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A nonlinear robust HVDC control for a parallel AC/DC power system

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Abstract

In this paper, a parallel AC/DC power system is investigated, and a nonlinear robust controller is proposed to improve transient stability of the power system and to damp out any prolonged oscillation after a fault is cleared. Lyapunov's direct method is used to synthesize the control, and asymptotic stability of the closed loop system and improved dynamic performance are shown by both theoretical proof and simulation results.

Keywords: Robust control; HVDC; Power system; Stability; Lyapunov method

1. Introduction

It has been recognized that, through an HVDC transmission line, fast electronic control can be applied on the DC power flow and that proper use of DC power control enhances the AC system dynamic performance. In particular, controlling DC line transmission power can improve stability of the AC system after a major disturbance. The problem of improving stability of AC/DC parallel power system has been studied, and the results obtained so far are encouraging. In Refs. [10,11], detailed discussions were given to illustrate the effectiveness of DC control in improving system stability. Methods based on eigenvalue analysis have been used to design the AC/DC control in Refs. [2,15]. Control designs using optimal and sub-optimal control strategies have been reported in Refs. [3,4,8,14]. One common feature of these algorithms dealing with the DC power

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control in AC/DC parallel system is that they are based on a linearized system model. The use of linearization could severely limit dynamic performance of the controllers designed as the operating condition of the system varies and the system topology often changes.

There are several reports on DC power control designs using a nonlinear model. In Refs. [5,9], feedback linearization technique is employed. Basically, this method utilizes a nonlinear transformation that maps the nonlinear model of an AC/DC system into a linear one. As long as system dynamics are known, the transformation can be found, and all system nonlinearities can be directly compensated for. Since a power system is distributed network, its global information is not available real time, and system parameters and topology often change.

Robust control is a control that guarantees stability for systems in which there are significant uncertainties such as parameter variations, unknown functions, and unmodeled dynamics. The objective of this paper is to design a robust DC power control. It will be designed based on the nonlinear model of the system and thus can be applied to an arbitrary fault. And, it ensures robust stability and performance. Lyapunov direct method is used to synthesize the control and to prove stability for the closed-loop nonlinear system. It has been shown in Refs. [6,12,13] that the Lyapunov direct method provides a unified framework by which various controls such as continuous robust control and sliding model control can be devised. Other design methods such as feedback linearization, recursive design, periodic control, and averaging technique can be used (whenever applicable) to generate Lyapunov functions and thus can be embedded into the Lyapunov framework.

This paper is organized as follows. In Section 2, a mathematical formulation of the control problem is given. In Section 3, a robust control is proposed, and stability is analyzed using the Lyapunov direct method. Numerical simulation results are presented in Section 4, and effectiveness of the proposed control is demonstrated. Section 5 contains several conclusions.

2. Problem formulation

The parallel AC/DC system under consideration is shown in Fig. 1, and it can be described by the following model:

$$\dot{\delta} = \omega, \quad \dot{\omega} = \frac{\omega_0}{H}\left(P_{\mathrm{m}} - \frac{D}{\omega_0}\omega - P_{\mathrm{e}}\right), \tag{1}$$

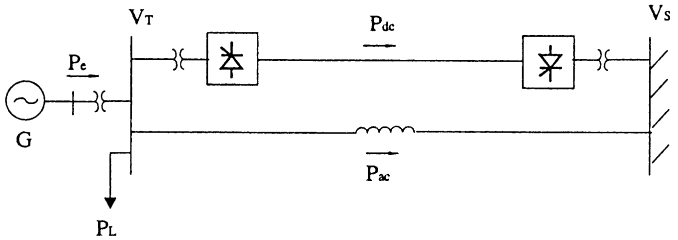

Fig. 1. A parallel AC/DC power system.

where δ is rotor angle, ω is the angular speed, ω_0 is a pre-specified steady-state angular speed, D is the generator damping coefficient, P_m is the mechanical input of the generator, P_e is the generator electric power output given by

$$P_e = P_L + P_{dc} + P_{ac}, \quad \text{and} \quad P_{ac} = \frac{V_T V_s}{x_L} \sin\theta. \tag{2}$$

In Ref. [2], P_{dc} is the active power on the DC line, P_{ac} is the active power on the AC line, P_L is the total of local loads, V_T is the generator terminal voltage magnitude, θ is the angle of terminal voltage, V_s is the voltage of the infinite bus (of angle 0°), and x_L is the equivalent reactance of the AC transmission line. The parallel AC/DC power system is nonlinear, and it contains uncertainties. Specifically, the value of x_L may be changed due to a disturbance; for instance, a part of parallel AC line may becomes isolated as a result of clearing a fault.

Now, let state variables x_1 and x_2 be defined as $x_1 = \delta^d - \delta$ and $x_2 = \omega^d - \omega$, where δ^d and ω^d are the desired values of δ and ω, respectively. Obviously, one should choose $\omega^d = 0$. It follows from Eq. (1) that the error system for the parallel AC/DC power system is

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = \frac{1}{M}[-D'x_2 - P_m + P_e] \tag{3}$$

with $M = H/2\pi f_0 = H/\omega_0$ and $D' = D/\omega_0$.

