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Brief Paper

Robust fault-tolerant self-recovering control of nonlinear uncertain systems☆

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Abstract

In this paper, the problem of devising a fault-tolerant robust control for a class of nonlinear uncertain systems is investigated. Possible failures of the sensor measuring the state variables are considered, and a robust measure is developed to identify the stability- and performance-vulnerable failures. Based on evaluation of the robust measure, a fault-tolerant robust control will switch itself between one robust control strategy designed under normal operation and another under the faulty condition. It is shown that, under two input-to-state stability conditions, the proposed scheme guarantees not only the desired performance under normal operations but also robust stability and best achievable performance when there is a sensor failure of any kind.

Keywords: Fault-tolerant control; Self-recovering; Lyapunov direct method

1. Introduction

Over the last several years, robust control of nonlinear uncertain systems has became a very active area of research. Classes of stabilizable uncertain systems and the corresponding robust controls have been found (Gutman, 1979; Corless & Leitmann, 1981; Qu, 1992; Qu, 1993; Krstic, Kanellakppoulos, & Kokotovic, 1995; Freeman & Kokotovic, 1996; Sepulchre, Jankovic, & Kokotovic, 1997; Qu, 1998), and most of these results are based on Lyapunov direct method. The objective of robust control is to stabilize dynamic systems in the presence of significant, bounded uncertainties (which include parametric uncertainty, unknown dynamics, time variant disturbances, etc.) In this paper, the problem of designing fault-tolerant robust control is studied so that the resulting control is also robust against sensor failures. It is shown that, under input-to-state stability conditions, robust fault-tolerant control exists for systems with matched uncertainties. The basic idea and the proposed design process can also be applied to the class of systems consisting of two (or more) cascaded subsystems, for example, systems composed of an actuator and a plant under control, the subsystems are nonlinear and uncertain, and the sensors measuring their state variables could become faulty.

Fault diagnosis and fault-tolerant control (or reconfigurable control) have been studied primarily for linear and/or parameterizable systems, see Bodson and Groszkiewicz (1997), Noura et al. (2000), and Patton (1997) and the references therein. For nonlinear systems, fault-tolerant control is needed as in the linear case but its design is more complicated. Furthermore, the presence of uncertainties in uncertain systems makes fault diagnosis more difficult. To overcome these difficulties, we propose in this paper to derive a robust fault-detection measure and to design robust control strategies using the Lyapunov direct method. Because of the uncertainties, minor faults that do not jeopardize stability or performance may not be diagnosed, and the proposed robust measure is to detect those faults that hinder system performance or potentially de-stabilize the system. When a sensor is judged to have major failure, a state observer can be employed to estimate the state and to determine its subsequent recovery. Fault-tolerant robust control is designed to combine the features of fault detection, robustness against uncertainty, and self recovery.

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Conceptually, the proposed method is different from both the nonlinear observer design in (Ahmed-Ali & Lamnabhi-Lagarrigue 1999; Aloliwi and Khalil, 1997a,b; Khalil, 1996a; Khalil and Esfandiari, 1993) and the recent result on fault detection and isolation using an observer/filter (Persis & Isidori, 2001). It is shown in Persis and Isidori (2001) that a single-channel fault can be detected and isolated if there exists an observer whose dynamics are known and, no matter what is the control input, are driven by the fault signal but decoupled from all other faults/uncertainties/disturbances. In the proposed approach, an observer is used to determine fault clearance and may be used to reconstruct intermediate state variables (for the cascaded systems), while fault detection is done using stability/performance measure(s). As such, using the proposed approach, only those faults that alter stability or performance can be detected, and input-to-state stability conditions are required. Note that input-to-state stability with respect to measurement noises has been studied in Jiang, Mareels, and Hill (1999).

The paper is organized into the following sections. System description, necessary assumptions, and the problem of designing fault-tolerant robust control are described in Section 2. Robust measures for identifying sensor failure, nonlinear observer, robust control design, and fault-tolerant robust control are organized sequentially as the subsections of Section 3. A simulation example is presented in Section 4.

2. Problem formulation

The class of nonlinear uncertain systems considered in the paper and the proposed robust control structure are shown in Fig. 1. To present the main idea without undue complication, system dynamics are given mathematically by the following differential equation:

$$\dot{x} = f(x,t) + B(x,t)[\Delta f(x,v,t) + u], \tag{1}$$

where $x(t) \in \Re^n$ is the state, $u(t) \in \Re^m$ is the control to be designed, $\Omega \subset \Re^p$ is any bounded set, $v(t) \in \Omega$ denotes the vector of significant uncertainties/unknowns, $f(x,t)$ and $B(x,t)$ are known parts of system dynamics, and $\Delta f(x,v,t)$ is the lumped vector of uncertainties. Due to the presence of uncertainties/unknowns, a successful control must be robust.

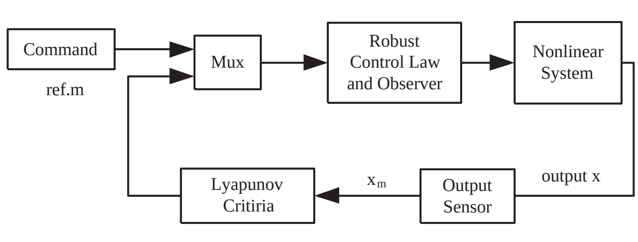

Fig. 1. A class of nonlinear uncertain systems and the proposed fault-tolerant robust control structure.

