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Loadability of power systems with steady-state and dynamic security constraints☆

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Abstract

Estimating loadability of a generation and transmission system is of practical importance in power system operations and planning. This paper presents a new formulation for the problem using mathematical programming theory. Both steady-state and dynamic security are taken into account in the proposed formulation. The difference between the proposed formulation and existing ones is that dynamic security is handled by an integration method. Using the new formulation, an iterative solution procedure is developed to solve the corresponding mathematical programming problem numerically. The method normally yields a slightly conservative estimate on the loadability of a generation/transmission system. Simulation results of a test power system are provided. © 2002 Elsevier Science Ltd. All rights reserved.

Keywords: Power systems; Loadability; Dynamic security; Steady-state security

1. Introduction

Construction and enhancement of generation/transmission systems generally require huge amount of capital investment. Regardless of whether or not capital investment is available for constructing systems and enhancing their capacities, efficient utilization of existing power facilities is always desired for both economical and environmental concerns. In fact, efficient utilization, which can be divided into such issues as optimal system scheduling or optimal facility maintenance, becomes increasingly important for the utilities as they face deregulation. On the other hand, optimal facility allocation known as optimal power system planning, is the method addressing how to optimize capital investment while meeting demand and security requirements [15]. To address these two problems, one has to estimate loadability of the transmission system under consideration.

Garver and Horne [9] proposed a method to compute loadability of generation/transmission networks based on linear programming. The method presented in Ref. [13] takes stability limit into account through the use of energy function method. Recently, the relationship between voltage stability and loadability has been explored in Refs. [1,10,14]. Conceptually, estimating loadability of a generation/transmission system is a generalized mathematical programming problem. It is not a standard mathematical programming problem because some of the constraints (specifically, dynamic security constraints) have to be expressed not in algebraic forms but in the form of differential equations [4,5].

Analytically, estimating the loadability of power systems is somewhat similar to the so-called generation rescheduling problem. However, there are a few important distinctions. First, computational effort of estimating loadability is several times more than that of generation rescheduling. Second, loadability of power systems is dependent upon the pattern of load increasing. In our previous results [5,6], a general framework for generation rescheduling problems is proposed based on mathematical programming theory. In this paper, the iterative procedure proposed in Refs. [5,6] is modified to estimate the loadability of power systems so that loadability of power systems can be solved using the mathematical programming method. To reduce the computational effort, we propose here the use of pseudo-inverse based security analysis in dealing with thermal limits under normal operations (i.e. steady-state security), while dynamic security (i.e. whether or not the power system under study remains to be stable after suffering from a major disturbance) is checked by fast integration combined with an automatic contingency selection. The method takes into account the effects of both steady-state and dynamic security, and the proposed algorithm can easily be applied to take into account a number of line contingencies.

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Nomenclature

P_{mi}, Q_{mi} real and reactive power generation by the ith generator
Q^0_{mi}, P^0_{Li} and Q^0_{Li} initial values of reactive power generation, active power load, and reactive power load at the ith bus, respectively
$\delta_i(t)$, ω_i, M_i, P_{gi} rotor angle, angular velocity, inertial constant, transient electrical power output of the ith machine, respectively
T_s time span of transient response considered
$\theta_i(t)$, V_i, θ, V angle at the ith bus, voltage magnitude at the ith bus, and column vectors formed by elements θ_i and V_i, respectively
G_{ij}, B_{ij} conductance and susceptance of the transmission line between the ith and jth buses
E_i voltage of the ith machine behind its transient reactance
g_{ij}, b_{ij} conductance and susceptance of the reduced admittance matrix corresponding to the ith and jth machines
$\bar{\delta}$ the maximum limit on relative swing angle allowed between any pair of machines (for instance, 180°)
$P_i(V,\theta)$, $Q_i(V,\theta)$ active and reactive power injections at the ith bus, respectively
$\underline{P}_{mi}$, $\underline{Q}_{mi}$ lower limits of P_{mi} and Q_{mi}, respectively
$\bar{P}_{mi}$, $\bar{Q}_{mi}$ upper limits of P_{mi} and Q_{mi}, respectively
T_i, $\bar{T}_i$ active power through the ith transmission line, and its thermal limit
N_B, N_L, N_g, N_c total number of buses, total number of transmission lines, total number of generators, and total number of contingencies under consideration, respectively
$\delta^l_i(t)$ rotor angle of the ith machine at time t under the lth contingency

Simulation results of a 6-machine 22-node test power system are reported.

