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Robust adaptive control of a class of nonlinearly parameterised time-varying uncertain systems

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Abstract: A robust adaptive control is presented for a class of time-varying nonlinear uncertain systems which have a fractional nonlinearly parameterised structure. The proposed design is based on robust adaptive backstepping and neural network approximation. The unknown time-varying parameters in the fractional nonlinear functions are estimated using a smooth projection algorithm and estimation errors are robustly compensated for by the additive terms in the proposed virtual and actual controls. Neural networks are employed to approximate the completely unknown bounding functions of the disturbance terms, and their weights as well as approximation errors are adaptively tuned. It is proved that the proposed robust adaptive control can ensure the semi-global uniform ultimate boundedness of all the closed-loop system signals. The control performance can be improved by an appropriate choice of the design parameters. Simulation results are provided to verify the effectiveness of the proposed design.

1 Introduction

Robust and adaptive control of nonlinear uncertain systems using the Lyapunov direct method has seen a significant progress in the past decades (see [1–4] and the references therein). Particularly, robust control has been popular to deal with feedback linearisable nonlinear systems with various uncertainties (unmodelled dynamics and disturbance) satisfying certain known bounding conditions, and it is this kind of available bound information that allows the development of robust control by dominating the uncertainties. In contrast, adaptive control is based on the estimation of unknown parameters in system, and control design is completed by incorporating the parameter adaptation law. Although the convergence of parameter estimates is not guaranteed in most cases, adaptive control may achieve the improved closed-loop performance due to the direct estimation and compensation of system uncertainties.

For adaptive control, fruitful results have been obtained for nonlinear systems where the unknown constant parameters appear linearly [1, 5, 6]. However, it is quite common that many practical systems, such as fermentation processes [7], biochemical processes [8] and friction dynamics [9], often contain unknown time-varying parameters that enter systems nonlinearly and belong to the so called nonlinearly parameterised systems. Under such a situation, the adaptive control design becomes much more challenging since the certainty equivalence principle may not be straightforwardly applicable and the construction of adaptation law and Lyapunov function is generally difficult. When the discussed systems are first-order and/or satisfy the matching conditions, adaptive controls for nonlinearly parameterised systems have been developed in [7, 9–11], among others. For a class of first-order plants containing nonlinearities of the form of ratios of polynomials in the process output with unknown parameters, an adaptive control design was presented in [7] by choosing a suitable Lyapunov function for the closed-loop system. By using an integral-type Lyapunov function, adaptive control was proposed in [11] for a class of more general first-order nonlinearly parameterised systems than that studied in [7]. Adaptive tracking of nonlinearly parameterised systems has also been reported in [9] for concave or convex case. More recently, a new robust-observer-based adaptive control was designed under the assumption that the system uncertainties have a nonlinear parameterisation in [10]. When nonlinearly parameterised systems are in triangular form and do not satisfy the matching conditions, there are also some recent works available in literature, such as that in [12, 13]. In [12], robust adaptive control technique is incorporated in the backstepping control design with flat zones to tackle the nonlinear parameterisation together with a novel smooth projection algorithms for parameter estimation. In our recent work [13], a new adaptive control is proposed by introducing a biasing vector function into parameter estimate, which can also be used for a class of strict-feedback nonlinearly parameterised systems without disturbance.

As an alternative, stable adaptive neural network control of nonlinear uncertain systems has also been extensively studied in the past decade along the line of using linearly parameterised adaptive control techniques [14]. For instance, stable adaptive neural controllers were developed in [15] for nonlinear systems in a Brunovsky form. For nonlinear systems not satisfying matching conditions, stable adaptive neural baskstepping controls were proposed in [14, 16, 17]. The main ideas behind those developments are to approximate the unknown nonlinear functions by parameterised neural networks and then through the adaptive tuning of neural networks' weights to achieve the design objective.

In this paper, we consider the robust adaptive control for a class of perturbed time-varying nonlinearly parameterised systems of the form

$$\begin{aligned} \dot{x}_i &= g_i(\bar{x}_i, t)x_{i+1} + f_i(\bar{x}_i, t) + \Delta_i(t, x), \quad i = 1, \ldots, n-1 \\ \dot{x}_n &= g_n(x, t)u + f_n(x, t) + \Delta_n(t, x) \\ y &= x_1 \end{aligned} \tag{1}$$

where $x = [x_1, \ldots, x_n]^{\mathrm{T}} \in R^n$ is the state vector, $\bar{x}_i = [x_1, \ldots, x_i]^{\mathrm{T}}$, $i = 1, \ldots, n-1$, $u \in R$ is the control, $y \in R$ is the output, $f_i(\bar{x}_i, t)$ and $g_i(\bar{x}_i, t)$ are unknown nonlinear system functions which are parameterised not linearly but of a fractional expression, and Δ_i are unknown Lipschitz continuous functions. A new robust adaptive control is proposed by using neural networks to approximate the completely unknown bounding functions of Δ_i and using smooth parameter projection algorithms to estimate the unknown time-varying parameters associated with nonlinear functions f_i and g_i. Parameters estimation errors are robustly compensated for by the additive terms within the proposed virtual and actual controls. In addition, with the aid of Nussbaum gain [18, 19], the proposed design does not require the a priori knowledge of the signs of the unknown control coefficients g_i. It is rigourously proved that the proposed robust adaptive control can ensure the semi-global uniform ultimate boundedness of all the closed-loop system signals. The size of the bounding compact set can be reduced by suitably adjusting the control design parameters. The main contribution of the paper is 2-fold: 1. the proposed design can explicitly address the nonlinear system (1) with time-varying unknown parameters and functions f_i and g_i having a fractional structure; 2. the proposed control applies neural network approximation to handle unknown dynamics Δ_i without imposing any size information on them.

