COOPERATIVE ATTITUDE SYNCHRONIZATION FOR RIGID-BODY SPACECRAFT *VIA* VARYING COMMUNICATION TOPOLOGY

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Abstract

This paper studies the application of nonlinear cooperative control theory to the attitude synchronization problem. To this end, explicit conditions, in terms of vector nonlinear differential inequalities, is presented firstly to ensure both the Lyapunov stability and asymptotically cooperative stability for a certain class of heterogeneous nonlinear system. Then, decentralized control algorithm incorporated with standard backstepping scheme is developed to accomplish attitude synchronization for a group of spacecraft with rigid body dynamics described by unit quaternion. In particular, the proposed control system imposes the least restrictive requirement on the communication, a piecewise constant, intermittently available, and sequentially complete topology is theoretically enough to ensure the asymptotically cooperative stability. Simulation results demonstrate the effectiveness of the proposed control algorithms.

1. Introduction

Attitude control of rigid bodies has been studied extensively in the literature ([1] and references therein) with various attitude representations [2], among which the unit quaternion-based approaches have received significant attention because of its inherent immunity to the singularity, making it more appealling in large-angle manoeuver scenarios than classical Euler angle representations [3, 4]. However, either the quaternion or Euler angle based methods covers the special orthogonal group $\mathscr{SO}(3)$ multiple times, introducing ambiguities, this will lead to unwinding behaviours as noted in [5, 6], which preclude the existence of the globally asymptotically stable equilibrium point with continuous feedback controller. In this paper, we will take advantage of the distinguishable global non-singularity of the quaternion-based control system while trying to eliminate its ambiguities so as to ensure almost global stability under the requirement of large-angle attitude manoeuvers.

Attitude synchronization/coordination is defined as controlling a group of rigid bodies such that their orientations are synchronized in a particular manner [7]. Inspired by the recent advances in the cooperative control theory [8, 9], cooperative attitude synchronization of multiple rigid bodies attracts more and more attentions. In [9], a quaternion-based attitude consensus algorithm has been thoroughly discussed under an undirected graph. Kang and Sparks [10] discussed several attitude coordination strategies (i.e., leader–follower and a virtual desired attitude strategy). VanDyke and Hall [11] developed a decentralized control scheme to ensure all the agents converge to the same orientation. In [12], an attitude synchronization algorithm is introduced under a ring communication topology assumption, its results were extended to the more general topology case in [9, 13]. In addition, attitude coordination of multiple underwater vehicles is studied in [7, 14] using the energy shaping strategy, which shapes the potential and kinetic energy of a system to make the desired state a stable equilibrium.

However, in many of the existing results, the topology is either assumed to be continuous, or is treated as of certain patterns (strongly connected, balanced, or having a spinning tree). In the venue of time-varying or switching topologies, Moreau [15] stated that consensus could always be reached for single-integrator systems if the topology or the union of graphs within a finite time has a globally reachable node. For double integrator system, the sufficient conditions is that the topology has a directed spanning tree or is connected [16] at every time interval; a similar conclusion can also be found in [9]. In addition, Zhang and Tian [17] proved that in Markov-switching topologies, the network is mean square consentable under linear consensus protocol if and only if the union of graphs has a globally reachable node. Also, for both single and double integrator systems, it is proved that the asymptotically cooperative stability can be ensured if the system is feedback linearizable [18, 19] and the communication topologies is

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sequentially complete over time, which is consistent to the results in [8, 15]. Counterpart study on a certain class of nonlinear system could be found in [8].

This paper studies the application of nonlinear cooperative control theory [8] to attitude synchronization problem for a group of spacecraft with rigid body dynamics described by the unit quaternion. Note that such application is nontrivial because of the inherent nonlinearity associated with the attitude dynamics. More specifically, decentralize control scheme incorporated with standard backstepping procedure is introduced to accomplish attitude consensus and tracking. Compared to prior work in this venue, the proposed scheme does not need feedback linearization and requires less restrictive communication as well, a piecewise constant, intermittently available and sequentially complete network is theoretically enough to guarantee the cooperative and Lyapunov stability.

2. Spacecraft Attitude Dynamics and Control

In this paper, we consider the attitude synchronization problem of a group of n spacecraft modelled as rigid bodies; we assume the attitudes and angular velocities of all the spacecraft used hereafter have been transferred to the appropriate frames using proper transformation matrices, and the orientation of the ith spacecraft, with respect to the inertia frame $\mathscr{I}$, is described by the unit quaternion $\mathbf{q}_i = [q_i^T, \eta_i] \in \Re^3 \times \Re$, satisfying [2]

$$q_i^T q_i + \eta_i^2 = 1 \tag{1}$$

where η_i and q_i are the scalar and vector parts of quaternion, respectively.