2. Mathematical formulation

In this section, the problem of estimating loadability of transmission systems is formulated using the terminology of mathematical programming theory. We begin with two remarks pertaining to the new formulation. The first one is about an assumption on changes of active and reactive power. It is assumed that the changes of active and reactive power at each power station are proportional to each other in the proposed formulation. While this assumption is not necessarily required (as one may simply impose any other rule on changes of power generation), such an assumption is made for the ease of presentation. The second one is about numerical algorithm. Although the proposed formulation is given in the form of mathematical programming, typical algorithms such as simplex algorithm and steepest descent algorithm, etc. cannot be used to solve the problem. This is because the formulation, as will be discussed later, is much more complicated than conventional mathematical programming models. Consequently, heuristics-based algorithms will have to be developed to solve the formulation. Now we are in a position to present the formulation.

Objective:

$$\text{Max } \alpha \tag{1}$$

subject to the following constraints:

$$P_i(V,\theta) + S_i(P_g) - \alpha P^0_{Li} = 0, \qquad i = 1,2,\ldots,N_B, \tag{2}$$

$$Q_i(V,\theta) + W_i\left(\alpha, S_i\left(Q^0_m\right)\right) - \alpha Q^0_{Li} = 0, \qquad i = 1,2,\ldots,N_B, \tag{3}$$

$$\underline{P}_{mi} \le P_{mi} \le \bar{P}_{mi}, \qquad i = 1,2,\ldots,N_g, \tag{4}$$

$$\underline{Q}_{mi} \le Q_{mi} \le \bar{Q}_{mi}, \qquad i = 1,2,\ldots,N_g, \tag{5}$$

$$-\bar{T}_i \le T_i \le \bar{T}_i, \qquad i = 1,2,\ldots,N_L, \tag{6}$$

and

$$\left|\delta^l_i(t) - \delta^l_j(t)\right| \le \bar{\delta}, \quad t = [0, T_s], \; i,j = 1,2,\ldots,N_g, \; l = 1,2,\ldots,N_c, \tag{7}$$

where

$$S_i(P_g) = \begin{cases} 0 & \text{if no machine is attached to bus } i \\ P_{gj} & \text{if the } j\text{th machine is attached to bus } i \text{ for some } j \in \{0,\ldots,N_g\} \end{cases},$$

function $S_i(Q^0_m)$ is defined in the same way, $W_i(\cdot)$ is a user-defined function to specify the change of reactive power generation:

$$P_i(V,\theta) = \sum_{j=1}^{N_B} V_i V_j [G_{ij}\cos(\theta_i - \theta_j) + B_{ij}\sin(\theta_i - \theta_j)], \tag{8}$$

$$Q_i(V,\theta) = \sum_{j=1}^{N_B} V_i V_j [G_{ij}\sin(\theta_i - \theta_j) - B_{ij}\cos(\theta_i - \theta_j)]. \tag{9}$$

δ_i is the solution to the following second-order ordinary differential equation:

$$\frac{d\delta_i}{dt} = \omega_i, \quad M_i \frac{d\omega_i}{dt} = P_{mi} - P_{gi}, \qquad i = 1, 2, \ldots, N_g, \quad (10)$$

and

$$P_{gi} = \sum_{j=1}^{N_G} E_i E_j [g_{ij} \cos(\delta_i - \delta_j) + b_{ij} \sin(\delta_i - \delta_j)]. \qquad (11)$$

All symbols in Eqs. (2)–(11) are defined in the nomenclature. More detailed explanations of the above expressions and their parameters can be found in Refs. [11,12,16].

In the problem statement (1), α is the percentage of the overall system real load versus its initial load value. Quantity α is referred to as loadability factor, and it is also the objective function of the programming problem. In the formulation, α and P_{mi} are adjustable variables. During the transient period, electrical power P_{gi} changes, while mechanical power P_{mi} does not. The objective of mathematical programming is to find the largest α and the corresponding mechanical power P_{mi}, $i = 1, 2, \ldots, N_g$, such that various steady-state and dynamic constraints are satisfied.

In this paper, both steady-state and dynamic security issues are considered. Inequalities (2)–(6) are thermal limits and form the so-called *steady-state security constraints*. Additional steady-state constraints such as line contingencies can be added, and inequality (7) together with Eq. (10) is a *dynamic security constraint*. Constraints (2)–(4), (6) and (7) are imposed in our study, but constraint (5) is not enforced as reactive power is little related to transient stability. Should single-axis and/or two-axis machine models be used, swing equations (10) and (11) can be changed accordingly. Thus, the proposed formulation is generic in the sense that various extensions can be made.