To study transient stability of the AC/DC system, dynamic of the DC transmission line are usually ignored. That is, the DC power P_{dc} is considered to be the control variable in the system. Furthermore, it can be assumed that, during the transient period, P_m and P_L in Eq. (2) are constants. Consequently, the steady-state operating point of system (1) or (3) are the solutions ω^r, P^r_{dc}, V^r_T and θ^r to the algebraic equations: given a fixed topology,

$$\omega^r = 0, \quad P_m - D'\omega^r - P_L - P^r_{dc} - P^r_{ac} = 0, \quad P^r_{ac} = \frac{V^r_T V_s}{x_L} \sin\theta^r. \tag{4}$$

Therefore, error system (3) can be rewritten as

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = \frac{1}{M}[-D'x_2 - (P^r_{ac} - P_{ac}) - (P^r_{dc} - P_{dc})]. \tag{5}$$

It is worth pointing out that, as shown in Eq. (4), the equilibrium point of the system after a disturbance may not be the pre-fault one or known a priori because of the unpredictable nature of the fault and the subsequent topology change of the system. Nevertheless, the post-fault steady state is completely controllable. DC power P^r_{dc} at any steady state can be measured although one may not be able to pre-calculate it from Eq. (4). Therefore, we can adjust the system steady state by measuring and then adjusting P^r_{dc} to a desired preset value, P^d_{dc}. This kind of adjustment is routine in operating power systems, therefore $P^r_{dc} = P^d_{dc}$ can be assumed. In the next section, we will focus upon the design of transient (and hence incremental) control. Using the Lyapunov direct method, we will search for a control that makes error system (5) robustly stable.

3. Nonlinear robust DC control design

Let the transient control variable be

$$u = P^r_{\mathrm{dc}} - P_{\mathrm{dc}} = P^{\mathrm{d}}_{\mathrm{dc}} - P_{\mathrm{dc}}, \tag{6}$$

where $P^{\mathrm{d}}_{\mathrm{dc}}$ is the desired steady state DC power. The control law for u is to be described shortly. Using the definition of u, error system (5) can be expressed as

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = \frac{1}{M}[-D'x_2 - u - f(x,u)], \tag{7}$$

where $f(x,u) \triangleq P^r_{\mathrm{ac}} - P_{\mathrm{ac}}$ is the lumped nonlinearity term given by

$$\begin{aligned} f(x,u) &= P^r_{\mathrm{ac}} - P_{\mathrm{ac}} \\ &= \frac{V^r_{\mathrm{T}} V_{\mathrm{s}}}{x_{\mathrm{L}}}(\sin\theta^r - \sin\theta) + \left(\frac{V^r_{\mathrm{T}} V_{\mathrm{s}}}{x_{\mathrm{L}}} - \frac{V_{\mathrm{T}} V_{\mathrm{s}}}{x_{\mathrm{L}}}\right)\sin\theta \\ &= \frac{V^r_{\mathrm{T}} V_{\mathrm{s}}}{x_{\mathrm{L}}} 2\cos\left(\frac{\theta^r + \theta}{2}\right)\sin\left(\frac{\theta^r - \theta}{2}\right) + \frac{(V^r_{\mathrm{T}} - V_{\mathrm{T}})V_{\mathrm{s}}}{x_{\mathrm{L}}}\sin\theta. \end{aligned} \tag{8}$$

It is shown in the Appendix A.2 that V_{T} is bounded as

$$V_{\mathrm{T}} \leqslant c_1 + c_2\sqrt{c_3 + c_4|u|}. \tag{9}$$

Evaluating the magnitude of Eq. (8) and then substituting Eq. (9) yield

$$\begin{aligned} |f(x,u)| &\leqslant 2\frac{V^r_{\mathrm{T}} V_{\mathrm{s}}}{x_{\mathrm{L}}} + \frac{V^r V_{\mathrm{s}}}{x_{\mathrm{L}}} + \frac{V_{\mathrm{T}} V_{\mathrm{s}}}{x_{\mathrm{L}}} \\ &\leqslant 3\frac{V^r_{\mathrm{T}} V_{\mathrm{s}}}{x_{\mathrm{L}}} + \frac{V_{\mathrm{s}}}{x_{\mathrm{L}}}\left(c_1 + c_2\sqrt{c_3 + c_4|u|}\right) \\ &\leqslant \eta_1 + \eta_2\sqrt{\eta_3 + \eta_4|u|}, \end{aligned} \tag{10}$$

where η_i are constants given by

$$\eta_1 = 3\frac{V^r_{\mathrm{T}} V_{\mathrm{s}}}{x_{\mathrm{L}}} + c_1\frac{V_{\mathrm{s}}}{x_{\mathrm{L}}}, \quad \eta_2 = c_2\frac{V_{\mathrm{s}}}{x_{\mathrm{L}}}, \quad \eta_3 = c_3, \quad \text{and} \quad \eta_4 = c_4.$$

Upon having the bounding function in Eq. (10) on nonlinear uncertainty $f(x,u)$, the following theorem can be concluded as the main result of this paper.