For the purpose of designing a fault-tolerant control, potential failure of the sensor measuring state x is considered. To this end, denote the measurement of x by x_m. Without loss of any generality, we can model x_m by the following algebraic equation:

$$x_m = x + \Delta h(x,t), \tag{2}$$

where $\Delta h(x,t)$ is the unknown function representing both occurrence and magnitude of possible faults by the sensor. In the normal mode of operation, $\Delta h(x,t) = 0$. While sensor failures can be different in each channel of measurement equation (2), three cases of sensor failure will be considered:

- Inaccurate measurement;
- Intermittent failure;
- Major failure.

The main difference among the three cases is the magnitude and duration of sensor failures. Magnitude of a fault could range from a small offset, to an unknown scaling factor (for example, $\Delta h(x,t) = \delta_x x$ with $|\delta_x| < 1$), and even to a total failure (for instance, $|\Delta h(x,t)| \geqslant |x|$). In terms of severity, the worst fault(s) will be that with $|\Delta h(x,t)|$ being larger than or equal to $|x|$, in which case the measurement feedback is of no advantage or use and, upon detecting its occurrence, the control structure has to become physically open-loop.

The *fault-tolerant robust control* problem is to design a control $u(x_m,t)$ such that the resulting closed loop system, as seen in Fig. 1, has the following properties:

- In the presence of significant (but bounded) uncertainties/unknowns $v(t) \in \Omega$, stability and performance (in the sense of either asymptotic stability or stability of uniform ultimate boundedness (Corless & Leitmann, 1981; Qu, 1998)) are guaranteed whenever the sensor is in good condition.
- When there is a sensor failure, stability is always maintained while performance may be degraded. The proposed fault detector is capable of detecting performance-degrading or potentially destabilizing faults so that the controller can reconfigure itself.
- After a fault is cleared and the corresponding transient settles, the fault detector can identify the recovery and enable the controller to switch back to its normal law.

2.1. Technical conditions

Fault-tolerant robust control design requires several technical assumptions, and most of them are in line with the standard ones in Khalil (1996b). Typically, robust control design is based upon stability or stabilizability of known dynamics. Specifically, the system consisting of known dynamics

$$\dot{x} = f(x,t) + B(x,t)u, \tag{3}$$

is referred to as the nominal system of system (1). The first condition is to ensure existence of a classical and unique solution for system (3) if the control u to be designed has the same property (Khalil, 1996b).

Assumption 1. *The functions in system* (3), $f(x,t)$ *and* $B(x,t)$ *are Caratheodory, locally Lipschitzian with respect to* x, *uniformly bounded with respect to* t, *and locally uniformly bounded with respect to* x.

The second assumption, given below, is on stability of the nominal system. If the nominal system is not stable, it is equivalent to require that the known system be asymptotically stabilized under a known nominal control.

Assumption 2. *The origin*, $x=0$, *is globally asymptotically stable for the uncontrolled nominal system of* (3).

The third assumption is regarding the nature and property of the uncertainties/unknowns.

Assumption 3. *The uncertainties are bounded in Euclidean norm as follows: for all* $(x,v,t)\in\Re^n\times\Omega\times\Re^+$,

$$\Delta f(x,v,t)=W_1(x,t)\phi_1+r_f(x,t), \tag{4}$$

where vector ϕ_1 *contains unknown constant parameters bounded by a known constant* $c_1>0$ *as* $\|\phi_1\|\leqslant c_1$, $W_1(\cdot)$ *is a known matrix function*, $r_f(\cdot)$ *is the un-parameterizable part of unknown dynamics bounded by a known constant as, for all* (x,t),

$$\|r_f(x,t)\|\leqslant c_r,$$

and $W_1(\cdot)$ *and* $r_f(\cdot)$ *are Caratheodory, uniformly bounded with respect to* t, *and locally uniformly bounded with respect to* x.

The above three assumptions are typical for robust control design. Two additional assumptions are needed for designing the proposed fault-tolerant control in order for the system to sustain major sensor faults. Specifically, Assumption 4 is introduced to meet the minimum requirement of all signals being bounded during the worst faults, and Assumption 5 is made to achieve control recovery and performance improvement after relieving the fault. Both of them are in terms of known dynamics and thus can be verified.

Assumption 4. *The nominal system* $\dot{x}=f(x,t)+B(x,t)u$ *is input-to-state stable* (*Krstic et al*., 1995; *Sontag*, 1990). *In addition, the uncontrolled uncertain system* $\dot{x}=f(x,t)+B(x,t)W_1(x,t)\phi_1$ *is also input-to-state stable with respect to "uncertainty input"* ϕ_1.

Mathematically, Assumption 4 implies that, as the converse-Lyapunov-like theorem (Khalil, 1996b), there exists a C^1 function $V(x,t):\Re^n\times\Re\to\Re^+$ such that

$$\gamma_1(\|x\|)\leqslant V(x,t)\leqslant\gamma_2(\|x\|),$$

$$\frac{\partial V(x,t)}{\partial t}+\nabla_x^{\mathrm{T}}V(x,t)[f(x,t)+B(x,t)u]$$
$$\leqslant-\gamma_3(\|x\|)+\|\nabla_x^{\mathrm{T}}V(x,t)B(x,t)u\|, \tag{5}$$

where $\gamma_i:\Re^+\to\Re^+$ are class $\mathscr{K}_\infty$ functions. Since $V(x,t)$ must be bounded for all bounded controls including the fictitious choice of

$$u=\frac{B^{\mathrm{T}}(x,t)\nabla_x V(x,t)}{\|B^T(x,t)\nabla_x V(x,t)\|},$$

one must be able to find constants $0<\lambda<1$ and $b_1>0$ such that

$$\|\nabla_x^{\mathrm{T}}V(x,t)B(x,t)\|\leqslant b_1\gamma_3^{\lambda}(\|x\|). \tag{6}$$