The motion of the ith spacecraft can be expressed as follows [1, 20]

$$J_i \dot{\omega}_i = -\omega_i^{\times} J_i \omega_i + \tau_i \tag{2}$$

$$\dot{q}_i = \frac{1}{2}(q_i^{\times} + \eta_i I_3)\omega_i \quad \dot{\eta}_i = -\frac{1}{2} q_i^T \omega_i \tag{3}$$

where $\omega_i \in \Re^3$ denotes angular velocity of the ith spacecraft, I_m is the m-dimensional identity matrix, $J_i \in \Re_+^{3\times 3}$ is the constant diagonal inertial matrix of the ith spacecraft with respect to the body frame $\mathscr{B}_i$, $\tau_i \in \Re^3$ is the control torque applied to the ith spacecraft expressed in $\mathscr{B}_i$. The subscript $\times$ of any vector $\zeta \in \Re^3$ denotes the skew-symmetric matrix formed by ζ, and is given by

$$\zeta^{\times} = \begin{bmatrix} 0 & -\zeta_3 & \zeta_2 \\ \zeta_3 & 0 & -\zeta_1 \\ -\zeta_2 & \zeta_1 & 0 \end{bmatrix}$$

Consequently,

$$\ddot{q}_i = -\frac{1}{4}\omega_i^T q_i \omega_i + \frac{1}{2}(\eta_i I_3 + q_i^{\times})\dot{\omega}_i + \frac{1}{2}\dot{q}_i \times \omega_i$$

After some algebraic manipulations, and using the fact $q_i \times \omega_i \times \omega_i - \omega_i^T q_i \omega_i = -\omega_i^T \omega_i q_i$, results

$$\ddot{q}_i = -\frac{1}{4}\omega_i^T \omega_i q_i + \frac{1}{2}(\eta_i I_3 + q_i^{\times})\dot{\omega}_i \tag{4}$$

In addition, the angular velocity of the ith spacecraft could be described by its quaternion [2]

$$\omega_i = 2(\eta_i \dot{q}_i - q_i \dot{\eta}_i) - 2 q_i^{\times} \dot{q}_i \tag{5}$$

The discrepancy between the current the desired attitude of the ith spacecraft defines the attitude tracking error. That is

$$e_i = \eta_d q_i - \eta_d q_d + q_i^{\times} q_d \quad \tilde{\omega}_i = \omega_i - \omega_d \tag{6}$$

Also, e_i and $\tilde{\omega}_i$ are error quaternions and angular velocities, satisfying the attitude dynamics relations as in (2) and (3), and the subscript d represents the desired value.

As is well known, the unit quaternion, although globally nonsingular, contains a sign ambiguity in that $(q_i,\ \eta_i)$ and $(-q_i, -\eta_i)$ represent the same attitude. In many quaternion extraction algorithms, however, the uniqueness can be achieved by restricting the Euler angle to $[0,\ \pi]$, such that $\eta_i \geq 0$ [1], [9].

Because of the inherent passivity between the control torque τ_i and ω_i (due to the fact $\omega_i^T J_i \dot{\omega}_i = \tau_i^T \omega_i$) as well as q_i and ω_i [21, 23], it is thus straightforward to use angular velocity and quaternion as feedback state in attitude control problems, to ensure the resulted closed-loop system is dissipative in a way that rendering $\omega_i \to 0$ as $t \to \infty$. In particular, the error quaternion and error angular velocity are often used in attitude tracking control problem, provided that the reference attitude is available to the group in a continuous manner. In this case, the tracking controller is [20]

$$\tau_i = -\kappa_{e_i} e_i - \kappa_{\omega_i} \tilde{\omega}_i + J_i \dot{\omega}_d + \omega_d \times J_i \omega_d \tag{7}$$

where $\kappa_{e_i} \in \Re^{3\times 3}$ and $\kappa_{\omega_i} \in \Re^{3\times 3}$ are positive control gain matrices, and they could be simply selected as $\kappa_{e_i} = kJ_i$ and $\kappa_{\omega_i} = cJ_i$ with $k, c > 0$. Alternatively, if the reference attitude is defined as $q_d = [0\ 0\ 0]^T, \eta_d = 1$, (7) becomes [9]

$$\tau_i = \omega_i \times J_i \omega_i - \kappa_{q_i} q_i - \kappa_{\omega_i} \omega_i \tag{8}$$

In essence, (8) could provide a rest-to-rest reorientation manoeuver (i.e., $\omega_i(t_f) = 0$) about an eigenaxis along the initial quaternion vector, this property can be proved using the following candidate Lyapunov function [1, 22]

$$V = \frac{1}{2\kappa_{q_i}} \omega_i^T J_i \omega_i + 2(1 - \eta_i) \tag{9}$$

Note that the attitude control objective $\lim_{t\to\infty}[q_i\ \omega_i] = \mathbf{0}$ can only be achieved almost globally, since $\dot{V} = -\kappa_{\omega_i} \omega_i^T \omega_i$ is negative semidefinite due to the well-known fact that $\mathscr{SO}(3)$ is not a contractible space, and hence quaternion-based control scheme does not offer globally continuous stabilizing results [5, 6, 23]. Or in other words, the unwinding problem is unavoidable in continuous, feedback-based attitude control problems, even of the controller is well-defined. Thus, the objective of global stability should be relaxed to almost global stability in this regard.

Remark 1 *The gyroscopic term in (8) can be further ignored without compromising the stability of the equilibrium (because of $\omega_i^T \omega_i^\times J_i \omega_i = 0$) [1]. As such, (8) becomes*

$$\tau_i = -\kappa_{q_i} q_i - \kappa_{\omega_i} \omega_i \tag{10}$$

Clearly that (10) is a model-independent control law, but its achievable performance (measured by the maximum tracking error), under (10), for a given set of gains depends on the body inertia [24]. In addition, the first term on the right-hand side of (4) has an opposite sign with the control torque τ_i, and it is commonly recognized as a damping term to $\ddot{q}_i$ and does not need to be compensated in quaternion based control design [19].

3. Preliminary Results on Cooperative Control

Without loss of generality, considering a group of agents with linear dynamics

$$\dot{x}_i = u_i \tag{11}$$

where x_i is the state of the ith agent, u_i is the input to the ith subsystem.

