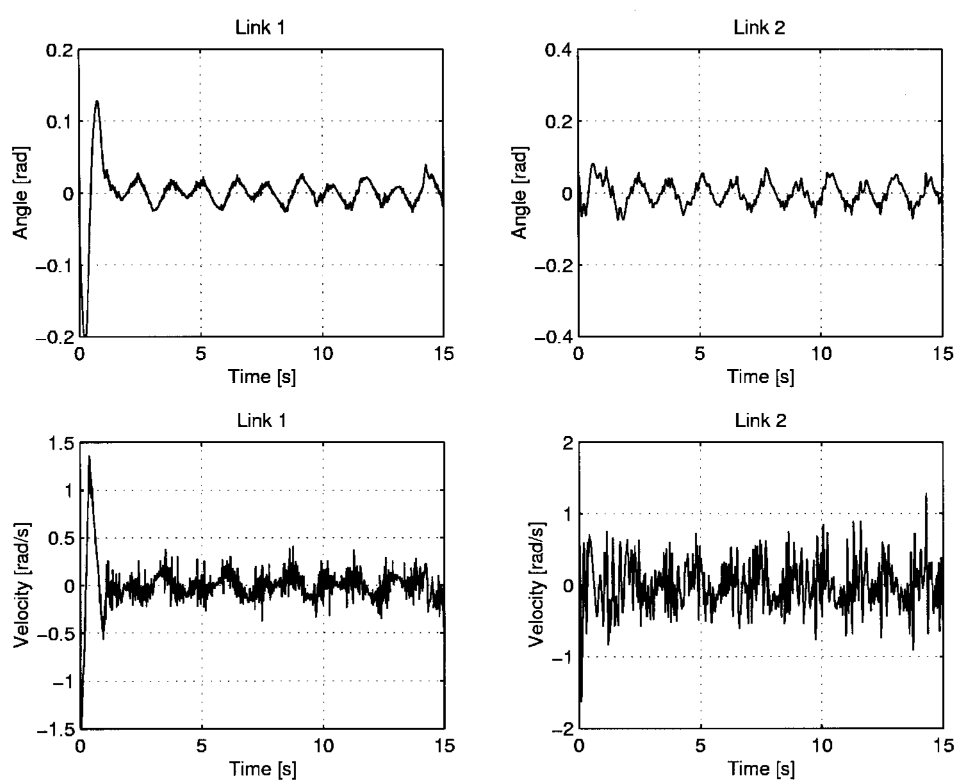


Figure 10. Position errors and velocity errors: RAL controller ($\gamma = 0.15$).