Similarly, function $W_1(\cdot)$ in (4) is bounded from above as, for all (x,t) and for known constants $b_2>0$ and $\beta\in(0,1-\lambda)$,

$$\|W_1(x,t)\|\leqslant b_2\gamma_3^{1-\lambda-\beta}(\|x\|). \tag{7}$$

Assumption 4 is made to ensure input-to-state stability with respect to both the control input and unknown vector ϕ_1, and it includes Assumption 2 as a special case. By contradiction, one can show that Assumption 4 is necessary during the presence of major sensor failure as feedback information would be too corrupted to be useful and the overall system would have to be made open-loop. Technically, the assumption is also important as it provides the Lyapunov function that will be used to analyze stability and synthesize robust control. To proceed with robust control design, Lyapunov function $V(x,t)$ in (5) should have been found. Note that this assumption ensures little performance other than boundedness, and the performance guarantee will be achieved by the proposed fault-tolerant control design.

Assumption 5. *Function* $f(x,t)$ *has the property that, for some* C^1 *function* $L(x,t):\Re^n\times\Re\to\Re^+$ *and for all* $x,z\in\Re^n$,

$$\gamma_4(\|x\|)\leqslant L(x,t)\leqslant\gamma_5(\|x\|),$$

$$\frac{\partial L(x-z,t)}{\partial t}+\nabla_{(x-z)}^{\mathrm{T}}L(x-z,t)[f(x,t)-f(z,t)]$$
$$\leqslant-\gamma_6(\|x-z\|),$$
$$\|\nabla_x^{\mathrm{T}}L(x,t)\|\leqslant b_0\gamma_6^{\lambda_0}(\|x\|), \tag{8}$$

where $b_0>0$ *and* $0<\lambda_0<1$ *are constants, and* $\gamma_j:\Re^+\to\Re^+$ *are class* $\mathscr{K}_\infty$ *functions.*

Physically, Assumption 5 says that, given any bounded perturbation $d(t)$, fictitious system $\dot{z}=f(z,t)+d$ has the so-called perturbation stability. That is, fictitious system $\dot{z}=f(z,t)+d$ approximately follows the nominal system $\dot{x}=f(x,t)$ and, with $z(t_0)=x(t_0)$, the tracking error is uniformly and uniformly ultimate bounded by the perturbation magnitude. In other words, the error system $\dot{e}=f(x,t)-f(z,t)-d$ with state $e=x-z$ is input-to-state stable (Krstic et al., 1995; Sontag, 1990). It is straightforward to show that, if Assumption 2 is strengthened to be exponentially stable, Assumption 4 holds locally provided that $\partial f(x,t)/\partial x$ is locally uniformly bounded.

Inequalities (8), (6), and (7) can be relaxed so that they need to hold only in the region of $\|x\| \geqslant \varepsilon_x$ for some constant $\varepsilon_x > 0$. With these five assumptions, one can proceed with stability- and performance-based fault detection and with the design of a fault-tolerant robust control. The proposed design will ensure stability and achieve performance enhancements whenever feasible.

3. Fault-tolerant control design

The proposed fault-tolerant robust control consists of three parts: a stability- and performance-based measure to monitor sensor health, a state observer, and robust control law for reconfiguration. It is novel that Lyapunov direct method is used to develop a robust measure for fault detection while being used to conduct system stability analysis and control design.

3.1. A robust measure for identifying sensor failure

In control implementation, state measurement $x_m(t)$ is available, but the state itself. Due to the presence of significant uncertainties in the system dynamics, it is not effective to identify faulty sensors by open-loop state estimation, i.e., by first generating estimate $\hat{x}(t)$ and then comparing them to the measurement (as the estimate will not be asymptotically convergent). A closed loop observer should only be used if the sensor measuring x is known to be accurate. Our approach is to develop a stability/performance based measure by which a faulty condition will be diagnosed if it causes either stability problem or performance degradation. Specifically, the following criteria will be used: x_m being uniformly bounded and

$$V_m(x_m, t) \leqslant V_c(t), \tag{9}$$

where $V(\cdot)$ is the Lyapunov function defined in Assumption 4; $V_m(\cdot)$ is the measured value of $V(\cdot)$, that is,

$$V_m(x_m, t) \triangleq V(x_m, t);$$

$V_c(\cdot)$ is a conservatively computed value of $V(\cdot)$, and it is defined by the differential equation

$$\dot{V}_c = -\frac{1}{3}\gamma_3 \circ \gamma_2^{-1}(V_c) + 3^{(1/\beta)-1}\beta(1-\beta)^{(1/\beta)-1}(c_1 b_1 b_2)^{1/\beta} + 3^{\lambda/(1-\lambda)}(1-\lambda)\lambda^{\lambda/(1-\lambda)}[b_1(c_r + |u|)]^{1/(1-\lambda)}. \tag{10}$$

The following lemma shows that condition (9) describes the expected closed-loop and open-loop performance under normal operation (despite of the presence of uncertainties). Inequality (9), together with x_m being uniformly bounded (as defined by (14) and shown by the subsequent stability analysis), can be used to diagnose sensor health.