Theorem. *The error system* (5) *is globally and asymptotically stable under the control*:

$$u = -\mathrm{sign}(x_1 + \lambda_2 x_2)v = -\mathrm{sign}(x_1 + \lambda_2 x_2)\frac{w^2(x_1) - \eta_3}{\eta_4}, \tag{11}$$

where constant λ and function $w(x_1)$ are chosen such that

$$\lambda_2 > \frac{M}{D}, \quad \textit{and} \quad w(x_1) \geqslant \frac{1}{2}\left[\eta_2\eta_4 + \sqrt{(\eta_2\eta_4)^2 + 4\left(\eta_3 + \eta_1\eta_4 + \frac{M\eta_4}{\lambda_2}|x_1|\right)}\right]. \tag{12}$$

That is,

$$\lim_{t\to\infty}\delta = \delta^{\mathrm{d}}, \quad \textit{and} \quad \lim_{t\to\infty}\omega = 0.$$

Proof. Consider the Lyapunov function candidate

$$V = \frac{1}{2}\left[\lambda_1 x_1^2 + (x_1 + \lambda_2 x_2)^2\right],$$

where $\lambda_i > 0$ are positive constants to be chosen. Then,

$$\begin{aligned}
\dot{V} &= \lambda_1 x_1 \dot{x}_1 + (x_1 + \lambda_2 x_2)(\dot{x}_1 + \lambda_2 \dot{x}_2) \\
&= (1+\lambda_1)x_1\dot{x}_1 + \lambda_2 x_1 \dot{x}_2 + \lambda_2 x_2 \dot{x}_1 + \lambda_2^2 x_2 \dot{x}_2 \\
&= (1+\lambda_1)x_1 x_2 + \lambda_2 x_1 \frac{1}{M}[-Dx_2 - u - f(x,u)] + \lambda_2 x_2^2 + \lambda_2^2 x_2 \frac{1}{M}[-Dx_2 - u - f(x,u)] \\
&= \left(1 + \lambda_1 - \frac{\lambda_2 D}{M}\right)x_1 x_2 - \lambda_2\left(\frac{\lambda_2 D}{M} - 1\right)x_2^2 - \frac{\lambda_2}{M}(x_1 + \lambda_2 x_2)u - \frac{\lambda_2}{M}(x_1 + \lambda_2 x_2)f(x,u).
\end{aligned}$$

Choosing

$$\lambda_1 = \lambda_2 + \frac{\lambda_2 D}{M} - 1,$$

we have

$$\begin{aligned}
\dot{V} &= \lambda_2 x_1 x_2 - \lambda_2\left(\frac{\lambda_2 D}{M} - 1\right)x_2^2 - \frac{\lambda_2}{M}(x_1 + \lambda_2 x_2)u - \frac{\lambda_2}{M}(x_1 + \lambda_2 x_2)f(x,u) \\
&= -x_1^2 + x_1(x_1 + \lambda_2 x_2) - \lambda_2\left(\frac{\lambda_2 D}{M} - 1\right)x_2^2 - \frac{\lambda_2}{M}(x_1 + \lambda_2 x_2)u - \frac{\lambda_2}{M}(x_1 + \lambda_2 x_2)f(x,u).
\end{aligned}$$

Substituting control law (11) yields

$$\begin{aligned}
\dot{V} &\leqslant -x_1^2 - \lambda_2\left(\frac{\lambda_2 D}{M} - 1\right)x_2^2 + |x_1||x_1 + \lambda_2 x_2| - \frac{\lambda_2}{M}|x_1 + \lambda_2 x_2|v \\
&\quad + \frac{\lambda_2}{M}|x_1 + \lambda_2 x_2|\left(\eta_1 + \eta_2\sqrt{\eta_3 + \eta_4|v|}\right) \\
&\leqslant -x_1^2 - \lambda_2\left(\frac{\lambda_2 D}{M} - 1\right)x_2^2 + |x_1 + \lambda_2 x_2|\left[|x_1| - \frac{\lambda_2}{M}\left(\frac{w^2 - \eta_3}{\eta_4}\right) + \frac{\lambda_2}{M}(\eta_1 + \eta_2 w)\right] \\
&= -x_1^2 - \lambda_2\left(\frac{\lambda_2 D}{M} - 1\right)x_2^2 - \frac{\lambda_2|x_1 + \lambda_2 x_2|}{M\eta_4}\left[w^2 - \eta_2\eta_4 w - \left(\eta_3 + \eta_1\eta_4 + \frac{M\eta_4}{\lambda_2}|x_1|\right)\right].
\end{aligned}$$

It becomes obvious that $\dot{V}$ is negative definite if

$$\lambda_2 > 0, \quad \frac{\lambda_2 D}{M} - 1 > 0, \quad w^2 - \eta_2\eta_4 w - \left(\eta_3 + \eta_1\eta_4 + \frac{M\eta_4}{\lambda_2}|x_1|\right) \geqslant 0.$$

The above inequalities hold if λ_2 and $w(x_1)$ are chosen according to Eq. (12). Since V is positive definite and $\dot{V}$ is negative definite, global and asymptotic stability can be concluded [6]. □

Remark 3.1. Given various scenarios of faults, constant bounds η_1, η_2, η_3, and η_4 in Eq. (12) can be calculated using the ranges of system parameters. Similarly $w(x_1)$ can be found according to Eq. (12).