In what follows, all the agents/spacecraft are assumed to be operated by themselves most of the time and the exchange of output information occurs only intermittently and locally. To capture the nature of information flow, we define the following binary sensing/communication matrix and its corresponding time sequence $\{t_k : k \in \aleph\}$ as $S(t) \in \{0,1\}^{n\times n} = S(k) = S(t_k), \forall \ t \in [t_k, t_{k+1})$, where $\aleph = \{0, 1, \ldots, \infty\}$

$$S(t) = \begin{bmatrix} 1 & s_{12}(t) & \ldots & s_{1n}(t) \\ s_{21}(t) & 1 & \ldots & s_{2n}(t) \\ \vdots & \vdots & \ddots & \vdots \\ s_{n1}(t) & s_{n2}(t) & \ldots & 1 \end{bmatrix} \tag{12}$$

where $s_{ij}(t) = 1$ if information of the jth spacecraft is available to the ith spacecraft, and $s_{ij}(t) = 0$ if otherwise. Time sequence $\{t_k : k \in \aleph\}$ and the corresponding changes in the row $S_i(t)$ of $S(t)$ are detectable instantaneously by and locally at the ith subsystem, but they are not predictable or prescribed or known a priori or modelled in any way [8].

The standard cooperative control input for system (11) is [8]

$$u_i = \sum_{j=1}^{n} \frac{s_{ij}(t) l_{ij}}{\sum_{l=1}^{n} s_{il}(t) l_{il}} (x_j - x_i) \triangleq \sum_{j=1}^{n} d_{ij}(t)(x_j - x_i) \tag{13}$$

where $l = [l_{ij}] \in \Re_+^{n\times n}$ is a row-stochastic gain matrix.

If the state x_i is scalar, substituting (13) into (11), we have close-loop dynamics as

$$\dot{x} = [-I_n + D(t)]x = -L(t)x \tag{14}$$

where $L(t)$ is essential equivalent to graph Laplacian, and $D(t) = [d_{ij}(t)]$.

It is known that system (14) is both Laypunov stable and asymptotically cooperative stable if and only if (12) is sequentially complete over time, or from graph point of view, from any t_k on, the union of all the future graphs has at least one globally reachable node [8]. This conclusion can be extended to the high-order, heterogeneous linear systems, whose dynamics can be mapped into the canonical form. Moreover, the resulted $D(t)$ is non-negative, piecewise constant, row-stochastic, and has the same sequential completeness property as $S(t)$.

However, the results of the linear system cannot be applied directly to the nonlinear system, simply because most nonlinear systems are too sophiscated to be feedback linearized, or the resulted cooperative input fails in practical implementation because of its complicated feedback terms. Taking attitude control problem for instance, it is nonlinear in a sense that the higher order derivatives of quaternion are cross-coupled with the angular velocity and the control torque as indicated in (4), any attempt to linearize the system will lead to complicated control structure [19].

Given a nonlinear system

$$\dot{x}_i = f_i(x_i) + g_i(x_i)u_i \qquad y_i = h_i(x_i) \tag{15}$$

where $y_i \in \Re^l$ is the output, $f_i(x_i)$, $g_i(x_i)$, and $h_i(x_i)$ are the system matrices with proper dimensions.

Note that the input u_i to system (15) often consists of self feedback terms and cooperative control terms, that is

$$u_i(t) = g^{-1}(x_i)\left\{-\alpha_i(x_i) + \left[\frac{\partial \beta_i(x_i)}{\partial x_i}\right]^{-1} \sum_{j=1}^{n} \overline{d}_{ij}\beta_j(x_j)\right\} \tag{16}$$

where $\alpha_i(x_i)$ denotes the self-feedback term, $\beta_i(\cdot)$ is scalar function, $\overline{D}(t) = [\overline{d}_{ij}]$ is the resulted network matrix determined by $S(t)$ and the physical property of heterogeneous nonlinear systems (i.e., relative degree), it is nonnegative, piecewise constant as $D(t)$ except $\overline{D}(t)$ is not necessarily to be row-stochastic.

Therefore, under input (16), the closed-loop system becomes

$$\dot{x}_i = f_i^c(\mathbf{x}, \overline{D}_i(t)) = f_i^c(\overline{d}_{i1}x_1, \overline{d}_{i2}x_2, \ldots, \overline{d}_{in}x_n) \tag{17}$$

where $f_i^c(.)$ is the closed-loop dynamics of the ith subsystem, $\overline{D}_i(t)$ are the ith row entries of $\overline{D}(t)$.

Before proceeding further, the following two conditions are introduced to address the stability properties of system (17). In what follows, $V_i(\cdot)$ with $i \in [1, \ldots, n]$ and

$L_{\mu,\kappa}(.)$ with $\mu, \kappa \in [1, \ldots, n]$ are positive definite, radially unbounded and differentiable functions.

Condition 1 [8]. System (17) is said to be amplitude dominant on the diagonal if, for all i, the differential inequality

$$\frac{d}{dt}V_i(x_i) \leq -\xi_i(|x_i|) + \gamma_i(x_i)\sum_{l=1}^{n}\overline{d}_{il}(t)\beta_{i,l}(x_l - x_i) \quad (18)$$

holds for nonnegative function $\xi_i(\cdot)$, and strictly monotonically increasing function $\gamma_i(\cdot)$ and $\beta_{i,l}(\cdot)$ with $\gamma_i(\mathbf{0}) = \beta_{i,l}(\mathbf{0}) = \mathbf{0}$.