Lemma 1. *Consider system* (1) *satisfying Assumptions* 3 *and* 4. *Then, if $x_m = x$, inequality* (9) *holds along every trajectory of the system for all t and for any choice of control $u(t)$.*

Proof. It follows from Lyapunov function $V(x,t)$ that

$$\begin{aligned}\dot{V} &= \frac{\partial V(x,t)}{\partial t} + \nabla_x^{\mathrm{T}} V(x,t) f(x,t) + \nabla_x^{\mathrm{T}} V(x,t) B(x,t) \\ &\quad \times [\Delta f(x,v,t) + u] \\ &\leqslant -\gamma_3(\|x\|) + b_1 \gamma_3^{\lambda}(\|x\|)[b_2 c_1 \gamma_3^{1-\lambda-\beta}(\|x\|) + c_r + |u|] \\ &\leqslant -\frac{1}{3}\gamma_3(\|x\|) + 3^{(1/\beta)-1}\beta(1-\beta)^{(1/\beta)-1}(b_1 b_2 c_1)^{1/\beta} \\ &\quad + 3^{\lambda/(1-\lambda)}(1-\lambda)\lambda^{\lambda/(1-\lambda)}[b_1(c_r + |u|)]^{1/(1-\lambda)},\end{aligned}$$

where the Hölder inequality is used to derive the last inequality. Therefore, inequality (9) is concluded by applying the comparison theorem (Khalil, 1996b) to the above inequality and differential equation (10). □

Scalar differential equation (10) provides a robust performance measure, it can easily be integrated online, and its initial condition, $V_c(t^*) = V_m(x_m(t^*), t^*)$, can be set to be one of the following three choices: $t^* = t_0$, $t^* = t - d$ for a constant $d > 0$, and $t^* = t - \Delta t$ with $0 < \Delta t \ll 1$. Therefore, condition (9) can be used to simultaneously diagnose long-term, intermediate-term, and instantaneous health of the sensor. If the condition is violated, the feedback sensor must be faulty.

It should be noted that the proposed measure is stability/performance oriented and hence they may not be able to detect minor faults (for instance, a small offset). Since the system itself is uncertain, detecting all possible faults is not achievable. From a practical point of view, only the faults that impact stability and performance must be identified, which is accomplished by employing condition (9). Also note that, since Lyapunov function is not unique, differential equation (10) associated with condition (9) is not unique either.

3.2. Open-loop state observer

Should a major sensor failure is detected, feedback of measurement x_m must be put aside, and the system will be operated in an open loop mode until a recovery is detected. During the period, stability of boundedness has to be maintained as the minimum requirement, and the state variables should be estimated in the presence of uncertainties so that the system can recover automatically once the faults get cleared. To this end, consider open-loop observer:

$$\dot{\hat{x}} = f(\hat{x}, t) + B(\hat{x}, t)u. \tag{11}$$

Lemma 2. *Consider that, under Assumptions* 3–5, *open-loop observer* (11) *is used to estimate the state of system* (1). *Then, estimation error between x and $\hat{x}$ will* *converge to zero globally and asymptotically as bounds c_1 and c_r as well as control u approach zero.*

Proof. It follows from Assumption 4 that, given a bounded u, system (1) and observer (11) are uniformly bounded. That is, there are constants M_x, $M_{\hat{x}}$ and M_b such that

$$\|x\| \leqslant M_x, \quad \|\hat{x}\| \leqslant M_{\hat{x}}, \quad \|B(\hat{x},t)\| \leqslant M_b \text{ and}$$

$$\|B(x,t)\| \leqslant M_b.$$

It follows from (1) and (11) that estimation error dynamics are

$$\dot{\tilde{x}} = f(x,t) - f(\hat{x},t) + [B(x,t) - B(\hat{x},t)]u + B(x,t)\Delta f(x,v,t),$$

where $\tilde{x} = x - \hat{x}$. Applying Assumptions 3 and 5 yields

$$\frac{\mathrm{d}L(\tilde{x},t)}{\mathrm{d}t} \leqslant -\gamma_6(\|\tilde{x}\|) + b_0\gamma_6^{\lambda_0}(\|\tilde{x}\|)[2M_b\|u\| + M_b b_2 \gamma_3^{1-\lambda-\beta}(M_x)c_1 + c_r],$$

which, according to the Hölder inequality, shows asymptotic convergence (with respect to c_1, c_r, and $\|u\|$) of observer estimation. □

As will be described in the next subsection, control $u = 0$ will be set when the open-loop state observer is invoked. As such, the magnitude of estimation error will be in terms of c_1 and c_r, which can be used as the indicator whether a severe sensor fault has been cleared. However, open-loop state estimate itself cannot be used to determine sensor failure due to the presence of uncertainties.

Despite of input-to-state stability of the plant and the observer, stability of the closed loop system in Fig. 1 and its performance can only be guaranteed by a fault-tolerant robust control. If $u(t)$ is properly designed, value of $V(x,t)$ will be forced to decrease eventually, so will be intermediate and instantaneous values of $V_c(t)$, and so will be values of $V_m(x_m,t)$ if there is no more sensor fault.

3.3. Fault-tolerant robust control

To achieve robustness against sensor failure, a fault-tolerant robust control is designed to incorporate the robust fault detection mechanism, a nonlinear observer, and a robust control law. Depending upon detection outcomes, the proposed control reconfigures itself. The robust control law, given below, combines several standard results in Gutman (1979), Corless and Leitmann (1981), and Qu (1998):

$$u_r(x_m,t) = -W_1(x_m,t)\hat{\phi}_1 - \frac{B^{\mathrm{T}}(x_m,t)\bigtriangledown_{x_m} V(x_m,t)c_r}{\|B^{\mathrm{T}}(x_m,t)\bigtriangledown_{x_m} V(x_m,t)\|c_r + \varepsilon_r}c_r, \tag{12}$$

$$\dot{\hat{\phi}}_1 = W_1^{\mathrm{T}}(x_m,t)B^{\mathrm{T}}(x_m,t)\bigtriangledown_{x_m} V(x_m,t) - k_a\hat{\phi}_1, \tag{13}$$

where $k_a > 0$ and $\varepsilon_r > 0$ are design gain/parameter, $\hat{\phi}_1$ is the estimate of ϕ_1, and $\hat{\phi}_1(t_0) = 0$.