The paper is organised as follows. Section 2 formulates the control problem and provides the basic preliminaries. Section 3 presents the proposed robust adaptive neural control design procedures and gives the rigorous stability analysis of the closed-loop system. Simulation results are given in Section 4, and Section 5 concludes the paper.

2 Preliminaries

2.1 *Problem formulation*

Consider the control problem of time-varying nonlinear uncertain systems given by (1). The control objective is to construct a robust adaptive control such that: (i) all the closed-loop signals remain semi-globally uniformly ultimately bounded and (ii) the output y of system (1) follows the desired reference signal $y_d(t) \in \Re$, which is a smooth bounded signal with bounded time derivatives $y_d^{(i)}(t)$, $1 \le i \le n$. In order to design adaptive control for system (1), the following assumptions are introduced.

Assumption 1: Uncertain dynamics in the system have a fractional parameterisation of the form

$$f_i(\bar{x}_i, t) = \frac{\theta_{f_{in}}^{\mathrm{T}}(t)\psi_{f_{in}}(\bar{x}_i, t)}{\theta_{f_{id}}^{\mathrm{T}}(t)\psi_{f_{id}}(\bar{x}_i, t)}, \quad g_i(\bar{x}_i, t) = \frac{\theta_{g_{in}}^{\mathrm{T}}(t)\psi_{g_{in}}(\bar{x}_i, t)}{\theta_{g_{id}}^{\mathrm{T}}(t)\psi_{g_{id}}(\bar{x}_i, t)} \tag{2}$$

where $\theta_{f_{in}} \in R^{p_{1i}}$, $\theta_{g_{in}} \in R^{p_{2i}}$, $\theta_{f_{id}} \in R^{p_{3i}}$, and $\theta_{g_{id}} \in R^{p_{4i}}$ are unknown time-varying parameters belonging to known compact sets $\Omega_{ji} \subset R^{p_{ji}}$ for $j = 1, \ldots, 4$, respectively, $\psi_{f_{in}}(\bar{x}_i, t)$, $\psi_{g_{in}}(\bar{x}_i, t)\psi_{f_{id}}(\bar{x}_i, t)$, and $\psi_{g_{id}}(\bar{x}_i, t)$ are dimensionally compatible smooth known functional vectors. In addition, for the well-defined non-singular functions f_i and g_i, without loss of generality, we assume that there exist some functions $c_{1i}(\bar{x}_i)$ and $c_{2i}(\bar{x}_i)$ and constants $\underline{c}_{1i} > 0$ and $\underline{c}_{2i} > 0$ such that, for all $(\bar{x}_i, t)$ and for all $\theta_{f_{id}} \in \Omega_{2i}$ and $\theta_{g_{id}} \in \Omega_{4i}$

$$\begin{aligned} \theta_{f_{id}}^{\mathrm{T}}(t)\psi_{f_{id}}(\bar{x}_i, t) &\ge c_{1i}(\bar{x}_i) \ge \underline{c}_{1i}, \\ \theta_{g_{id}}^{\mathrm{T}}\psi_{g_{id}}(\bar{x}_i, t) &\ge c_{2i}(\bar{x}_i) \ge \underline{c}_{2i} \end{aligned} \tag{3}$$

For simplicity, let Ω_{ji} be a closed ball of known radius r_{ji} centred in the origin, for $j = 1, \ldots, 4$.

Assumption 2: The state-dependent and time-varying control coefficients $g_i(\bar{x}_i, t) \neq 0$ for all $\bar{x}_i \in \Re^i$ and for all t.

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Without loss of generality, we assume that $g_i(\bar{x}_i, t) > 0, \forall \bar{x}_i \in \Re^i, \forall t$.

Assumption 3: For $1 \le i \le n$, there exists unknown non-negative smooth function $\phi_i(\bar{x}_i)$ such that $\forall (t, x) \in R_+ \times R^n$

$$|\Delta_i(t, x)| \le \phi_i(\bar{x}_i) \tag{4}$$

Remark 1: The considered nonlinear systems (1) belong to a more general class of nonlinear uncertain systems. Specifically, systems (1) contain two classes of uncertainties: one is the time-varying parameters uncertainty as in the fractional parameterisation (2), and in this sense the systems (1) belongs to the so-called nonlinearly parameterised nonlinear systems; another is the completely unknown nonlinearity Δ_i, which could be due to many factors [20], such as measurement noise, modelling errors, external disturbances, modelling simplifications or changes due to time variations. Hence, it is generally difficult to absorb the completely unknown nonlinearity Δ_i into f_i if not impossible.