Remark 3.2. Due to the operation of taking norm (or magnitude), application of the Lyapunov direct method often produces a conservative result. This enhances robust stability, but the actual control gains for implementation could be made smaller. In other words, less-conservative values of the gains can be found by simulation or by using an optimization technique.

Remark 3.3. It follows from Eqs. (10) and (12) that control parameters in the proposed control (11) can be selected provided that, in the presence of perturbations, upper bounds on system parameters are known. In the case that upper bounds are not unknown, it has been shown in Ref. [13] that an adaptive control algorithm can be introduced into the proposed control to estimate the upper bounds. Thus, the proposed control is robust with respect to parameter perturbations.

Remark 3.4. The proposed control (11) happens to be of switching type and hence may be chattering under some operating conditions. In case that there is a limit of bandwidth on switching frequency, one can modify the control to be of saturation type by trading off asymptotic stability. See chapter 4 in Ref. [13] and references therein.

4. Simulations

To carry out quantitative performance evaluation of the proposed control, the AC/DC parallel system in Fig. 1 is simulated with the following parameters.

D	2.5	H	10.0
ω_0	376.99	x_T	0.12
x_L	0.8	x_d	0.2
E_q	1.25	P_m	0.9
P_L	0.1	V_s	$1\angle 0$

In the simulation, two disturbance scenarios are tested:

- *Case 1:* A three phase short circuit fault occurs on the AC transmission line and near the infinite bus, and it is cleared after five cycles (about 83 ms).
- *Case 2:* A three phase short circuit fault occurs on the AC transmission line and near the generator bus. The fault is isolated after five cycles by cutting off one of the parallel AC lines. It is assumed that the equivalent AC line reactance after the fault is changed from 0.8 to 1.2.

In the first case, the fault only causes a temporary short circuit, and system parameters are not affected by the fault. Hence, the post-fault steady state of error system (5) will be the pre-fault stable equilibrium point if the desired values δ^d and P_{dc} are kept unchanged. In other words, the proposed control will stabilize the system and bring it back to its previous steady state. In the second case, one of the system parameters has been changed after the fault. Therefore, the post-fault equilibrium point is no longer the pre-fault steady state unless the steady state of post-fault DC line power P_{dc} is reset properly.

Open-loop transient responses of the system (without the proposed control) for cases 1 and 2 are depicted in Figs. 2 and 3, respectively. It is clear that there exist sustained oscillations (for the generator angle) after the faults. As expected, the generator angle in Fig. 2 converges slowly to the pre-fault steady state angle; and in Fig. 3 the machine angle drifts away from its pre-fault value.

To simulate the proposed control in a practical setting, magnitude and bandwidth saturations should be considered. In what follows, a saturation function on magnitude is introduced as $|u| < u_{max}$ where u_{max} is a constant size limit on control signal u. Similarly, a rate limit can be imposed by introducing a saturation function and an integrator, in which case the robust control should be redesigned using the recursive design (i.e., the backstepping design). The latter is omitted for briefness, and interested readers are referred to Ref. [13] and the references therein.

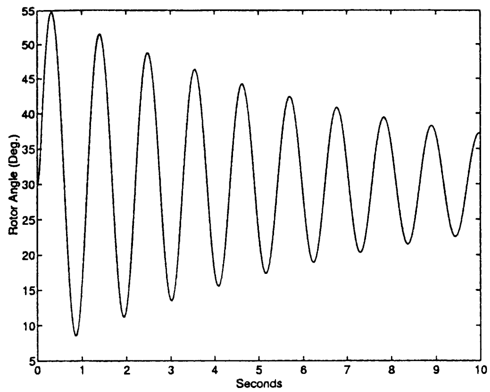

Fig. 2. Generator angle under the case 1 fault and without the proposed control.

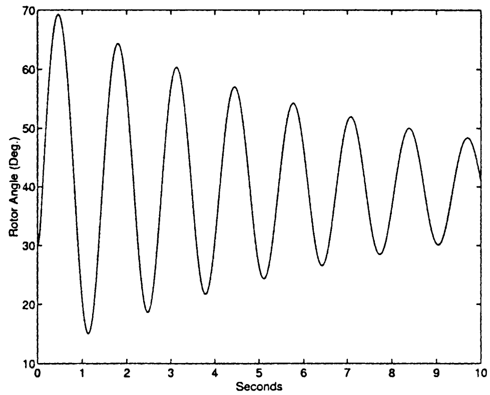

Fig. 3. Generator angle under the case 2 fault and without the proposed control.