For multi-machine power systems, there has been no result on how to conclude analytically their exact stability regions. Instead, whether an initial condition is in the stability region can be determined by integrating the system trajectory. Without knowing the stability boundary, one can resort him/herself to a relative stability measure (which may lead to a somewhat conservative result), and inequality (7) is one of such choices. In the proposed formulation, $\bar{\delta}$ is a user-supplied parameter and, when its value is set to be smaller, a more conservative solution will be generated. In the simulation to be presented, $\bar{\delta}$ is set to be 180°.

It should be noted that in the proposed formulation, dynamic security is explicitly considered through a robust procedure. Specifically, dynamic security is checked by the Runge–Kutta method which is most reliable at present. We also employ the so-called sensitivity factor (or PTDF) method to calculate line flows. The exact equation, which is based on linearized AC load flow, is to be stated in the following sections. In addition, the programming problem given by Eqs. (1)–(10) is not solvable using any existing mathematical programming package because of the presence of constraints (7) and (10). To solve this programming problem, one has to rely on engineering judgment. In Section 3, an algorithm based on such an approach is provided.

3. The proposed solution procedure

An overview of the proposed solution procedure is given first, followed by its detailed descriptions.

3.1. Overview of the procedure

It is obvious that the mathematical programming formulation, given by Eqs. (1)–(7), has three major components: the loadability factor and P_{mi}, steady-state security constraints given by Eqs. (2)–(6), and dynamic security constraints given by Eq. (7). The proposed procedure explores the relationship among these three components.

The objective of maximizing loadability factor α and its corresponding active power generation distribution can be met by using an iterative algorithm. The proposed iteration scheme, shown by Fig. 1, consists of two layers. Numerically, it is a two-kernel procedure. The first is *security assessment procedure*, and the second is *generation adjustment procedure*. The first layer is the outer-loop iteration within which loadability factor α is increased each

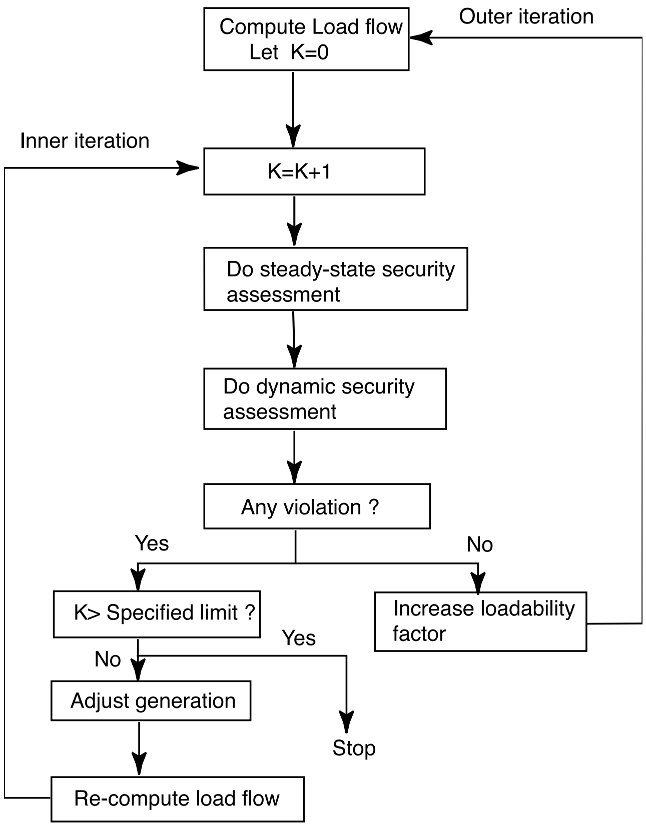

Fig. 1. Flowchart of the solution procedure.

time by a small increment. This layer of iteration is always repeated unless the first kernel fails a number of times consecutively. The second layer is the inner-loop iteration within which distribution of active power generation is adjusted through the second kernel to meet the steady-state and dynamic security constraints specified by the first kernel. Overall, iteration is continued until no meaningful improvement on loadability factor can be achieved. By nature, computational effort of the problem is several times more than that of a generation rescheduling problem (for which *outer-loop iteration* is not needed).

It is apparent that efficiency of the inner-loop iteration depends on the algorithm chosen for generation adjustment and that performance of the security assessment routine has major impacts on the overall computation time. Procedures used for security assessment and generation adjustment will be discussed in Sections 3.2 and 3.3, respectively.