The objective of robust fault-tolerant control is twofold: identify and recover from major faults, reconfigure itself accordingly to ensure stability of uniform ultimate boundedness (with respect to a threshold ε) whenever achievable and to guarantee uniform boundedness in the worst case (i.e., major faults and significant uncertainties). To this end, let us define the following set:

$$\|x_m(t)\| \leqslant (\gamma_1^{-1}\circ\gamma_2)(\max\{\|x_m(t^*)\|, \gamma_3^{-1}(2k_a c_1^2 + 2\varepsilon_r)\}) \triangleq \bar{C}_{x_m}. \tag{14}$$

Then, the proposed *fault-tolerant robust control* is defined by

u is set to be u_r in (12)
 if (9) and (14) are valid for x_m
u is switched to $u = 0$ and open-loop observer (11) is invoked
 if one of the following conditions is violated :
 (9)
 or (14) when u_r is being applied
u switches back to u_r
 if (9) and (14) become valid and if $\|x_m - \hat{x}\|$ is relatively small

Note that the threshold on $\|x_m - \hat{x}\|$ should be set to be small as compared to the transient but not too small due to the presence of uncertainties and noise.

Performance under the proposed control is summarized in the following theorem which represents the main result of the paper.

Theorem. *Consider system* (1) *satisfying Assumptions* 1, 3, 4, *and* 5. *Then, the proposed fault-tolerant control ensures that x is always uniformly bounded and that, whenever the sensor measuring x is operating properly, x is ultimately bounded with respect to any given small threshold* $\varepsilon > 0$.

Proof. Consider first the case that $u = u_r$. In this case, the closed loop dynamics of system (1) are

$$\dot{x} = f(x,t) + B(x,t)\left[W_1(x,t)\phi_1 + r_f(x,t) - W_1(x_m,t)\hat{\phi}_1 - \frac{B^{\mathrm{T}}(x_m,t)\bigtriangledown_{x_m} V(x_m,t)c_r}{\|B^{\mathrm{T}}(x_m,t)\bigtriangledown_{x_m} V(x_m,t)\|c_r + \varepsilon_r}c_r\right].$$

Now, consider the Lyapunov function

$$L'(x,\hat{\phi}_1) = V(x,t) + \frac{1}{2}\|\tilde{\phi}_1\|^2,$$

where $\tilde{\phi}_1 = \phi_1 - \hat{\phi}_1$. Obviously, there exist class-$\mathscr{K}$ functions $\gamma_7(\cdot)$ and $\gamma_8(\cdot)$ that

$$\gamma_7(\|\Phi\|) \leqslant L'(x,\hat{\phi}_1,t) \leqslant \gamma_8(\|\Phi\|),$$

where $\|\Phi\|^2 = \|x\|^2 + \|\tilde{\phi}_1\|^2$. It follows from robust controllaw u_r in (12) and (13) that

$$\dot{L}' \leqslant -\gamma_3(\|x\|) + \varepsilon_r + k_a \tilde{\phi}_1^{\mathrm{T}} \hat{\phi}_1 + \Delta(x, x_m, t)$$

$$\leqslant -\gamma_3(\|x\|) - \frac{k_a}{2}\|\tilde{\phi}_1\|^2 + k_a c_1^2 + \varepsilon_r + \Delta(x, x_m, t) \quad (15)$$

$$\leqslant -\gamma_9(\|\Phi\|) + k_a c_1^2 + \varepsilon_r + \Delta(x, x_m, t), \quad (16)$$

where $\gamma_9(\cdot)$ is a class-$\mathscr{K}$ function, and

$$\begin{aligned}\Delta(x, x_m, t) \triangleq\; & [\nabla_x^{\mathrm{T}} V(x,t)B(x,t)W_1(x,t) \\ & - \nabla_{x_m}^{\mathrm{T}} V(x_m,t)B(x_m,t)W_1(x_m,t)]\phi_1 \\ & -[\nabla_x^{\mathrm{T}} V(x,t)B(x,t) \\ & - \nabla_{x_m}^{\mathrm{T}} V(x_m,t)B(x_m,t)]W_1(x_m,t)\phi_1 \\ & +[\nabla_x^{\mathrm{T}} V(x,t)B(x,t) \\ & - \nabla_{x_m}^{\mathrm{T}} V(x_m,t)B(x_m,t)]W_1(x_m,t)\tilde{\phi}_1 \\ & +[\| \nabla_x^{\mathrm{T}} V(x,t)B(x,t)W_1(x,t)\| \\ & - \| \nabla_{x_m}^{\mathrm{T}} V(x_m,t)B(x_m,t)W_1(x_m,t)\|]c_r \\ & + \| \nabla_x^{\mathrm{T}} V(x,t)B(x,t) - \nabla_{x_m}^{\mathrm{T}} V(x_m,t)B(x_m,t)\|c_r.\end{aligned}$$