Condition 2 [8]. System (17) is said to be relative amplitude dominant on the diagonal if, for any index pair $\{\mu, \kappa\}$, the following differential inequality holds:

$$\begin{aligned}\frac{d}{dt}L_{\mu,\kappa}(x_\mu - x_\kappa) \;\leq\; \gamma'_{\mu,\kappa}(x_\mu - x_\kappa)\sum_{l=1}^{n}[\overline{d}_{\mu l}(t)\beta'_{\mu,\kappa,l}(x_l - x_\mu)\\ -\overline{d}_{\kappa l}(t)\beta''_{\mu,\kappa,l}(x_l - x_\mu)] - \xi'_{\mu,\kappa}(|x_\mu - x_\kappa|)\end{aligned} \quad (19)$$

where scalar function $\xi'_{\mu,\kappa}(\cdot)$ is non-negative, and $\gamma'_{\mu,\kappa}$, $\beta'_{\mu,\kappa}$, and $\beta''_{\mu,\kappa}$ are strictly monotonically increasing functions with $\gamma'_{\mu,\kappa}(\mathbf{0}) = \beta'_{\mu,\kappa}(\mathbf{0}) = \beta''_{\mu,\kappa}(\mathbf{0}) = \mathbf{0}$.

In addition, Condition 2 can be verified if the following relation is satisfied [8]

$$\begin{aligned}[\beta_\mu(x_\mu) - \beta_k(x_k)]^T \Big\{ \frac{\partial \beta_\mu(x_\mu)}{\partial x_\mu}[f_\mu(x_\mu) - \alpha_\mu(x_\mu)]\\ -\frac{\partial \beta_k(x_k)}{\partial x_k}[f_k(x_k) - \alpha_k(x_k)] + \frac{\partial \beta_\mu(x_\mu)}{\partial t} - \frac{\partial \beta_k(x_k)}{\partial t}\Big\}\\ \leq -\|\beta_\mu(x_\mu) - \beta_k(x_k)\|^2\end{aligned} \quad (20)$$

where functions $\beta_i(\cdot)$, $\alpha_i(\cdot)$, and $f_i(\cdot)$ are the same functions as in (15) and (16).

In essence, Condition 1 renders the Lyapunov stability of any nonlinear system by ensuring the closed-loop system is dominant in diagonal, under the sequentially complete communication matrix $S(t)$. Condition 2 guarantees the cooperative stability of the overall closed-loop system. The following lemma addresses the relation between these two conditions and the cooperative stability of the nonlinear system:

Lemma 1 [8]. *System (17) is both Lyapunov stable and asymptotically cooperative stable, if it satisfies both the Condition 1 and Condition 2, and the communication matrix $S(t)$ is sequentially complete over time. Furthermore, whenever the element $\overline{d}_{ij}(t) \neq 0$ of the matrix $\overline{D}(t)$, it is uniformly bounded from below by a positive constant.*

Using Lemma 1, systematic development can be done for several classes of nonlinear systems. In addition, it should be noted that, while Lyapunov function components $V_i(\cdot)$ and $L_{\mu,k}(\cdot)$ can always be chosen, it is often too difficult to find or assume a differentiable Lyapunov function because of the nonlinear dynamics and the time-varying topologies, whose changes are sequentially complete but otherwise unknown a priori. However, despite of the unpredictable changes in $S(t)$ and hence in $\overline{D}(t)$, the two conditions in Lemma 2 can always be checked, thus they can be used to guide the cooperative control design. The details of its application to cooperative attitude synchronization problem is provided in the Section 4.

4. Output-Feedback Cooperative Attitude Synchronization

Cooperative attitude control in general requires both feedback and cooperative control terms to achieve the desired group behaviour, while, in the existing results [9, 12], the cooperative control term is determined by the topology and often consists of both attitude and angular velocity tracking error, imposing serious challenge in practical implementation since the rate information is always hard to be estimated online due to the physical limitations, and the discrepancy caused by the exchange of the angular velocities will accumulate with evolution and eventually jeopardize the whole mission. In this section, a simple and effective cooperative control algorithm is proposed to accomplish the attitude synchronization requiring only the exchange of quanternion/attitude information among neighbours. In addition, a virtual leader will be introduced such that the attitude consensus can be controlled to any particular value.

According to (6), the attitude tracking or reorientation for the ith spacecraft is accomplished when q_i converges to q_d and ω_i synchronizes ω_d. That is, if $q_i \to q_d$, then $e_i \to 0$ and $\tilde{\omega}_i \to 0$. In addition, since the attitude discrepancy between the ith and jth spacecraft [20] is

$$q_{ij} = \eta_j q_i - \eta_j q_j + q_i^{\times} q_j \qquad \tilde{\omega}_{ij} = \omega_i - \omega_j \quad (21)$$

Clearly that the attitude coordination will be achieved if, $\forall i, j$ and $i \neq j$, the attitude discrepancy q_{ij} vanishes as well as $\tilde{\omega}_{ij} \to \mathbf{0}_{3\times 1}$ under continuously or intermittently available communication between the ith and jth spacecraft.

As such, the attitude synchronization is accomplished if and only if $q_i \to q_j$, which yields $\tilde{\omega}_{ij} = \mathbf{0}_{3\times 1}$ according to (5).