In getting the second sets of simulation results, size limit u_{max} is set to be 0.1. Figs. 4 and 5 illustrate the generator angle and the DC power P_{dc} under the case 1 fault; and Figs. 6 and 7 show the system dynamic response under the case 2 fault. In both simulation runs, the pre-fault DC line power P^o_{dc} is chosen to be $P^d_{dc} = 0.4$ in order to minimize the change of DC line power and to keep the pre-fault generator angle δ^o at its desired value $\delta^d = 30°$.

It can be easily observed that the oscillations decay quickly under the proposed control. Furthermore, the oscillation amplitude during the first half cycle is also decreased, which allows more real power transmission. In Fig. 4, the generator angle converges back to its pre-fault value, and in Fig. 5 the DC line power also goes back to its pre-fault value. For the case 2 fault, parameter variation in the system changes the equilibrium state. It is clear from Fig. 8 that the bus voltage approaches a new steady state. In the control algorithm, P^d_{dc} is chosen to remain its pre-fault value. Unless P^d_{dc} is adjusted, P^r_{dc} shown in Fig. 7 is different from the pre-fault value. As shown in Fig. 6, the oscillation has been largely suppressed but there are small ripples in the machine angle for quite a while, which is due to the fact that the control has reached its magnitude limit and hence become less effective.

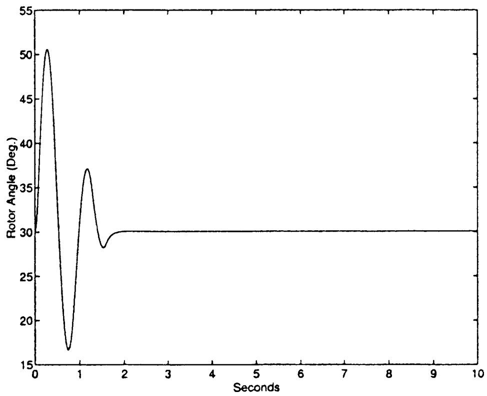

Fig. 4. Generator angle under the case 1 fault and constraint $u_{max} = 0.1$.

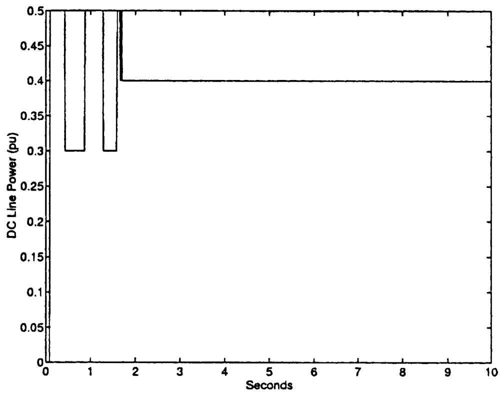

Fig. 5. DC line power P_{dc} under the case 1 fault and constraint $u_{max} = 0.1$.

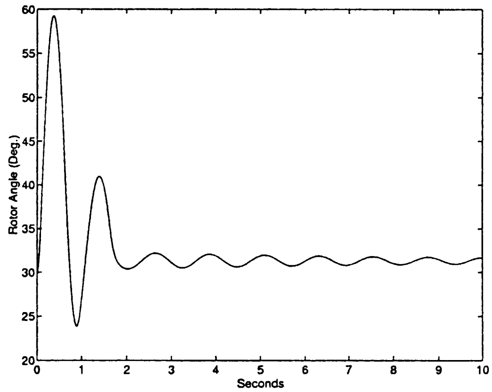

Fig. 6. Generator angle under the case 2 fault and constraint $u_{max} = 0.1$.

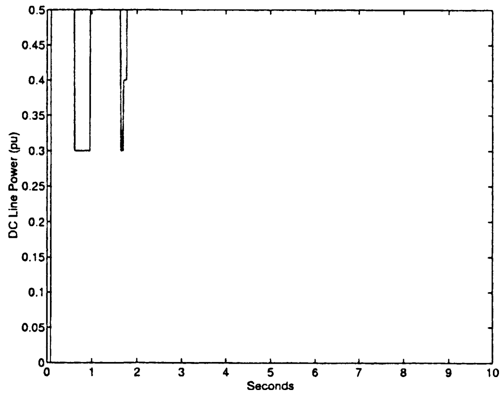

Fig. 7. DC line power P_{dc} under the case 2 fault and constraint $u_{max} = 0.1$.

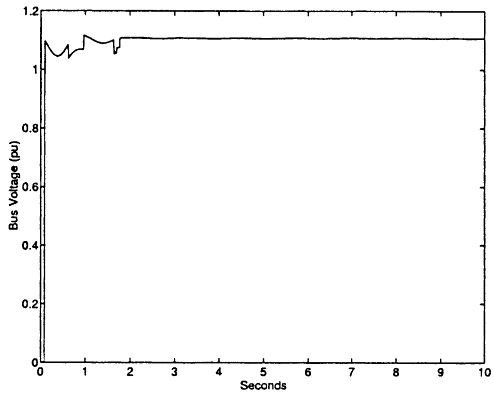

Fig. 8. The terminal bus voltage V_T under the case 2 fault and constraint $u_{max} = 0.1$.