If $x_m = x$ (while $u = u_r$), inequality (9) holds, $\Delta(x, x_m, t) = 0$ in (16), and hence stability of being both uniformly bounded and uniformly ultimately bounded can be concluded using the stability theorems in Corless and Leitmann (1981), and Qu (1998). Specifically, the uniform bound on Φ (i.e., on both x and $\tilde{\phi}_1$) is

$$\gamma_7^{-1} \circ \gamma_8 \circ \gamma_9^{-1}(\max\{\|\Phi(t_0)\|, k_a c_1^2 + \varepsilon_r\}),$$

and the ultimate bound is

$$\varepsilon_c \triangleq \gamma_7^{-1} \circ \gamma_8 \circ \gamma_9^{-1}(k_a c_1^2 + \varepsilon_r).$$

For on-line monitoring on $\|x\|$, the uniform bound can be tightened to be that in (14). Note that the ultimate bound on $\|x\|$ is of form

$$\varepsilon \triangleq \gamma_7^{-1} \circ \gamma_8 \circ \gamma_3^{-1}(k_a c_1^2 + \varepsilon_r),$$

which can be made arbitrarily small by selecting ε_r and k_a.

If $\|x_m - x\|$ remains bounded while inequality (9) holds, it follows from Assumption 4 and the expression of $\Delta(x, x_m, t)$ that the bound on $\|\Delta(x, x_m, t)\|$ would be of first order in $\|\tilde{\phi}_1\|$ and of lower order in $\|x\|$ than that of $\gamma_3(\|x\|)$. Hence, whenever $\|\Phi\|$ exceeds certain value, the right hand side of (15) becomes negative. In other words, if $\|x_m - x\|$ is bounded, robustness of x and $\tilde{\phi}_1$ being bounded is ensured.

As $\|x_m - x\|$ becomes larger, inequality (9) or (14) (or both) will become invalid, but the proposed control will also maintain uniform boundedness by its design. This is because, unless $u = 0$, conditions (9) and (14) are being imposed and because, if $u = 0$, the open-loop system with uncertainties and the open-loop observer are bounded. □

Clearly, the proposed fault-tolerant control is stability- and performance-based as it is synthesized using the Lyapunov direct method. It is novel that Lyapunov method is used not only to analyze stability and design control but also to detect faults. Reconfiguration of the control ensures that, in the worst case, uniform boundedness is guaranteed as the minimum performance. The control law of $u = 0$ is necessary because faulty sensor(s) can make the system physically open-loop and, due to the presence of significant uncertainties, any open-loop control could be counter-productive. For the overall system to be more fault tolerant, redundancy needs to be built into so that the system can maintain input-to-state stability by itself and that any faulty sensor can be switched off and replaced by a redundant sensor. What the proposed fault-tolerant control does is to maintain the minimum performance during all faulty conditions and to recover the normal operation after faults are cleared. This is the motivation of the paper, and many applications such as space systems are of the nature.

3.4. Extensions

It can be shown that the above fault-tolerant robust control design can be applied to the class of uncertain systems consisting of cascaded subsystems, for instance, a system consisting of actuator dynamics and plant dynamics and being of form

$$\begin{aligned}\dot{x} &= f(x,t) + B(x,t)[\Delta f(x, v_x, t) + z], \\ \dot{z} &= g(z,t) + \Delta g(z, v_z, t) + u,\end{aligned} \quad (17)$$

where $f(x,t)$, $g(z,t)$ and $B(x,t)$ are known parts of system dynamics, and $\Delta f(x, v_x, t)$ and $\Delta g(z, v_z, t)$ are uncertainties in the two subsystems. Using the cascaded structure, inequalities of form (9) can be developed as robust performance measures to identify and isolate faults for individual subsystems. Robust fault-tolerant control laws can then be designed accordingly.

Note that, in making the above extension, the system structure provides additional options in guaranteeing performance when some sensors are faulty. For example, the system structure of (17) generally ensures observability of z as long as measurement of x is valid. In other words, when only the sensor on z is faulty, a closed-loop nonlinear robust observer can be constructed to generate z with sufficient accuracy, and an observer-based robust controller can be designed for the overall system. According to detection outcomes, the fault-tolerant control will switch among the normal state-feedback control law, observer-based control laws, and an open-loop control law.

4. Simulation example

To illustrate the fault tolerance robust control, a simple system is simulated, and its dynamics are

$$f(x,t) = -x^3,\ B(x,t) = 1 + 0.5\sin(x),$$

$$W_1(x,t) = [x\sin(x)\ x^2].$$

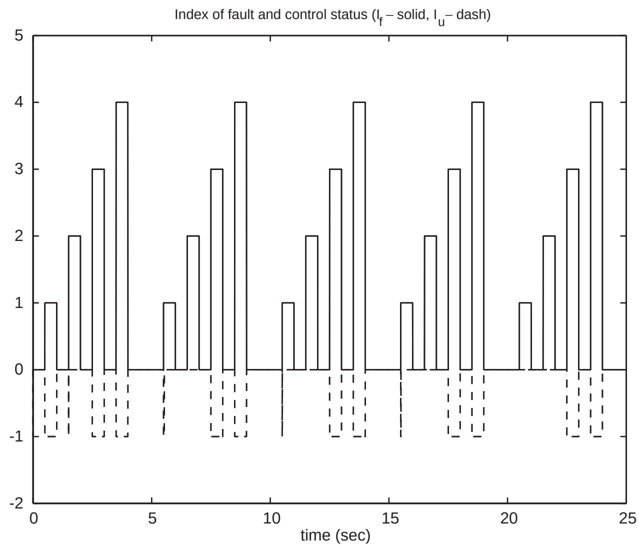

Fig. 2. Index values of sensor fault and the subsequent control action.