Furthermore, since

$$e_i - e_j = \eta_d(q_i - q_j) + (q_i^{\times} - q_j^{\times})q_d \quad (22)$$

Hence, for any desired attitude q_d, if $q_i \to q_j$, then $e_i \to e_j$.

Therefore, the control objective for attitude synchronization is reduced to find a decentralized control scheme, utilizing local measurements only, such that

$$\lim_{t\to\infty}(q_i - q_j) = \mathbf{0}_{3\times 1}$$

Inspired by the above analysis and the attitude dynamics indicated in (2) and (3), we choose quaternion and the angular velocity as the state of the nonlinear control system for the ith spacecraft. Also, it is a common experience using quaternion as output in attitude control problem since it can be measured/estimated online using onboard sensor (i.e., IMU). In addition, the scalar part of the quaternion (i.e., η_i) could be disregarded since it is bounded and given by $\sqrt{1-q_i^T q_i}$, and to avoid the underactuated property inherent with quaternion as well.

As such, the nonlinear attitude dynamics is derived *via* the following transformations:

$$x_i = [q_i^T \;\; \omega_i^T]^T, \quad y_i = q_i, \quad u_i = \tau_i \tag{23}$$

Consequently, the system matrices related to the form (15) are

$$f(x_i) = \begin{bmatrix} \frac{1}{2}(q_i^\times + \eta_i I_3)\omega_i \\ -J_i^{-1}\omega_i^\times J_i \omega_i \end{bmatrix}_{6\times 3} \quad g(x_i) = \begin{bmatrix} \mathbf{0}_{3\times 3} \\ J_i^{-1} \end{bmatrix}_{6\times 3}$$
$$h(x_i) = \begin{bmatrix} I_3 & \mathbf{0}_{3\times 3} \end{bmatrix} x_i \tag{24}$$

Obviously, the proposed nonlinear system has an open-loop equilibrium at $x_i = \mathbf{0}_{6\times 1}$, and from (4) it has a relative degree of two. Moreover, since the attitude dynamics (3) is highly nonlinear and cannot be expressed explicitly as linear combination of monotone increased/decreased functions [8], the standard backstepping procedure is introduced to simplify the design procedure without linearizing the system and sacrificing the overall performance [3]. In particular, this recurring scheme is used to stabilize both the attitude and angular velocity so as to ensure the almost global stability of the closed-loop system.

Because of its inherent passivity between q_i and ω_i, it is thus beneficial choosing ω_i as the virtual input to system (3) and selecting $[q_i^T \; z_i^T]^T$ and $[\omega_i^T \; \tau_i^T]^T$ as the new state and input of the resulted system, where $z_i = \omega_i - \omega_i^*$, ω_i^* is the desired ω_i. As such, the new system matrices are

$$f'(x_i) = \begin{bmatrix} \mathbf{0}_3 \\ -J_i^{-1}\omega_i^\times J_i \omega_i \end{bmatrix}_{6\times 3} \quad h'(x_i) = \begin{bmatrix} I_3 & \mathbf{0}_{3\times 3} \end{bmatrix} x_i$$
$$g'(x_i) = \begin{bmatrix} \frac{1}{2}(q_i^\times + \eta_i I_3) & \mathbf{0}_{3\times 3} \\ \mathbf{0}_{3\times 3} & J_i^{-1} \end{bmatrix}_{6\times 6} \tag{25}$$

Hence, the virtual input to (3) can be designed as

$$\omega_i^* = -\kappa_{q_i} q_i \tag{26}$$

As such, we have $\dot{q}_i = -\frac{\kappa_{q_i}\eta_i}{2} q_i$. Since $\eta_i \geq 0$, system (3) will be stabilized almost globally and exponentially under (26). As such,

$$\dot{z}_i = \dot{\omega}_i - \dot{\omega}_i{}^* = -J_i^{-1}\omega_i^\times J_i \omega_i + J^{-1}\tau_i - \dot{\omega}_i{}^* \tag{27}$$

Hence, z_i will also be stabilized exponentially with the following feedback controller

$$\tau_i = \omega_i^\times J_i \omega_i + J\dot{\omega}_i^* - \kappa_{z_i} z_i \tag{28}$$

where κ_{z_i} is positive definite gain matrix.

In what follows, cooperative attitude synchronization as well as its stability analysis will be conducted based on (28). As mentioned earlier, since global stability is impossible to be attained in quaternion-based attitude control problems, all the results in this section under alias of (28) are relaxed to almost global stability.

4.1 Cooperative Attitude Consensus

In essence, the attitude consensus is achieved among n networked rigid bodies, if for all $q_i(\mathbf{0})$, $\omega_i(\mathbf{0})$ with $\forall i, j$, $\|q_i - q_j\| \to \mathbf{0}$ and $\|\omega_i - \omega_j\| \to \mathbf{0}$, as $t \to \infty$ [9]. In this section, the attitude consensus problem is treated as an output cooperation problem, in which the main objective is finding an implementable algorithm to ensure $y_i(t) \to c\mathbf{1}$, $c \in \Re$, $\forall i$.

With the nonlinear system defined in (25) and (27), we propose the following input for the ith spacecraft

$$\tau_i = \omega_i^\times J_i \omega_i + J\dot{\omega}_i^* - \kappa_{z_i} z_i + \sum_{j=1}^{n} s_{ij}(t)(q_j - q_i) \tag{29}$$

where $s_{ij}(t)$ are the entries of communication matrix (12).