3.2. Security assessment

In general, steady-state security assessment should include $(n-1)$ contingency analysis. In this paper, evaluation of steady-state security is simplified to be a standard load flow analysis, which is familiar to power audience. Contingency analysis, if desired, can easily be incorporated into the proposed framework.

The algorithm used for dynamic security assessment is a step-by-step integration procedure. It fully exploits sparse matrix/vector techniques and contains an automatic contingency selection approach developed previously by the authors. Details about the algorithm can be found in Ref. [7].

3.3. Adjustment of generation

The algorithm used to adjust generation is a numerical implementation of the following steps:

Step 1. Classify and group the available generators in the system into three sets: those machines that are severely disturbed, machines that are slightly disturbed, and generators connected to a swing bus. Criteria for classification should be based on machine acceleration at the instant when a fault occurs and on machine kinetic energy at the instant that the fault is cleared.

Step 2. Re-dispatch active power generation of the machines according to the following guidelines. For severely disturbed generators, reduce their active power generations by a small percentage (say 5%). For slightly disturbed machines, do not change their generations. For swing machines, increase their active power generation to compensate for the total reduction of generation at disturbed generators. Let the active power generations of the ith machine before and after the adjustment be denoted by P_{mi} and P'_{mi} $(i = 1, 2, \dots, N_g)$, respectively.

Step 3. Evaluate line flow using the linearized line flow equation:

$$T'_l = T_l + \sum_{i=1}^{N_g} H_{li}(P'_{mi} - P_{mi}), \tag{12}$$

where T_l and T'_l are the line flows of the lth transmission line before and after re-dispatching active power generation, respectively. Weighting H_{li} is the so-called sensitivity factor [16], and it relates active power injection to the line flow.

Step 4. Check inequality (6) to see if thermal limit of transmission lines has been violated. If $|T'_l| < \bar{T}_l$ for all $l = 1, 2, \dots, N_L$, stop adjusting generation and go to step 'Re-compute load flow' defined at the bottom of Fig. 1. Otherwise, proceed with Step 5.

Step 5. Rewrite the linearized load flow equation (12) as

$$\Delta T = H(P''_m - P'_m), \tag{13}$$

where vector ΔT consists of all the changes of transmission line active power flow as its components, elements of matrix H are H_{li}, $l = 1, 2, \dots, N_L$, $i = 1, 2, \dots, N_g$, vector P'_m is of dimension N_g and its elements are P'_{mi}, and P''_m denotes the new active power generation vector to be decided.

Form vector ΔT of appropriate dimension by defining its elements as $\Delta T_k = T'_k - T_k$, where $k \in \psi$, and ψ is the set containing the number of transmission lines at which violations of thermal limit on line flow are observed in Steps 3 and 4. Note that ΔT is a 'condensed' vector, and its dimension is denoted by N_T. In other words, the transmission lines without any line flow violation are excluded from ΔT.

Step 6. Construct $N_T \times N_g$ matrix S whose elements on the lth row are H_{li}, $i = 1, 2, \dots, N_g$. It follows from Eq. (13) that

$$P''_m = P'_m + S^T (S \cdot S^T)^{-1} \cdot \Delta T, \tag{14}$$

where superscript T denotes transpose. Eq. (14) is the so-called *pseudo-inverse based steady-state security control formulation* in Refs. [2,3].

Step 7. Set $P_m = P''_m$ and go to the step Re-compute load flow defined at the bottom of Fig. 1.

The procedure of generation adjustment involves verification of both dynamic security and steady-state security. Specifically, the steady-state security control algorithm given in Refs. [2,3] is extended so that, while steady-state security is studied in Steps 3–7 based on the extended version of pseudo-inverse method, dynamic security is verified in Steps 1 and 2 (based on heuristics). This extension makes it possible to handle steady-state and dynamic security in a unified way in the dispatch algorithm. Incremental generation adjustments chosen separately to alleviate either steady-state insecurity or dynamic insecurity may sometime conflict with each other. For example, the mechanical power of a group of generators need to be reduced to eliminate dynamic insecurity; but such a reduction could cause steady-state insecurity or make it worse. When this type of cases arises, our solution is: first