To further illustrate the relationship between magnitude limit and system response, the proposed control is also simulated under the constraint $u_{max} = 0.15$, and the results are shown in Figs. 9–12. For the case 1 fault, the pre-fault values δ^o and P^o_{dc} are chosen to be the desired values. But, for the case 2 fault, new values $\delta^r = 40°$ and $P^r_{dc} = 0.35$ are selected in order to provide the operation range for P_{dc}. In general, the system dynamic responses under $u_{max} = 0.15$ are better than those under $u_{max} = 0.1$. This is also expected as, the more the DC line power can be adjusted according to the proposed control algorithm, the better the system behavior becomes. With recent developments of HVDC electronics and with improvements of HVDC control devices, the proposed control can be practically implemented to have a significant impact.

As before, the angle and the DC power in Figs. 9 and 10 converge to their pre-fault values, respectively. In comparison, the angle and DC line power in Figs. 11 and 12 converge to the new values. Finally, Fig. 13 shows that the amplitude of the terminal bus also converges to a new value.

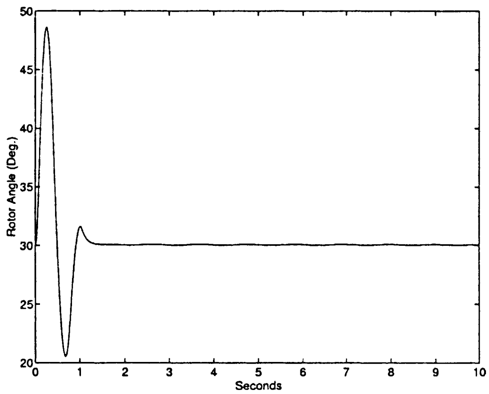

Fig. 9. Generator angle under the case 1 fault and constraint $u_{max} = 0.15$.

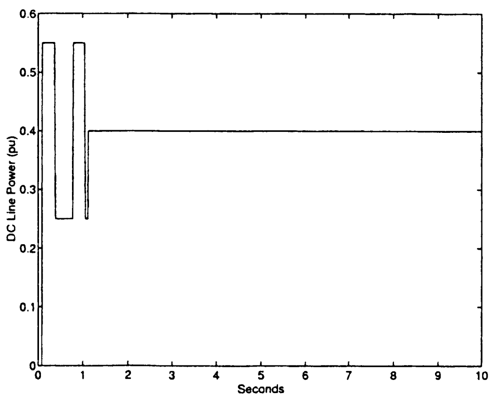

Fig. 10. DC line power P_{dc} under the case 1 fault and constraint $u_{max} = 0.15$.

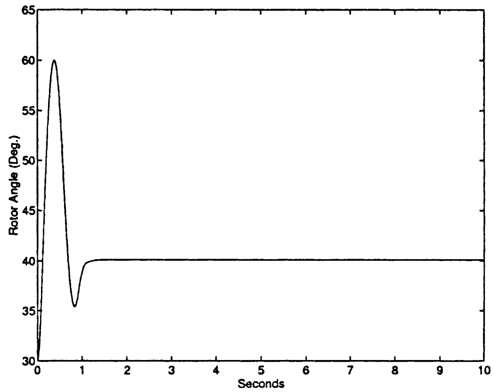

Fig. 11. Generator angle under the case 2 fault and constraint $u_{max} = 0.15$.

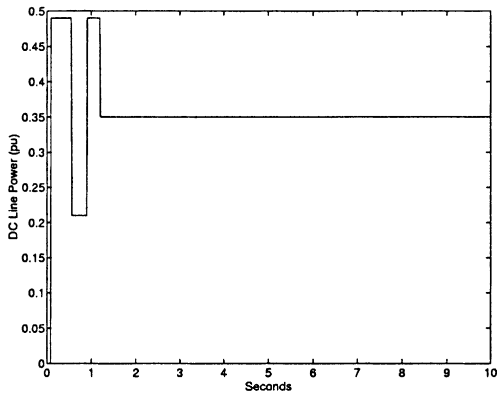

Fig. 12. DC line power P_{dc} under the case 2 fault and constraint $u_{max} = 0.15$.

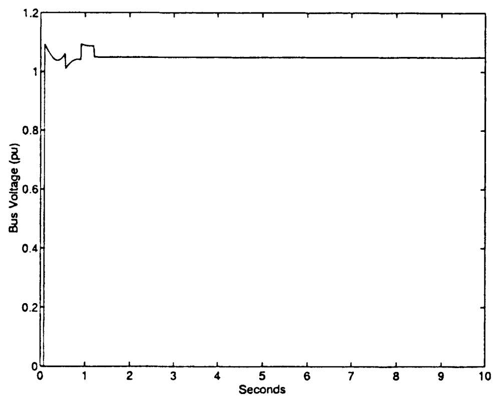

Fig. 13. The terminal bus voltage V_T under the case 2 fault and constraint $u_{max} = 0.15$.