It is clear that the first three terms in (29) are feedback terms, with which the dynamics equation for z_i becomes $\dot{z}_i = -\kappa_{z_i} z_i$, indicating ω_i approaches to ω_i^* in an exponential manner, which in turn means the attitude of the ith spacecraft is also stabilized exponentially.

Furthermore, from (3) and (26), we have

$$\dot{\omega}_i^* = -\frac{\kappa_{q_i}}{2}(q_i^\times + \eta_i I_3)\omega_i \tag{30}$$

Therefore, substituting (27) and (30) into (29), yields

$$\begin{aligned} \tau_i \; &= \omega_i^\times J_i \omega_i - \left[\frac{J\kappa_{q_i}}{2}(q_i^\times + \eta_i I_3) + \kappa_{z_i} I_3\right]\omega_i - \kappa_{z_i}\kappa_{q_i} q_i \\ &\quad + \sum_{j=1}^{n} s_{ij}(t)(q_j - q_i) \end{aligned} \tag{31}$$

With proper selection of the control gains, (31) becomes

$$\begin{aligned} \tau_i \; &= \omega_i^\times J_i \omega_i - \left[\frac{J\kappa_{\overline{q}_i}}{2}(q_i^\times + \eta_i I_3) + \kappa_{\overline{\omega}_i} I_3\right]\omega_i - \kappa_{\overline{q}_i} q_i \\ &\quad + \sum_{j=1}^{n} s_{ij}(t)(q_j - q_i) \end{aligned} \tag{32}$$

where $\kappa_{\overline{\omega}_i} \geq 0$ and $\kappa_{\overline{q}_i} \geq 0$ are control gains.

Clearly (32) includes PD control action of q_i, which will effectively stabilize the attitude, and if $\kappa_{\overline{q}_i}$ is sufficiently small or the attitude is slow-varying, (32) is rendered to

$$\tau_i = \omega_i^\times J_i \omega_i - \kappa_{\overline{\omega}_i}\omega_i - \kappa_{\overline{q}_i} q_i + \sum_{j=1}^{n} s_{ij}(t)(q_j - q_i) \tag{33}$$

It is apparent the first term on the right-hand side of (33) is used to cancel the nonlinear effect of the local angular velocity, the second two terms are used to stabilize the attitude so as to ensure a rest-to-rest manoeuver during the transition, and the last term is the cooperative control term using the intermittently available output information from its neighbours, to ensure the cooperative behaviour of the group.

In addition, the consensus value is determined by the topology and the initial state, as is well known [8, 25]. Suppose the communication matrix $D(t)$ is row-stochastic, and $\gamma = [\gamma_1, \ldots, \gamma_n]^T$ is the unity left eigenvector associated with eigenvalue $\lambda(D) = 1$, then the consensus value is

$$\lim_{t\to\infty} q_i(t) = q_f = \gamma_1 q_1(0) + \cdots + \gamma_n q_n(0) \tag{34}$$

In particular, if the topology is at its lower-triangularly form at every t_k, the first spacecraft will act as the leader (*i.e.*, $\gamma_1 = 1$) while all other spacecraft converge to its attitude asymptotically within finite time and the convergence rate is in general justified by the Fiedler value. Furthermore, if the topology is balanced or $\sum_{j=1}^{n} d_{ij} = \sum_{k=1}^{n} d_{ik}, \forall j \neq k$, the average consensus will be ensured since in this case $\gamma_i = \gamma_j, \forall i \neq j$ [25].

Consequently, substituting (32) into the system yields an overall networked system as of (17), in which the combined system structure and network matrix is

$$\overline{D}(t) = [\overline{d}_{ij}] = \left[\begin{array}{cc|cc|c|cc} \multicolumn{2}{c|}{\Lambda_1} & 0 & 0 & \ldots & 0 & 0 \\ & & s_{12} & 0 & \ldots & s_{1n} & 0 \\ \hline 0 & 0 & \multicolumn{2}{c|}{\Lambda_2} & \ldots & 0 & 0 \\ s_{21} & 0 & & & \ldots & s_{2n} & 0 \\ \hline \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \hline 0 & 0 & 0 & 0 & \ldots & \multicolumn{2}{c}{} \\ s_{n1} & 0 & s_{n2} & 0 & \ldots & \multicolumn{2}{c}{\Lambda_n} \end{array}\right] \tag{35}$$

where Λ_i is the binary matrix representing the internal dynamical interconnections of the ith subsystem, of the form:

$$\Lambda_i = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{36}$$

Apparently, Λ_i are nonnegative and irreducible matrices. Therefore, $\overline{D}(t)$ has the same irreducibility/reducibility as $S(t)$. As such, $\overline{D}(t)$ is uniformly sequentially complete if and only if $S(t)$ is uniformly sequentially complete over time [8].

Theorem 1. Consider the spacecraft dynamics given in (23) under the control (32), with restriction that $S(t)$ is uniformly sequentially complete. Then, all the spacecraft in the group reach their attitude consensus asymptotically.

Proof: The proof of Theorem 1 can be found in Appendix.

Remark 2. *In case of linearized design, the state for the ith subsystem is often chosen as* $x_i = [q_i^T \ \dot{q}_i^T]^T$. *Using (4), the system is rendered to Jordan canonical form and thereby input-state feedback linearizable [18] [19].*

Consequently, applying the linear cooperative control approach [8], we have

$$\tau_i^l = \omega_i^\times J_i \omega_i + 2J_i(q_i^\times + \eta_i I_3)^{-1} \times \left[\sum_{j=1}^{n} d_{ij}(t)(q_j - q_i) - \kappa_{\omega_i}\omega_i\right] \tag{37}$$

Compared with (32), the main difference between the linear and nonlinear design schemes is that, except using inverse of internal quaternion dynamics to counteract the nonlinear term, which may cause singularity problem, the resulted interconnection matrix $D(t)$ needs to be row-stochastic in the linear case, while in the nonlinear case, the resulted $\overline{D}(t)$ is of the form (35); this distinction provides extra freedom in control design.