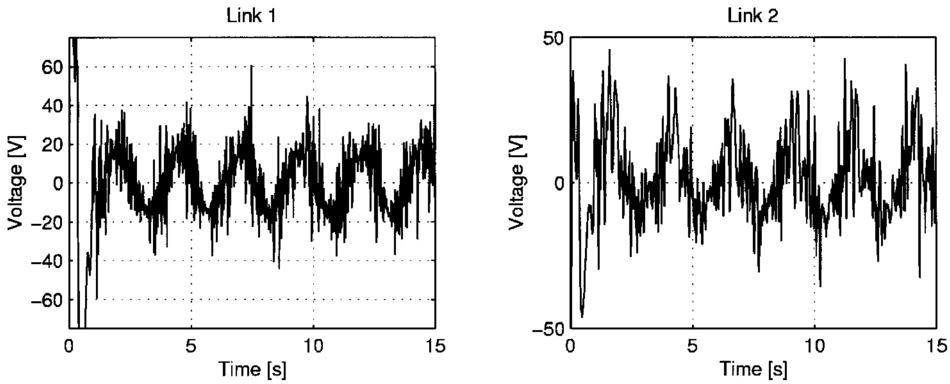
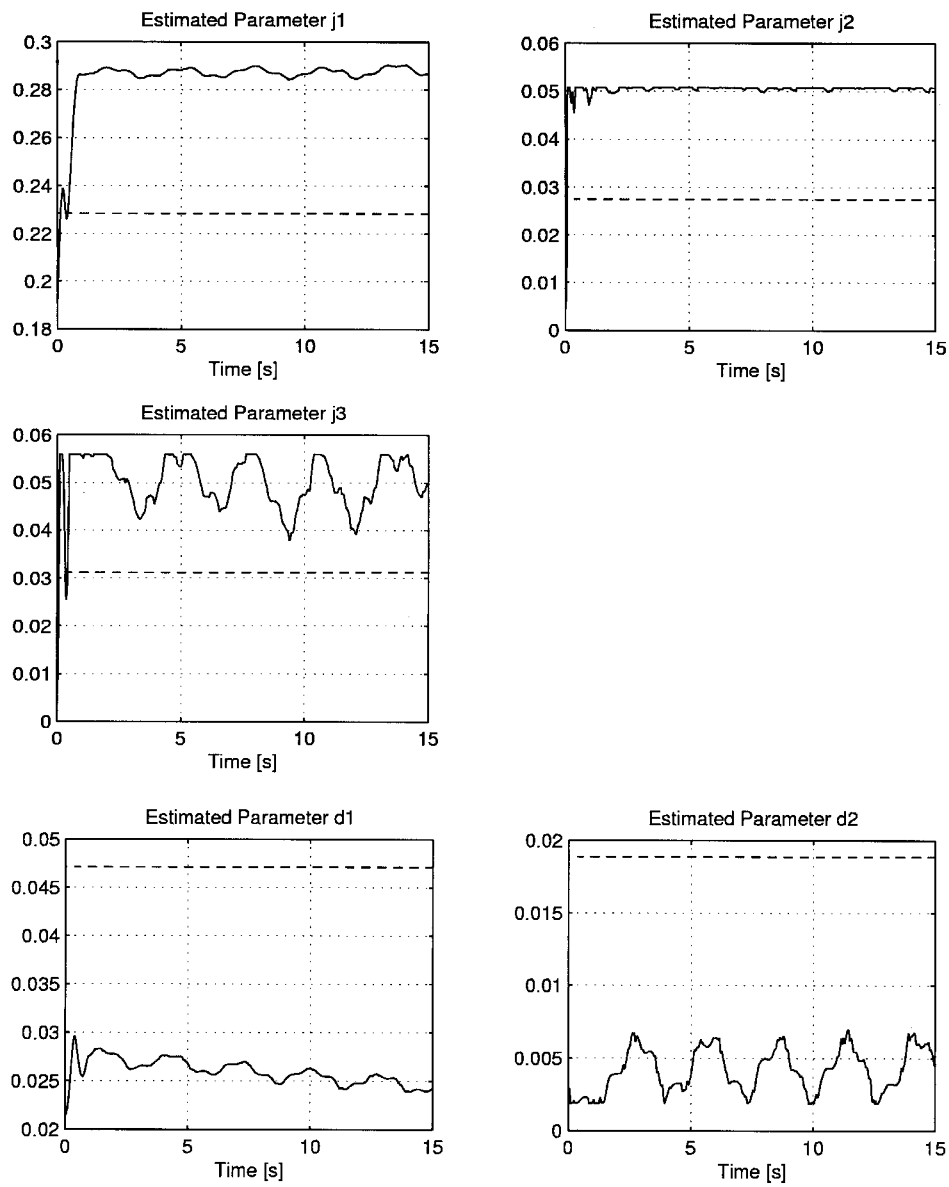


Figure 11. Input voltages: RAL controller ($\gamma = 0.15$).



··· : Real Parameter Value
— : Estimated Parameter Value

Figure 12. Estimated parameters: RAL controller ($\gamma = 0.15$).

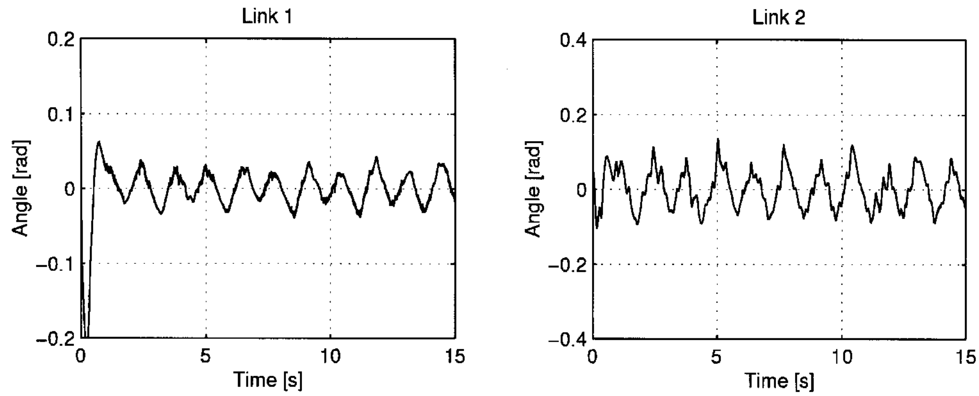


Figure 13. Position errors: RAL controller ($\gamma = 0.3$).