In this paper, instead of seeking the complete approximation of system nonlinearities, a robust adaptive control is proposed by fully taking into consideration the system structure properties, that is, the parameters uncertainty in (2) is handled directly using adaptation law together with Nussbaum gain technique to cope with the unknown control directions in g_i, while unknown nonlinearity Δ_i is handled using neural network approximation. The proposed design expands the class of nonlinear systems for which robust adaptive control approaches have been studied.

Remark 2: The fractional parameterisation structures in (2) show that parameters $\theta_{f_{in}}$ and $\theta_{g_{in}}$ enter the system (1) linearly, but $\theta_{f_{id}}$ and $\theta_{g_{id}}$ appear nonlinearly. Some practical systems in biochemical engineering can be expressed into such a structure [7, 8]. For example, a fermentation process model given in [7] has the expression of

$$\dot{x} = \frac{\rho k_1 x^2}{k_2 + k_3 x} + g(x)u$$

where ρ, k_1, k_2 and k_3 are unknown parameters.

Remark 3: In Assumption 1, inequality (3) is introduced to avoid the crossing of zero of the denominators of functions $f_i(\bar{x}_i, t)$ and $g_i(\bar{x}_i, t)$, which is a standard assumption in order to ensure that uncertainties to be compensated for are locally uniformly bounded. In addition, the lower bounds $\underline{c}_{1i}$ and $\underline{c}_{2i}$ in (3) are only used for analysis purpose in the proposed robust adaptive neural control, and their values are not necessarily known.

Remark 4: In the literature, $g_i(\bar{x}_i, t)$, $i = 1, \ldots, n$ are referred to as virtual control coefficients [1], whose values should not be zero for the purpose of controllability of system (1). Therefore, Assumption 2 is a standard one as shown in literature of adaptive nonlinear control [1]. Apparently, to be consistent with the inequality with respect to $g_i(\bar{x}_i, t)$ in (3), Assumption 2 also implies that numerator of $g_i(\bar{x}_i, t)$ satisfies $\theta_{g_{in}}^{\mathrm{T}}(t)\psi_{g_{in}}(\bar{x}_i, t) > 0$.

Remark 5: Assumption 3 implies that the allowed class of uncertainties Δ_i satisfy a triangularity condition in terms of $\bar{x}_i$ which is standard in the literature of robust adaptive control of nonlinear systems [20–22]. In this paper, a linearly parameterised approximator [radial basis function (RBF) neural network] will be applied to approximate the unknown bounding function $\phi_i(\bar{x}_i)$. It is worth mentioning that if Δ_i is time-independent, we can directly approximate Δ_i using neural network instead of approximating the bounding function and then same control design procedures in this paper follow.

In this paper, we address the robust adaptive control design problem for system (1) with completely unknown virtual control coefficients $g_i(\bar{x}_i, t)$ by using Nussbaum gain technique. A function $N(\zeta)$ is called a Nussbaum-type function if it has the following properties [18]

$$\lim_{s\to\infty} \sup \frac{1}{s}\int_0^s N(\zeta)\mathrm{d}\zeta = \infty, \quad \lim_{s\to\infty} \inf \frac{1}{s}\int_0^s N(\zeta)\mathrm{d}\zeta = -\infty$$

Commonly used Nussbaum functions include: $k^2\cos(k)$, $k^2\sin(k)$ and $\exp(k^2)\cos((\pi/2)k)$ [23]. In this paper, the even Nussbaum function, $\exp(\zeta^2)\cos((\pi/2)\zeta)$, is exploited.

Lemma 1: [19]: Let $V(\cdot)$ and $\zeta(\cdot)$ be smooth functions defined on $[0, t_f)$ with $V(t) \ge 0, \forall t \in [0, t_f)$. If the following inequality holds

$$\begin{aligned} V(t) \le c_0 + e^{-c_1 t}\int_0^t g_1(x)N(\zeta)\dot{\zeta}e^{c_1\tau}\mathrm{d}\tau \\ + e^{-c_1 t}\int_0^t \dot{\zeta}e^{c_1\tau}\mathrm{d}\tau, \quad \forall t \in [0, t_f) \end{aligned} \tag{5}$$

where constant $c_1 > 0$, $g_1(x)$ takes values in the unknown closed intervals $I_1 := [l_1^-, l_1^+]$ with $0 \notin I_1$, and c_0 represents some suitable constant, then $V(t)$, $\zeta(t)$ and $\int_0^t g_1(x)N(\zeta)\dot{\zeta}\mathrm{d}\tau$ must be bounded on $[0, t_f)$.

According to Prop. 2 of [24], if the solution of the resulting closed-loop system is bounded, then $t_f = \infty$.

A smooth project algorithm as given by Definition 1 is used in this paper for parameters estimation.