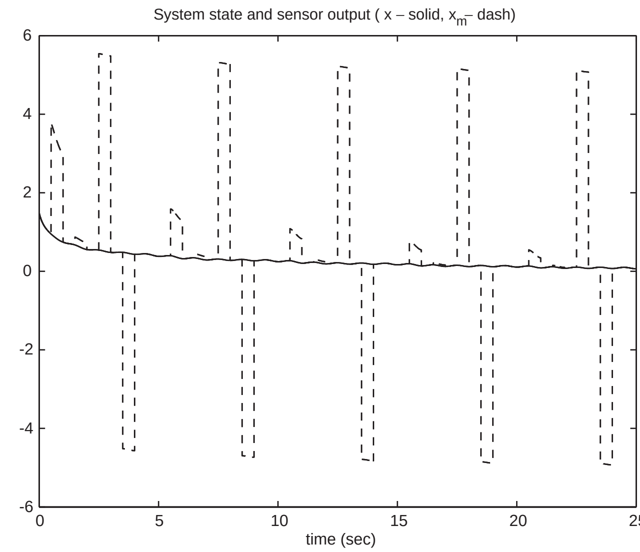

Fig. 3. System state x versus measured output x_m.

It is easy to verify that all the assumptions are satisfied with

$$V(x,t) = \tfrac{1}{2}x^2, \quad \gamma_1(\|x\|) = \gamma_2(\|x\|) = V(x,t), \quad \text{and} \quad \gamma_3(\|x\|) = x^4.$$

On the other hand, design parameters and control gain are chosen to be

$$b_1 = 2, \quad b_2 = 2, \quad c_1 = 0.2236, \quad c_r = 0.1, \quad \lambda = \tfrac{1}{4}, \quad \beta = \tfrac{1}{4}, \quad \varepsilon_r = 0.1 \quad \text{and} \quad k_a = 0.4.$$

In the simulation, the initial conditions are

$$x(0) = 1.5, \quad \hat{x}(t^*) = 0, \quad \hat{\phi}_1(t_0) = 0;$$

the “uncertainties” are set to be

$$\Delta f(x,v,t) = W_1(x,t)\phi_1 + r_f, \quad \phi_1^{\mathrm{T}} = [0.2\ 0.1], \quad \text{and } r_f = c_r \sin(2\pi t);$$

and the simulated sensor failure is represented by $\Delta h(x,t)$ which assumes the following values sequentially and repeatedly:

$$\begin{aligned} &\Delta_1 h(x) = \delta_1 x && \delta_1 = 3.0 \\ &\Delta_2 h(x) = \delta_2 x && \delta_2 = 0.3 \\ &\Delta_3 h(x) = \delta_3 \operatorname{sign}(x) && \delta_3 = 5 \\ &\Delta_4 h(x) = \delta_3 \operatorname{sign}(x) && \delta_4 = -5. \end{aligned} \tag{18}$$

The third case is the most serious as the output of the faulty sensor jumps from its current value to its maximum value of its range and stays there.

The simulation results are shown in Figs. 2–5. To make better connection between fault occurrences and subsequent actions by the proposed fault-tolerant control, values of the following two indices are plotted in Fig. 2:

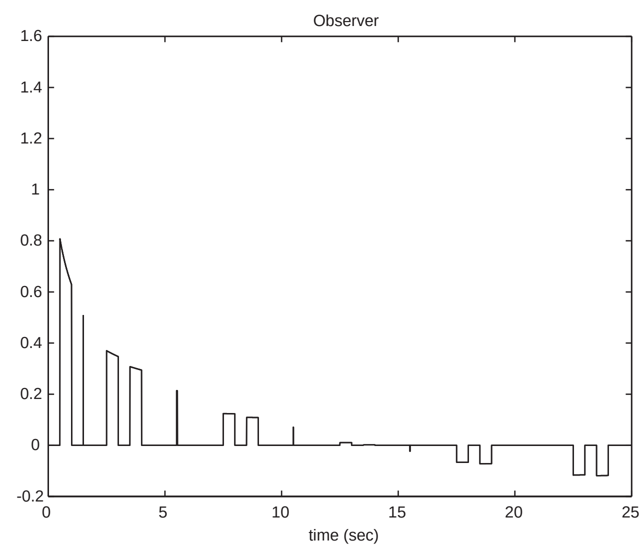

Fig. 4. Observer output.

$$I_f(t) = \begin{cases} 0 & \text{if } \Delta h(x,t) = 0 \\ 1 & \text{if } \Delta h(x,t) = \Delta h_1(x) \\ 2 & \text{if } \Delta h(x,t) = \Delta h_2(x) \\ 3 & \text{if } \Delta h(x,t) = \Delta h_3(x) \\ 4 & \text{if } \Delta h(x,t) = \Delta h_4(x) \end{cases} \quad \text{and}$$

$$I_u(t) = \begin{cases} 0 & \text{if } u(t) = u_r, \\ -1 & \text{if } u(t) = 0. \end{cases}$$

Simulation shows that the proposed control maintains robust stability during the faults, is capable to restore the

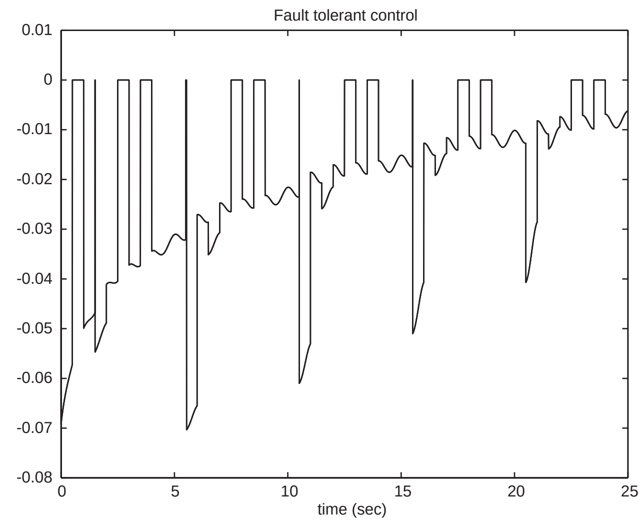

Fig. 5. Fault-tolerant robust control.