5. Conclusion

A nonlinear robust control is proposed for a parallel AC/DC power system. The stability proof by Lyapunov direct method shows that global and asymptotic stability can be achieved under any disturbance occurred anywhere in the system. The proposed control is robust against both faults and parameter variations. Simulation results confirm the analytic conclusions and demonstrate effectiveness of the control and significant improvement in system dynamic responses. Magnitude saturation is also considered in simulation.

Appendix A

A.1. Relationships between AC and DC quantities

Symbols used:

E_{LL} – rms line–line voltage of the converter ac bus.
I_1 – rms value of the fundamental frequency component of the converter ac current.
α – valve firing delay angle (from the instant that the valve voltage is positive).
μ – overlap angle (also called commutation angle).
ϕ – phase angle between V and I.
$\cos\phi$ – displacement power factor.
V_{d0} – ideal no-load dc voltage (when $\alpha = 0$ and $\mu = 0$).

Voltages and currents on ac and dc sides of the converter are related, and they are functions of several converter parameters including the converter transformer. Detailed derivations are given in Ref. [7]. The following expressions on voltages and currents are included here for easy reference.

When $\alpha = 0$ and $\mu = 0$,

$$V_{\mathrm{d}} = V_{\mathrm{d0}} = \frac{3\sqrt{2}}{\pi} E_{\mathrm{LL}} \approx 1.35 E_{\mathrm{LL}}.$$

When $\alpha > 0$ and $\mu = 0$,

$$V_{\mathrm{d}} = V_{\mathrm{d0}} \cos\alpha.$$

In theory, $\alpha \in [0, 180°]$ (when $\mu = 0$). Hence, V_{d} can vary from $+V_{\mathrm{d0}}$ to $-V_{\mathrm{d0}}$. It follows that

$$I_1 = \frac{\sqrt{6}}{\pi} I_{\mathrm{d}} = 0.78 I_{\mathrm{d}}, \quad \text{and} \quad \cos\phi = \cos\alpha = \frac{V_{\mathrm{d}}}{V_{\mathrm{d0}}}.$$

When $\alpha = 0$ and $0 < \mu = 60°$,

$$V_{\mathrm{d}} = V_{\mathrm{d0}} \frac{\cos\alpha + \cos(\alpha + \mu)}{2} = \frac{3\sqrt{2}}{\pi} E_{\mathrm{LL}} \frac{\cos\alpha + \cos(\alpha + \mu)}{2} \tag{A.1}$$

and

$$I_1 \approx \frac{\sqrt{6}}{\pi} I_d = 0.78 I_d. \tag{A.2}$$

The approximation error in Eq. (A.2) is 4.3% at $\mu = 60°$ (the maximum overlap angle for normal steady-state operation), and it will be smaller (around 1.1%) for most practical cases when μ is 30° or less. It can be seen from Eqs. (A.1) and (A.2) that the ratio between ac and dc currents is almost fixed, but the ratio between ac and dc voltages varies as a function of α and μ. Hence, the HVDC converter can be viewed as a variable-ratio voltage transformer, with an almost fixed current ratio. On the other hand, we have

$$P_{dc} = V_d I_d, \tag{A.3}$$

$$P_{ac} = \sqrt{3} E_{LL} I_1 \cos\phi. \tag{A.4}$$

Substituting the expressions of V_d and I_d into Eq. (A.3) and comparing the result with Eq. (A.4), we know

$$\cos\phi \approx \frac{\cos\alpha + \cos(\alpha+\mu)}{2}.$$

It follows from Eq. (A.1) that

$$\cos\phi \approx \frac{V_d}{V_{d0}}.$$

Therefore, we obtain

$$V_d \approx 1.35 E_{LL} \cos\phi, \quad \text{and} \quad Q_{ac} = \sqrt{3} E_{LL} I_1 \sin\phi.$$

A.2. Derivation of bounding function in Eq. (9)

The complex current on the AC transmission line is given by

$$\mathbf{I_{ac}} = \frac{V_T\angle\theta - V_s}{jx_L},$$

and the current supplied to the DC line can be expressed as [1,7]

$$\begin{aligned}\mathbf{I_{dc}} &= \left(\frac{\mathbf{S_{dc}}}{V_T\angle\theta}\right)^* \\ &= \left(\frac{jQ_{dc}}{V_T\angle\theta}\right)^* + \left(\frac{P_{dc}}{V_T\angle\theta}\right)^* \\ &= \left(\frac{j0.78\sqrt{3} I_d V_T \sin\phi}{V_T\angle\theta}\right)^* + \left(\frac{P_{dc}}{V_T\angle\theta}\right)^*,\end{aligned} \tag{A.5}$$

where Q_{dc} is the reactive power consumed by rectifier/inverter, I_d is the DC current, and ϕ is the displacement power angle. Under the constant current control mode, which is usual mode of operation of the rectifier/inverter and is also the control mode used for the stability control, I_d is fixed to a preset value, say, I_d^o. Therefore, Eq. (A.5) can be written as

$$\mathbf{I}_{dc} = \left(\frac{j0.78\sqrt{3}I_d^o V_T \sin\phi}{V_T\angle\theta}\right)^* + \left(\frac{P_{dc}}{V_T\angle\theta}\right)^*.$$