Definition 1: [25] Let $\theta(t) \in \Omega$ be an unknown time-varying parameter vector, $\hat{\theta}$ be the estimate, and $\Omega \subset R^p$ be a closed ball of known radius r_Ω. The projection algorithm $\mathrm{Proj}(y, \hat{\theta})$ is given by

$$\mathrm{Proj}(y, \hat{\theta}) = \begin{cases} y, & \text{if } p(\hat{\theta}) \le 0 \\ y, & \text{if } p(\hat{\theta}) \ge 0 \text{ and } \dfrac{\partial p}{\partial \hat{\theta}} y \le 0 \\ \left[I - \dfrac{p(\hat{\theta})(\partial p/\partial \hat{\theta})^{\mathrm{T}} (\partial p/\partial \hat{\theta})}{\|\partial p/\partial \hat{\theta}\|^2} \right] y, & \\ \quad \text{if } p(\hat{\theta}) \ge 0 \text{ and } \dfrac{\partial p}{\partial \hat{\theta}} y > 0 & \end{cases} \tag{6}$$

$$p(\hat{\theta}) = \frac{\|\hat{\theta}\|^2 - r_{\Omega}^2}{\epsilon^2 + 2\epsilon r_{\Omega}} \tag{7}$$

where ϵ is an arbitrary positive real. From (6), if $\hat{\theta}(0) \in \Omega$, we have the following nice property: $\|\hat{\theta}(t)\| \le r_{\Omega} + \epsilon$, $\forall t \ge 0$.

It is worth mentioning that when virtual control coefficients g_i are known constants or unknown constants but with known signs of g_i's and $f_i(\bar{x}_i, t)$ have a linearly parameterised structure, that is, $f_i(\bar{x}_i, t) = \theta_{f_{in}}^{\mathrm{T}}(t)\psi_{f_{in}}(\bar{x}_i, t)$, robust adaptive controls for systems similar to (1) have been developed, such as that in [20, 21, 26] for the case of $g_i = 1$, and that in [1] for g_i's being constants with known signs. When there is no a priori knowledge about the signs of g_i's, the control design problem becomes much more difficult, and Nussbaum gain technique has been seen as an effective tools to solve this problem for certain classes of nonlinear systems [19, 27, 28]. In particular, Lemma 1 was applied in our recent work [19] to solve the robust adaptive tracking problem of a class of time-varying nonlinear system with linearly parameterised uncertainties and with completely unknown g_i's being only explicitly time-dependent functions. In this paper, we consider a more general and complicated class of nonlinear uncertain systems in (1), which has nonlinearly parameterised uncertainties and g_i's are both state-dependent and explicitly time dependent. In addition, disturbance terms Δ_i also do not assume a known linearly parameterised structure, while neural networks are employed to approximate their unknown bounding functions $\phi_i(\bar{x}_i)$.

2.2 *Linearly parameterised neural networks*

A linearly parameterised approximator will be used to approximate the unknown bounding functions $\phi_i(\cdot)$. Several function approximators can be applied for this purpose, such as, RBF neural networks [14, 29], high-order neural networks [30] and fuzzy systems [31], which can be described as $W^{\mathrm{T}}S(z)$ with input vector $z \in R^n$, weight vector $W \in R^l$, node number l, and basis function vector $S(z) \in R^l$. Universal approximation results indicate that, if l is chosen sufficiently large, then $W^{\mathrm{T}}S(z)$ can approximate any continuous function to any desired accuracy over a compact set [30]. In this paper, we use the RBF NN to approximate a smooth function. That is, for the unknown nonlinear functions $\phi_i(\bar{x}_i)$, $i = 1, \ldots, n$ in (4), we have the following approximation over the compact sets Ω_i

$$\phi_i(\bar{x}_i) = W_i^{*\mathrm{T}}\psi_i(\bar{x}_i) + \omega_i(\bar{x}_i), \quad \forall \bar{x}_i \in \Omega_i \subset R^i \tag{8}$$

where $W_i^* \in R^{l_i}$ is an unknown constant parameter vector, the NN node number $l_i > 1$, $\omega_i(\bar{x}_i)$ is the approximation error, and $\psi(\bar{x}_i) = [\psi_{i1}(\bar{x}_i), \ldots, \psi_{il_i}(\bar{x}_i)]^{\mathrm{T}}$ is the basis function vector, with $\psi_{ij}(\bar{x}_i)$ being chosen as the commonly used Gaussian functions, which have the form

$$\psi_{ij}(\bar{x}_i) = \exp\left[\frac{-(\bar{x}_i - \mu_{ij})^{\mathrm{T}}(\bar{x}_i - \mu_{ij})}{\eta_{ij}^2}\right], \quad j = 1, 2, \ldots, l_i \tag{9}$$

where $\mu_{ij} = [\mu_{ij1}, \mu_{ij2}, \ldots, \mu_{iji}]^T$ is the centre of the receptive field and η_{ij} is the width of the Gaussian function.