normal operation once the faults are cleared, and enures performance after fault clearances. Clearly, faults such as $\Delta_2 h(x)$ and, when $|x|$ is small, $\Delta_1 h(x)$ cannot be detected as they do not cause much performance degradation. When a fault is detected, the control becomes open loop, and the open loop observer is invoked to estimate the state. After the fault is cleared, the recovery is detected by using the inequality $\|x_m - \hat{x}\| \leqslant 0.015$, and the control switches back to the robust control law. The threshold values should not be chosen to be too small, otherwise no automatic recovery will be initiated. If the initial threshold values are chosen to be too large, the fault-tolerant control may attempt to switch back to 0 or u_r before the fault is relieved. Even in this case, robustness and stability of boundedness will be maintained as the robust fault detection measure will always switch off any incorrect control action. In short, the threshold value can be set properly off line or be tuned on line and automatically.

5. Conclusion

Fault-tolerant robust control of a class of nonlinear uncertain systems is studied. In addition to nonlinear uncertainties, the system may also experience sensor failure, and the proposed robust control is made fault-tolerant by integrating a traditional robust control with a robust measure capable of detecting major faults that are liable in either stability or performance. The robust measure, robust control strategy and the fault-tolerant control are synthesized using the Lyapunov's direct method. Under ISS-stability like conditions, robust boundedness stability is guaranteed for all operating conditions and, as long as the sensor measuring the system output operates properly, the desired performance of uniform ultimate boundedness of any accuracy can be achieved.

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References

Ahmed-Ali, T., & Lamnabhi-Lagarrigue, F. (1999). Sliding observer-controller design for uncertain triangular nonlinear systems. *IEEE Transactions on Automatic Control*, *44*, 1244–1249.

Aloliwi, B., & Khalil, H. K. (1997a). Adaptive output feedback regulation of a class of nonlinear systems: Convergence and robustness. *IEEE Transactions on Automatic Control*, *42*, 1714–1716.

Aloliwi, B., & Khalil, H. K. (1997b). Robust adaptive output feedback control of nonlinear systems without of persistence of excitation. *Automatica*, *33*, 2025–2032.

Bodson, M., & Groszkiewicz, J. (1997). Multivariable adaptive algorithms for reconfigurable flight control. *IEEE Transactions on Control Systems Technology*, *5*(2), 217–229.

Corless, M. J., & Leitmann, G. (1981). Continuous state feedback guaranteeing uniform ultimate boundedness for uncertain dynamic systems. *IEEE Transactions on Automatic Control*, *26*, 1139–1144.

Freeman, R. A., & Kokotovic, P. V. (1996). *Robust nonlinear control design: State space and lyapunov techniques*. Boston: Birkhauser.

Gutman, S. (1979). Uncertain dynamical systems—A Lyapunov min–max approach. *IEEE Transactions on Automatic Control*, *24*, 437–443.

Jiang, Z. P., Mareels, I., & Hill, D. (1999). Robust control of uncertain nonlinear systems via measurement feedback. *IEEE Transactions on Automatic Control*, *44*(4), 807–812.

Khalil, H. K. (1996a). Adaptive output feedback control of nonlinear systems represented by input-output models. *IEEE Transactions on Automatic Control*, *41*(2), 177–188.

Khalil, H. (1996b). *Nonlinear systems* (2nd ed.). Englewood Cliffs, NJ: Prentice-Hall.

Khalil, K. H., & Esfandiari, F. (1993). Semiglobal stabilization of a class of nonlinear systems using output feedback. *IEEE Transactions on Automatic Control*, *38*(9), 1412–1415.

Krstic, M., Kanellakppoulos, I., & Kokotovic, P. V. (1995). *Nonlinear and adaptive control design*. New York: Wiley.

Noura, H., et al. (2000). Fault-tolerant control in dynamic systems: Application to a winding machine. *IEEE Control Systems Magazine*, *20*(1), 33–49.

Patton, R.J. (1997). Fault-tolerant control: The 1997 situation (survey). *IFAC Safeprocess* '97, Vol. 2, Hull, UK (pp. 1033–1055).

Persis, C. D., & Isidori, A. (2001). A geometric approach to nonlinear fault detection and isolation. *IEEE Transactions on Automatic Control*, *46*(6), 853–865.

Qu, Z. (1992). Global stabilization of nonlinear systems with a class of unmatched uncertainties. *Systems & Control Letter*, *18*, 301–307.

Qu, Z. (1993). Robust control of nonlinear uncertain systems under generalized matching conditions. *Automatica*, *29*, 985–998.

Qu, Z. (1998). *Robust control of nonlinear uncertain systems*. New York: Wiley Interscience.

Sepulchre, R., Jankovic, M., & Kokotovic, P. V. (1997). *Constructive nonlinear control*. Berlin: Springer.

Sontag, E. D. (1990). Further facts about input to state stabilization. *IEEE Transactions on Automatic Control*, *35*, 473–476.

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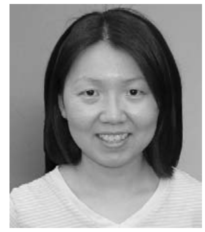

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