Similarly, the current going to the local load is given by

$$\mathbf{I_L} = \left(\frac{P_L}{V_T\angle\theta}\right)^*.$$

Thus, the complex terminal voltage $\mathbf{V_T}$ can be calculated by

$$\begin{aligned}\mathbf{V_T} &= V_T\angle\theta = E\angle\delta - (\mathbf{I_{ac}} + \mathbf{I_{dc}} + \mathbf{I_L})jx_T \\ &= E\angle\delta - \frac{x_T}{x_L}(V_T\angle\theta - V_s) - jx_T\left(\frac{j0.78\sqrt{3}I_d^o V_T \sin\phi}{V_T\angle\theta}\right)^* - jx_T\left(\frac{P_{dc}}{V_T\angle\theta}\right)^* - jx_T\left(\frac{P_L}{V_T\angle\theta}\right)^*.\end{aligned}$$

Solving for $V_T\angle\theta$ yields

$$V_T\angle\theta\left(1+\frac{x_T}{x_L}\right) = E\angle\delta + \frac{x_T}{x_L}V_s - jx_T\left(\frac{j0.78\sqrt{3}I_d^o V_T \sin\phi}{V_T\angle\theta}\right)^* - jx_T\left(\frac{P_{dc}}{V_T\angle\theta}\right)^* - jx_T\left(\frac{P_L}{V_T\angle\theta}\right)^*.$$

Taking magnitude on both sides of the above equation, we have

$$\begin{aligned}&V_T\left(1+\frac{x_T}{x_L}\right) \\ &\quad= \left|E\angle\delta + \frac{x_T}{x_L}V_s - jx_T\left(\frac{j0.78\sqrt{3}I_d^o V_T \sin\phi}{V_T\angle\theta}\right)^* - jx_T\left(\frac{P_{dc}}{V_T\angle\theta}\right)^* - jx_T\left(\frac{P_L}{V_T\angle\theta}\right)^*\right| \\ &\quad\leqslant E + \frac{x_T}{x_L}V_s + 0.78\sqrt{3}I_d^o x_T + x_T\frac{(|P_{dc}| + P_L)}{V_T}.\end{aligned}$$

Multiplying V_T on both sides and rearranging the inequality yield

$$V_T^2\left(1+\frac{x_T}{x_L}\right) - \left(E + \frac{x_T}{x_L}V_s + 0.78\sqrt{3}I_d^o x_T\right)V_T - x_T(|P_{dc}| + P_L) \leqslant 0.$$

Solving the inequality, we can obtain the bounding condition on V_T as

$$\begin{aligned} V_{\mathrm{T}} &\leqslant \frac{E + \frac{x_{\mathrm{T}}}{x_{\mathrm{L}}} V_{\mathrm{s}} + 0.78\sqrt{3} I_{\mathrm{d}}^{o} x_{\mathrm{T}}}{2\left(1 + \frac{x_{\mathrm{T}}}{x_{\mathrm{L}}}\right)} + \frac{\sqrt{\left(E + \frac{x_{\mathrm{T}}}{x_{\mathrm{L}}} V_{\mathrm{s}} + 0.78\sqrt{3} I_{\mathrm{d}}^{o} x_{\mathrm{T}}\right)^{2} + 4\left(1 + \frac{x_{\mathrm{T}}}{x_{\mathrm{L}}}\right) x_{\mathrm{T}} (|P_{\mathrm{dc}}| + P_{\mathrm{L}})}}{2\left(1 + \frac{x_{\mathrm{T}}}{x_{\mathrm{L}}}\right)} \\ &= c_1 + c_2 \sqrt{c_3 + c_4 |u|}, \end{aligned}$$

where c_i are positive constants defined by

$$c_1 = \frac{E + \frac{x_{\mathrm{T}}}{x_{\mathrm{L}}} V_{\mathrm{s}} + 0.78\sqrt{3} I_{\mathrm{d}}^{o} x_{\mathrm{T}}}{2\left(1 + \frac{x_{\mathrm{T}}}{x_{\mathrm{L}}}\right)}, \quad c_2 = \frac{1}{2\left(1 + \frac{x_{\mathrm{T}}}{x_{\mathrm{L}}}\right)},$$

$$c_3 = \left(E + \frac{x_{\mathrm{T}}}{x_{\mathrm{L}}} V_{\mathrm{s}} + 0.78\sqrt{3} I_{\mathrm{d}}^{o} x_{\mathrm{T}}\right)^{2} + 4\left(1 + \frac{x_{\mathrm{T}}}{x_{\mathrm{L}}}\right) x_{\mathrm{T}} (|P_{\mathrm{dc}}^{r}| + P_{\mathrm{L}}), \quad \text{and} \quad c_4 = 4\left(1 + \frac{x_{\mathrm{T}}}{x_{\mathrm{L}}}\right) x_{\mathrm{T}}.$$

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