Remark 6: The optimal weight vector W_i^* in (8) is an 'artificial' quantity required only for analytical purposes. Typically, W_i^* is chosen as the value of W_i that minimises $\omega_i(\bar{x}_i)$ for all $\bar{x}_i \in \Omega_i$, that is

$$W_i^* := \arg \min_{W_i \in R^{l_i}} \left\{ \sup_{\bar{x}_i \in \Omega_i} |\phi_i(\bar{x}_i) - W_i^{\mathrm{T}}\psi(\bar{x}_i)| \right\} \tag{10}$$

According to the universal approximation theorem [29, 30], approximation error $\omega_i(\bar{x}_i)$ must be bounded upon having the expression of (8). The following assumption on the approximation error is thus in order.

Assumption 4: Over a compact region $\Omega_i \in R^i$

$$|\omega_i(\bar{x}_i)| \le \delta_i^* \quad \forall \bar{x}_i \in \Omega_i, \quad i = 1, \ldots, n \tag{11}$$

where $\delta_i^* \ge 0$ is an unknown bound.

To this end, it can be seen that, the system described by (1) has two types of uncertainty: parametric uncertainty, which is due to unknown parameters $\theta_{f_{in}}$, $\theta_{g_{in}}$, $\theta_{f_{id}}$ and $\theta_{g_{id}}$, and the bounding uncertainty which is due to unknown Δ_i. In addition, the sign of uncertain control coefficients $g_i(\cdot)$ are completely unknown. In this paper, we present a robust adaptive control design for the class of uncertain nonlinear systems (1) by adaptively updating the estimates of unknown parameters $\theta_{f_{in}}$, $\theta_{g_{in}}$, $\theta_{f_{id}}$, $\theta_{g_{id}}$, W_i^* and δ_i^*, as will be detailed in the following section.

3 Robust adaptive neural control design

In this section, the robust adaptive neural control design procedure for nonlinear system (1) and the stability analysis

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of the closed-loop adaptive control system are presented. Our design consists of n steps. The design of both the control law and the adaptation laws is based on a change of coordinates

$$
\begin{aligned}
z_1 &= x_1 - y_d \\
z_2 &= x_2 - \alpha_1(x_1, \dot{y}_d, \hat{W}_{a,1}, \hat{\theta}_{f_{1n}}, \hat{\theta}_{f_{1d}}, t) \\
&\vdots \\
z_i &= x_i - \alpha_{i-1}(x_1, \ldots, x_{i-1}, \dot{y}_d, \ldots, y_d^{(i-1)}, \hat{W}_{a,1}, \ldots, \\
&\quad \hat{W}_{a,i-1}, \hat{\theta}_{f_{1n}}, \ldots, \hat{\theta}_{f_{(i-1)n}}, \hat{\theta}_{f_{1d}}, \ldots, \hat{\theta}_{f_{(i-1)d}}, \hat{\theta}_{g_{1n}}, \ldots, \\
&\quad \hat{\theta}_{g_{(i-2)n}}, \hat{\theta}_{g_{1d}}, \ldots, \hat{\theta}_{g_{(i-2)d}}, t)
\end{aligned} \tag{12}
$$

where $i = 2, \ldots, n$, the functions α_i, $i = 1, \ldots, n-1$ are referred to as intermediate control functions, which will be designed using backstepping, and $\hat{\theta}_{f_{in}}$, $\hat{\theta}_{f_{id}}$, $\hat{\theta}_{g_{in}}$ and $\hat{\theta}_{g_{id}}$ represent the estimates of unknown parameters $\theta_{f_{in}}$, $\theta_{f_{id}}$, $\theta_{g_{in}}$ and $\theta_{g_{id}}$, respectively. $\hat{W}_{a,i}$ represents the estimate of unknown parameter $W_{a,i}^*$ that is an augmented parameter and consists of $\delta_j^*, j = 1, \ldots, i-1$ and ideal neural network weight $W_j^*, j = 1, \ldots, i$ as clarified later. At each intermediate step i, we design the intermediate control function α_i using an appropriate Lyapunov function V_i, and give the adaptation laws $\dot{\hat{W}}_{a,i}$. The parameters adaptation laws for $\hat{\theta}_{f_{in}}$ and $\hat{\theta}_{g_{in}}$ are given below

$$
\dot{\hat{\theta}}_{f_{in}} = \gamma \mathrm{Proj}(z_i \psi_{f_{in}}, \hat{\theta}_{f_{in}}), \quad \dot{\hat{\theta}}_{g_{in}} = \gamma \mathrm{Proj}(z_i \psi_{g_{in}}, \hat{\theta}_{g_{in}}) \tag{13}
$$

where $\mathrm{Proj}(\cdot)$ is the smooth projection algorithm as given by Definition 1 and $\gamma > 0$ is a design parameter. The estimates $\hat{\theta}_{f_{id}}$ and $\hat{\theta}_{g_{id}}$ are selected such that inequalities $\hat{\theta}_{f_{id}}^{\mathrm{T}}(t)\psi_{f_{id}} > 0$ and $\hat{\theta}_{g_{id}}^{\mathrm{T}}(t)\psi_{g_{id}} > 0$ are satisfied. At the nth step, the actual control u appears and the design is completed.