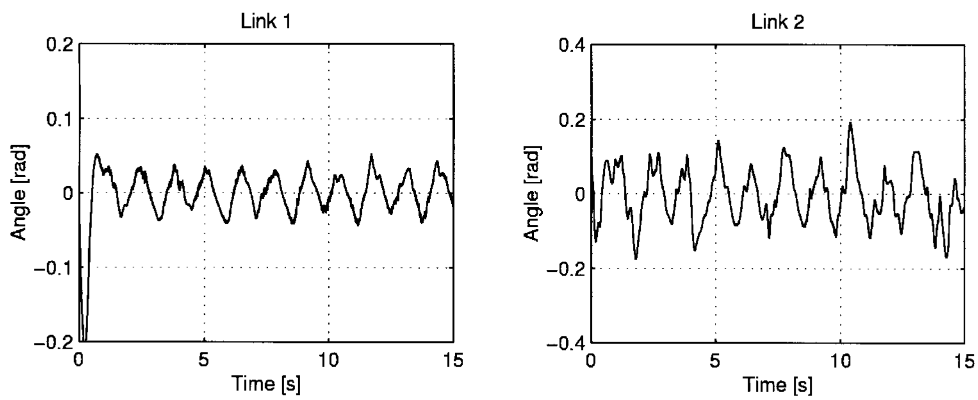


Figure 14. Position errors: RAL controller ($\gamma = 1.0$).

errors, while the RBL controller generates much smaller tracking errors. This illustrates that the RB controller is highly affected by the disturbance, however, the RBL controller attenuates the influences of the disturbance. Comparing the tracking performance of the RBL controller for different values of γ, from the link position errors in figures 6, 8 and 9, it is seen that the link position errors in steady-state response become considerably small in proportion to the decrease of value of γ. This demonstrates that enhanced efficiency of disturbance attenuation is achieved by selecting small γ.

The tracking performance of the RAL controller for $\gamma = 0.15$ (figure 10) is quite similar to that of the RBL controller for $\gamma = 0.15$ (figure 6) as well as input voltages (figures 7 and 11). However, as mentioned in Remark 4, since convergence of parameters is not guaranteed, the estimated parameters are not likely to converge to the real values as seen in figure 12. Comparing the tracking performance of the RAL controller for different values of γ, as shown in the link position errors in figures 10, 13 and 14, although overshoot is observed in transient position error for link 1 γ is small, the steady-state position errors result as smaller in proportion to the decrease of γ.

Comparing the tracking performance of the RAL controller and of the RBL controller, as mentioned above, when γ is small the tracking performance for the RAL controller is almost the same as that of the RBL controller. However, when γ becomes large the tracking errors in using the RAL controller are much smaller than those in using the RBL controller (see figures 9 and 14). This is caused by the reason that in the case of RAL controller implementation, the influence of the disturbance on tracking errors is eased due to the parameter adaptation independently of value of γ.

In terms of these observations, we conclude that the effectiveness of the proposed robust and robust adaptive tracking controllers for rigid-link electrically-driven robot manipulators was verified.

Remark 5: For vertical type robot manipulators driven by dc motors, it is considered that the gravity term is significant and the gravity compensation error degrade the control performance. We have executed simulation works for a two-link vertical type electrically-driven robot manipulator. Simulation results using the RBL controller are shown in Ishii *et al.* (1999 a) and simulation results using the RAL controller are shown in Ishii *et al.* (1999 b) respectively. In both cases, the tracking errors of each joint result fairly small without setting the attenuation level γ too small. In particular, the tracking errors of each joint in using RAL are extremely smaller than those in using RBL. From this point of view, also for the vertical type robot manipulators, the usefulness of the proposed control scheme is validated.

6. Conclusions

In this paper, we developed a new approach to a tracking control of rigid-link electrically-driven robot manipulators with uncertainty by incorporating the criterion of a tracking performance given by L_2-gain constraint in controller synthesis. We proposed a robust tracking control law with L_2-gain performance based on Lyapunov recursive design, which ensures the global uniform ultimate bounded stability of the tracking error system with the L_2-gain less than any given small level. Besides, the approach was extended to a robust adaptive tracking control law with L_2-gain performance so as to deal with uncertain payloads, in which the manipulator parameters are estimated by a parameter adaptation law. There are two features in our approach. The first one is that a Lyapunov function is constructed recursively so as to not only guarantee stability of the tracking error system but also satisfy dissipation inequality ensuring the L_2-gain constraint. The second one is that it does not require to solve any Hamilton–Jacobi inequality. Experimental works were carried out to evaluate the tracking performance of the proposed controllers. Experimental results showed an improved tracking performance of the proposed controllers compared with a controller given in Dawson *et al.* (1992).

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