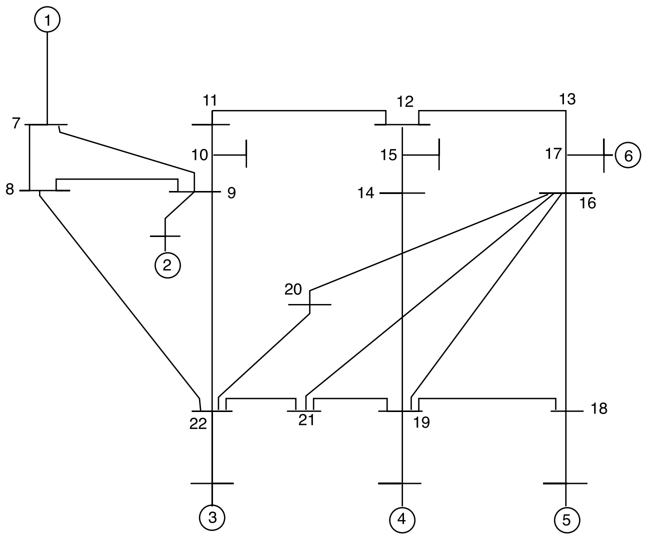

Fig. 2. Single-line diagram of test power system.

adjust mechanical power of some of the machines to ensure dynamic security and then adjust other generators to alleviate steady-state insecurity. Since the number of binding constraints is relatively small in real-world power systems, this heuristics procedure should work reasonably well, especially if the user performs his/her calculations in an interactive way.

4. Simulation results

The proposed formulation has been applied to the 6-machine, 22-node test power system defined by Fig. 2. Original data of this test system is available upon request. Note that machine 6 is not a generating unit but a var resource. Consequently, it is not considered in the process of generation adjustment.

In the simulation, the *specified limit* on iteration counter K in Fig. 1 is set to be 4. The initial active power generation and demand are listed in Table 1. Note that the test system is secure both in the steady-state and dynamically under this initial pattern of power injection. For briefness, detailed results of security assessment are not included here.

Table 1
Initial power injection mode of test power system

Location	Active power generation	Active power load
1	5.7000	
2	5.4990	
3	3.0000	
4	1.6000	
5	4.3000	
6	−0.0100	
8		2.8700
9		3.7600
16		5.0000
18		0.7190
19		2.2650
20		0.7000
21		0.8600
22		4.300

Table 2
Power injection mode of test power system when α is increased to 1.03

Location	Active power generation	Active power load
1	6.1454	
2	6.1766	
3	3.1929	
4	1.6475	
5	4.4271	
6	−0.0100	
8		2.9553
9		3.8709
16		5.1483
18		4.4275
19		0.8856
20		0.7406
21		0.7209
22		2.3316

To test the proposed numerical procedure, loadability factor is increased in increments of 1%. Our simulation shows that the test system remains to be secure at values of $\alpha = 1.0$, 1.01, and 1.02. However, when loadability equals 1.03, the test system is only dynamically secure but not steady-state secure. Three thermal limit violations are found: active power flow across lines 2–9 is 6.1766 which is over the limit 6.00; active power flow across lines 3–22 is 3.1929 (over the limit 3.00); and active power flow across lines 5–28 is 4.4271 (over the limit 4.00). Generation injections in this case are listed in Table 2.

Now the generation adjustment algorithm described in Section 3.3 is applied. After three inner iterations, the test system is made to be both steady-state and dynamically secure. The active power generations after adjustment are listed in Table 3, and some of the dynamic security assessment results are listed in Table 4.

Our simulation also shows that if α is increased to 1.04, secure active power generation configuration cannot be found even after generation adjustment. Therefore, the numerical simulation suggests that the maximum loadability factor of the test system is between 1.03 and 1.04. Further study is needed to see if this result is conservative.

Table 3
Active power generation after adjustment when α equals to 1.03

Location	Active power generation
1	7.0600
2	5.5000
3	3.0000
4	2.0000
5	4.0000
6	−0.0100

Table 4
Dynamic security assessment results of test system when α equals to 1.03

Contingency location		Maximum relative swing angle (°)
8	22	64.2
9	22	64.6
11	12	163.8
12	11	172.0
13	12	72.0
18	16	60.8

5. Conclusions

The problem of calculating loadability of generation/transmission systems has gained renewed interests in recent years partly because deregulation of utility industry are being undertaken in many countries. Finding an exact solution to the problem is formidable due to the limitations of existing mathematical methodologies. In this paper, a new algorithm is proposed to estimate the maximum loadability factor of a power system, and it is evolved from those algorithms developed previously by the authors for generation rescheduling. A successful application would involve judicious engineering judgment. The unique features of the method are that both steady-state and dynamic security are taken into account and that dynamic security is analyzed using an integration algorithm (which makes the proposed method more robust). In addition to the fact that simplification is employed in computing load flow equations and dynamic security constraints, our simulation results suggest that the proposed algorithm is quite promising. Research is under way to loose the assumptions currently employed in the algorithm.

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