Remark 7: The estimates $\hat{\theta}_{f_{in}}$ and $\hat{\theta}_{g_{in}}$ for unknown parameters $\theta_{f_{in}}$ and $\theta_{g_{in}}$ in the numerators of functions f_i and g_i in (2) are updated using adaptation laws in (13). While for unknown parameters $\theta_{f_{in}}$ and $\theta_{g_{in}}$ in the denominators of functions f_i and g_i, we simply choose their estimates $\hat{\theta}_{f_{id}}$ and $\hat{\theta}_{g_{id}}$ arbitrarily as long as inequalities $\hat{\theta}_{f_{id}}^{\mathrm{T}}(t)\psi_{f_{id}} > 0$ and $\hat{\theta}_{g_{id}}^{\mathrm{T}}(t)\psi_{g_{id}} > 0$ are satisfied in order to avoid the control singularity in the proposed virtual and actual controls given in (18), (34) and (45). This is generally not difficult since functions $\psi_{f_{id}}$ and $\psi_{g_{id}}$ are known, and by analysing their properties the suitable choices of $\hat{\theta}_{f_{id}}$ and $\hat{\theta}_{g_{id}}$ can be made to satisfy inequalities $\hat{\theta}_{f_{id}}^{\mathrm{T}}(t)\psi_{f_{id}} > 0$ and $\hat{\theta}_{g_{id}}^{\mathrm{T}}(t)\psi_{g_{id}} > 0$ for all t.

For example, in the simulation, we have $\psi_{g_{1d}} = 1 + x_1^2$ and $\psi_{f_{1d}} = 2 + \sin(t)$, thus we can simply let the estimates be constants as $\hat{\theta}_{g_{1d}}(t) = 1$ and $\hat{\theta}_{f_{1d}}(t) = 2$. It is worth noting that the choice of $\hat{\theta}_{f_{id}}$ and $\hat{\theta}_{g_{id}}$ is different from that in robust control design because this does not require to know the exact values of bounding information $\underline{c}_{1i}$ and $\underline{c}_{2i}$ in (3).

Remark 8: Backstepping is employed in the proposed control design, and this recursive design procedure generates a virtual control (34) and the corresponding adaptation law (35). Thus, for the given nth-order system (1), all the functions involved in the control design are required to be at least $(n-1)$ times continuously differentiable. Without loss of generality, we assume that the functions involved are sufficiently smooth.

Remark 9: The results obtained in this paper are semi-global, in the sense that they are valid as long as $\bar{x}_i$ remains in Ω_i, where the set Ω_i and bounding parameter δ_i^* can be arbitrarily large. In the special case that (11) holds for all $\bar{x}_i \in \Re^i$, the results become global.

Step 1: To start, let us study the following subsystem of (1)

$$
\dot{x}_1 = g_1(x_1, t)x_2 + \frac{\theta_{f_{1n}}^{\mathrm{T}}(t)\psi_{f_{1n}}(x_1, t)}{\theta_{f_{1d}}^{\mathrm{T}}(t)\psi_{f_{1d}}(x_1, t)} + \Delta_1(x, t) \tag{14}
$$

where x_2 is taken for a virtual control input. Let $V_0(z_1) = (1/2)z_1^2$. In light of Assumptions 3 and 4, the time derivative of V_0 along the solution of (14) satisfies

$$
\begin{aligned}
\dot{V}_0 &= z_1\left[g_1(x_1, t)x_2 + \frac{\theta_{f_{1n}}^{\mathrm{T}}(t)\psi_{f_{1n}}(x_1, t)}{\theta_{f_{1d}(t)}^{\mathrm{T}}\psi_{f_{1d}}(x_1, t)} - \dot{y}_d\right] + z_1\Delta_1(x, t) \\
&\le z_1\left[g_1(x_1, t)x_2 + \frac{\theta_{f_{1n}}^{\mathrm{T}}(t)\psi_{f_{1n}}(x_1, t)}{\theta_{f_{1d}}^{\mathrm{T}}(t)\psi_{f_{1d}}(x_1, t)} - \dot{y}_d\right] \\
&\quad + |z_1| W_{a,1}^{*\mathrm{T}} \bar{\psi}_{a,1}(x_1)
\end{aligned} \tag{15}
$$

where $W_{a,1}^* = [W_1^{*\mathrm{T}}, \delta_1^*]^{\mathrm{T}}$, $\bar{\psi}_{a,1}(x_1) = [\psi_1^{\mathrm{T}}, 1]^{\mathrm{T}}$. Consider the Lyapunov function candidate V_1

$$
V_1 = V_0 + \frac{1}{2}(\hat{W}_{a,1} - W_{a,1}^*)^{\mathrm{T}}\Gamma_1^{-1}(\hat{W}_{a,1} - W_{a,1}^*) \tag{16}
$$

where $\Gamma_1 = \Gamma_1^{\mathrm{T}} > 0$. Then, the time derivative of V_1 along (15) is

$$
\begin{aligned}
\dot{V}_1 &\le z_1\left[g_1(x_1, t)x_2 + \frac{\theta_{f_{1n}}^{\mathrm{T}}(t)\psi_{f_{1n}}(x_1, t)}{\theta_{f_{1d}}^{\mathrm{T}}(t)\psi_{f_{1d}}(x_1, t)} - \dot{y}_d\right] \\
&\quad + |z_1| W_{a,1}^{*\mathrm{T}} \bar{\psi}_{a,1}(x_1) + (\hat{W}_{a,1} - W_{a,1}^*)^{\mathrm{T}}\Gamma_1^{-1}\dot{\hat{W}}_{a,1}
\end{aligned} \tag{17}
$$