4.2 Cooperative Attitude Tracking

To simplify the cooperative attitude tracking problem, a virtual leader is introduced to the group and labelled as node 0 in the network; it will act as the leader to the group and perform accordingly with respect to the topology, in which the communication of all $n+1$ nodes are also intermittent and local; the overall communication matrix has the same properties as (12) with the extra row and column capturing the information exchange between the virtual leader and the other nodes. That is, $s_{0i}(t) = 1$ if the information of the ith rigid body is observed by the virtual leader and $s_{0i}(t) = 0$ if otherwise, $s_{i0}(t) = 1$ if the information of the virtual leader is known to the ith rigid body and $s_{i0}(t) = 0$ if otherwise.

Moreover, the attitude dynamics of the virtual leader is also represented by the unit quaternion, $\mathbf{q}_0 = \{\eta_0, q_0\}$ and

$$\dot{q}_0 = \frac{1}{2}(q_0^\times + \eta_0 I_3)\omega_0 \quad \dot{\eta}_0 = -\frac{1}{2} q_0^T \omega_0 \tag{38}$$

where $\omega_0 \in \Re^3$ is the angular velocity, η_0 and q_0 are, respectively, the scalar and vector part of the quaternion.

As such, the cooperative attitude tracking problem is rendered to attitude consensus problem as discussed previously. Therefore, (32) also applies in this case except one minor revision to the cooperative term to account for the inclusion of the virtual leader. That is

$$\tau_i = \omega_i^\times J_i \omega_i - \left[\frac{J\kappa_{\overline{q}_i}}{2}(q_i^\times + \eta_i I_3) + \kappa_{\overline{\omega}_i} I_3\right]\omega_i - \kappa_{\overline{q}_i} q_i + \sum_{j=0}^{n} \overline{s}_{ij}(t)(q_j - q_i) \tag{39}$$

where $\overline{s}_{ij}(t)$ are the entries to the augmented sensor/communication matrix $\overline{S}(t)$.

Compared with (32), with the inclusion of the virtual leader, the resulted input has an additional term for every rigid body, $\overline{s}_{i0}(q_0 - q_i)$, which, based on the communication matrix, is used to eliminate the attitude discrepancy between the ith spacecraft and the virtual leader.

In addition, with (39) yields

$$\dot{\omega}_0 = \left[\sum_{j=0}^{n} \overline{s}_{0j}(q_j - q_0)\right] - \omega_0 \tag{40}$$

As such, if virtual leader has no feedback from the group (i.e., $\overline{s}_{0i} = 0, \forall i$), ω_0 will converge to zero in an exponential manner, and then maintain unchanged at the rest of the engagement, which indicates q_0 will also remain constant according to (38). Likewise, in the case $\overline{s}_{0i} = 1$, as shown in (40), q_0 will change accordingly so as to achieve consensus within the group, the attitude tracking problem eventually becomes consensus seeking problem.

Substituting (39) into (15), the resulted network matrix $\overline{D}(t)$ takes a similar form as (35), except with two extra columns and rows characterizing the interaction with virtual leader. Moreover, since the virtual leader is assumed to have the same dynamics as the rigid bodies, adding a node to the topology does not change its sequentially complete property as well as the irreducibily/reducibility property of the network matrix $\overline{D}(t)$. Then, applying Theorem 1 to the resulting closed-loop system renders the following corollary:

Corollary 1. *Under input (39), system (15) is both almost global Lyapunov stable and asymptotically cooperative stable, if the augmented communication matrix sequence $\overline{S}(t_k)$ is sequentially complete over time.*

Proof: The proof of the *Corollary* 1 is straightforward from the proof of *Theorem* 1.

5. Numerical Simulations

In this section, the performance of the proposed cooperative attitude control schemes are examined in a group of three spacecraft. Moreover, cooperative attitude tracking is essentially the same as consensus seeking with the inclusion of the virtual leader. Thus, we will take cooperative attitude tracking case for instance, examining application of (39) to a group of three rigid bodies interacting with a virtual leader.

The inertial matrix for all the rigid bodies in the group is selected as

$$J = \begin{bmatrix} 1.2 & 0 & 0.2 \\ 0 & 1 & 0 \\ 0.2 & 0 & 1 \end{bmatrix} \mathrm{kg\,m^2}$$

The initial conditions for all the rigid bodies used in the simulation is specified in Table 1, where the first column stands for the index for the spacecraft (i.e., 0 for virtual leader).

Without loss of generality, we take the case where the virtual leader takes feedback from its neighbours for instance, In particular, the topology is chosen to be randomly switched between the following communication matrices

Table 1
Initial conditions

	q_1	q_2	q_2	ω_x	ω_y	ω_z
1	0.5	0.1	0.2	0	0	0
2	0.2	0.3	0.2	0	0	0
3	0.3	0.1	0.6	0	0	0
0	0.5	0.5	0.5	0.1	0.2	0

$$\overline{S}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \overline{S}_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$
$$\overline{S}_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \tag{41}$$

Their time sequence binary product is

$$S^*(t) = \overline{S}_3 \bigwedge \overline{S}_2 \bigwedge \overline{S}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \tag{42}$$

Obviously, $S^*(t)$ is reducible and sequentially lower-triangularly complete; it is clear that virtual leader has no feedback from its neighbours.