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Let the intermediate control function α_1 be

$$\begin{aligned}\alpha_1(x_1, \dot{y}_d, \hat{W}_{a,1}, \hat{\theta}_{f_{1n}}, \hat{\theta}_{f_{1d}}, t) &= N(\zeta_1)\Bigg[k_1 z_1 + \frac{\hat{\theta}^{\mathrm{T}}_{f_{1n}}\psi_{f_{1n}}(x_1, t)}{\hat{\theta}^{\mathrm{T}}_{f_{1d}}\psi_{f_{1d}}(x_1, t)} + \hat{W}^{\mathrm{T}}_{a,1}\,\mathrm{Tanh}\left(\frac{z_1\overline{\psi}_{a,1}}{\epsilon_1}\right) \\ &\quad \times \overline{\psi}_{a,1}(x_1) - \dot{y}_d - v_1(x_1, t)\Bigg] \end{aligned} \tag{18}$$

with

$$\begin{aligned}\dot{\zeta}_1 &= k_1 z_1^2 + z_1\frac{\hat{\theta}^{\mathrm{T}}_{f_{1n}}\psi_{f_{1n}}(x_1, t)}{\hat{\theta}^{\mathrm{T}}_{f_{1d}}\psi_{f_{1d}}(x_1, t)} + z_1\hat{W}^{\mathrm{T}}_{a,1}\,\mathrm{Tanh}\left(\frac{z_1\overline{\psi}_{a,1}}{\epsilon_1}\right) \\ &\quad \times \overline{\psi}_{a,1}(x_1) - z_1\dot{y}_d - z_1 v_1(x_1, t)\end{aligned} \tag{19}$$

$$v_1 = -\frac{1}{4}kz_1\psi^{\mathrm{T}}_{f_{1n}}\psi_{f_{1n}} - \frac{1}{4}kz_1\psi^{\mathrm{T}}_{f_{1d}}\psi_{f_{1d}}\left(\frac{\hat{\theta}^{\mathrm{T}}_{f_{1n}}\psi_{f_{1n}}}{\hat{\theta}^{\mathrm{T}}_{f_{1d}}\psi_{f_{1d}}}\right)^2 \tag{20}$$

where constants $k > 0$ and $k_1 > 1/4$, ϵ_1 is a small constant, $N(\zeta_1)$ is an even smooth Nussbaum-type function, and

$$\mathrm{Tanh}\left(\frac{z_1\overline{\psi}_{a,1}}{\epsilon_1}\right) \triangleq \mathrm{diag}\left\{\tanh\left(\frac{z_1\overline{\psi}_{a,11}}{\epsilon_1}\right), \tanh\left(\frac{z_1\overline{\psi}_{a,12}}{\epsilon_1}\right), \ldots, \tanh\left(\frac{z_1\overline{\psi}_{a,1(l_1+1)}}{\epsilon_1}\right)\right\}$$

with $\overline{\psi}_{a,1j}$ being the jth element of $\overline{\psi}_{a,1}$.

Let the parameters adaptation law be

$$\dot{\hat{W}}_{a,1} = \Gamma_1 z_1 \mathrm{Tanh}\left(\frac{z_1\overline{\psi}_{a,1}}{\epsilon_1}\right)\overline{\psi}_{a,1} - \Gamma_1\sigma_1(\hat{W}_{a,1} - W^0_{a,1}) \tag{21}$$

where $\sigma_1 > 0$ and $W^0_{a,1}$ are design constants.

Remark 10: In order to prevent parameter drifts due to inherent errors of neural network approximation, we incorporate a leakage term into the adaptation law for neural network weights based on a σ-modification, which is useful in establishing the differential inequality in terms of Lyapunov function V_i in each step for stability proof.

Using (18), a direct substitution of $x_2 = z_2 + \alpha_1$ and (20) into (17) gives

$$\begin{aligned}\dot{V}_1 &\le g_1 z_1 z_2 + g_1 z_1\alpha_1 + z_1\frac{\theta^{\mathrm{T}}_{f_{1n}}\psi_{f_{1n}}}{\theta^{\mathrm{T}}_{f_{1d}}\psi_{f_{1d}}} - z_1\dot{y}_d \\ &\quad + |z_1|W^{*\mathrm{T}}_{a,1}\overline{\psi}_{a,1}(x_1) + (\hat{W}_{a,1} - W^*_{a,1})^{\mathrm{T}}\Gamma_1^{-1}\dot{\hat{W}}_{a,1}\end{aligned} \tag{22}$$