Figures 1 and 2 show, respectively, the attitude and angular velocities of the spacecraft with input (39) and topology (42). Note that all spacecraft converge to the reference trajectory (labeled as $i = 0$ in the figures), and since the virtual leader does not take feedback from the its neighbours, so the reference attitude stays constant throughout the engagement. Figure 3 provides the time history of quaternion in the attitude consensus case under topology (42) with eliminating the first row and column; obviously all the spacecraft achieve attitude consensus in less than 25 s.

6. Conclusion

This paper presents the cooperative attitude synchronization algorithms for a group of rigid body spacecraft whose kinematics are described by unit quaternion. In view of the inherent nonlinearity related to attitude dynamics/kinematics, the nonlinear cooperative control theory is applied to develop decentralized control algorithm for both cooperative attitude consensus and tracking problems. Simulation results indicate clearly that the proposed schemes can achieve attitude consensus and ensure a fast and stable tracking of the reference attitude.

Figure 1. Time history of quaternion with virtual leader.

Figure 2. Time history of angular velocity with virtual leader.

While the proposed algorithm is effective in achieving attitude synchronization, however, the uncertainties often associated with spacecraft is not taken into account in the control development, and the attitude dynamics does not include the flexible part of the spacecraft, which should be considered in the future version. Also, control algorithm to improve the convergence rate of the network is also expected. Moreover, the resulted cooperative control algorithm (32) or (39) includes the terms of angular velocity. In most practical cases, certain filters or observers have to be designed to estimate the local angular velocity in a real-time manner. Therefore, how to compensate

Figure 3. Time history of quaternion without virtual leader.

for angular velocity without compromising the cooperative stability should be of future interest.

Appendix

Proof of Theorem 1

To prove the amplitude dominant of the resulted closed-loop system with input (32), designing the following Lyapunov-like function $V_i(z_i)$ as in

$$V_i = \frac{1}{2\kappa_{q_i}} z^T J_i z_i \tag{A.43}$$

Obviously, V_i is a positive definite and radially unbounded function.

Therefore, invoking the assumption $J_i = J_i^T$, taking the time derivative of V_i along the attitude dynamics, after some algebraic manipulations, we have

$$\frac{d}{dt} V_i = -\frac{\kappa_{\omega_i}}{\kappa_{q_i}} z_i^T z_i + \frac{1}{\kappa_{q_i}} z_i^T \sum_{j=1}^{n} s_{ij}(t)(q_j - q_i) \tag{A.44}$$

To verify Condition 1, define the following functions:

$$\xi_i(x) = 0 \quad \gamma_i(x) = \frac{1}{\kappa_{q_i}} [\mathbf{0} \;\; I_3] x \quad \beta_i(x) = x \tag{A.45}$$

Hence,

$$\frac{d}{dt} V_i \leq \frac{z_i^T}{\kappa_{q_i}} \sum_{j=1}^{n} \overline{d}_{ij}(t)(x_j - x_i)$$

where $\overline{d}_{ij}(t)$ *is the* ijth entries of $\overline{D}(t)$ as defined in (35).

It follows Condition 1 is verified and the resulted closed-loop system is amplitude dominant on the diagonal, which indicates (17) is Lyapunov stable.

Furthermore, as indicated in (32), select $\beta_i(x_i) = q_i$ in (20) as the output of the ith subsystem. Therefore

$$\frac{\partial \beta_i(x_i)}{\partial x_i} = [I_3 \;\; \mathbf{0}_{3\times 3}] \qquad \frac{\partial \beta_i(x_i)}{\partial t} = 0$$

Consequently, substituting the above relations and (25) into (20), yields

$$-\kappa_q (q_i - q_j)^T (q_i - q_j) \leq -\|q_i - q_j\|^2 \tag{A.46}$$

which in turn demonstrates (25) is relative amplitude dominant on the diagonal, or in other words, asymptotically cooperative stable.

Therefore, invoking Lemma 1, system (15) with input (32) is both almost global Lyapunov stable and asymptotically cooperatively stable provided that time sequence of $S(t)$ is sequentially complete.

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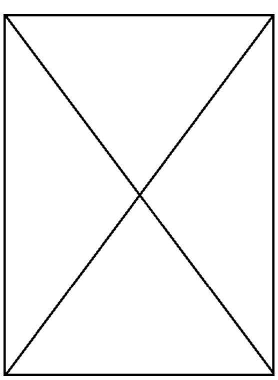
Biographies



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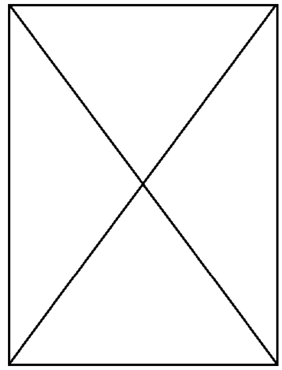

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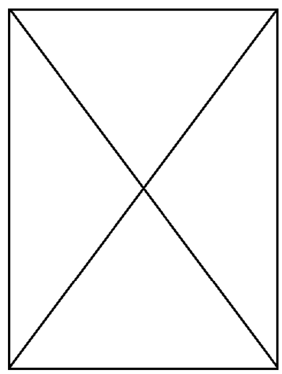

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