Adding and subtracting

$$k_1 z_1^2 + z_1\frac{\hat{\theta}^{\mathrm{T}}_{f_{1n}}\psi_{f_{1n}}(x_1)}{\hat{\theta}^{\mathrm{T}}_{f_{1d}}\psi_{f_{1d}}(x_1)} - z_1 v_1(x_1, t) + z_1\hat{W}^{\mathrm{T}}_{a,1}\mathrm{Tanh}\left(\frac{z_1\overline{\psi}_{a,1}}{\epsilon_1}\right)\overline{\psi}_{a,1}(x_1) + z_1\frac{\hat{\theta}^{\mathrm{T}}_{f_{1n}}\psi_{f_{1n}}}{\theta^{\mathrm{T}}_{f_{1d}}\psi_{f_{1d}}}$$

on the right hand of (22), and noting (19) and (21), we have

$$\begin{aligned}\dot{V}_1 &\le -k_1 z_1^2 + g_1 z_1 z_2 + g_1 N(\zeta_1)\dot{\zeta}_1 + \dot{\zeta}_1 + z_1 v_1 \\ &\quad + z_1\frac{\theta^{\mathrm{T}}_{f_{1n}}\psi_{f_{1n}}}{\theta^{\mathrm{T}}_{f_{1d}}\psi_{f_{1d}}} - z_1\frac{\hat{\theta}^{\mathrm{T}}_{f_{1n}}\psi_{f_{1n}}}{\hat{\theta}^{\mathrm{T}}_{f_{1d}}\psi_{f_{1d}}} + z_1\frac{\hat{\theta}^{\mathrm{T}}_{f_{1n}}\psi_{f_{1n}}}{\theta^{\mathrm{T}}_{f_{1d}}\psi_{f_{1d}}} - z_1\frac{\hat{\theta}^{\mathrm{T}}_{f_{1n}}\psi_{f_{1n}}}{\theta^{\mathrm{T}}_{f_{1d}}\psi_{f_{1d}}} \\ &\quad + |z_1|W^{*\mathrm{T}}_{a,1}\overline{\psi}_{a,1}(x_1) - z_1 W^{*\mathrm{T}}_{a,1}\mathrm{Tanh}\left(\frac{z_1\overline{\psi}_{a,1}}{\epsilon_1}\right)\overline{\psi}_{a,1}(x_1) \\ &\quad - \sigma_1(\hat{W}_{a,1} - W^*_{a,1})^{\mathrm{T}}(\hat{W}_{a,1} - W^0_{a,1}) \\ &= -k_1 z_1^2 + g_1 z_1 z_2 + g_1 N(\zeta_1)\dot{\zeta}_1 + \dot{\zeta}_1 + z_1 v_1 \\ &\quad + z_1\frac{\tilde{\theta}^{\mathrm{T}}_{f_{1n}}\psi_{f_{1n}}}{\theta^{\mathrm{T}}_{f_{1d}}\psi_{f_{1d}}} - z_1\frac{\hat{\theta}^{\mathrm{T}}_{f_{1n}}\psi_{f_{1n}}\tilde{\theta}^{\mathrm{T}}_{f_{1d}}\psi_{f_{1d}}}{\hat{\theta}^{\mathrm{T}}_{f_{1d}}\psi_{f_{1d}}\theta^{\mathrm{T}}_{f_{1d}}\psi_{f_{1d}}} \\ &\quad + |z_1|W^{*\mathrm{T}}_{a,1}\overline{\psi}_{a,1}(x_1) - z_1 W^{*\mathrm{T}}_{a,1}\mathrm{Tanh}\left(\frac{z_1\overline{\psi}_{a,1}}{\epsilon_1}\right)\overline{\psi}_{a,1}(x_1) \\ &\quad - \sigma_1(\hat{W}_{a,1} - W^*_{a,1})^{\mathrm{T}}(\hat{W}_{a,1} - W^0_{a,1})\end{aligned} \tag{23}$$

where $\tilde{\theta}_* = \theta_* - \hat{\theta}_*$. Noting v_1 in (20) and by completing the squares, we have

$$-\frac{1}{4}kz_1^2\psi^{\mathrm{T}}_{f_{1n}}\psi_{f_{1n}} + z_1\frac{\tilde{\theta}^{\mathrm{T}}_{f_{1n}}\psi_{f_{1n}}}{\theta^{\mathrm{T}}_{f_{1d}}\psi_{f_{1d}}} \le \frac{1}{k}\frac{\tilde{\theta}^{\mathrm{T}}_{f_{1n}}\tilde{\theta}_{f_{1n}}}{(\theta^{\mathrm{T}}_{f_{1d}}\psi_{f_{1d}})^2}$$

$$-\frac{1}{4}kz_1^2\psi^{\mathrm{T}}_{f_{1d}}\psi_{f_{1d}}\left(\frac{\hat{\theta}^{\mathrm{T}}_{f_{1n}}\psi_{f_{1n}}}{\hat{\theta}^{\mathrm{T}}_{f_{1d}}\psi_{f_{1d}}}\right)^2 - z_1\frac{\hat{\theta}^{\mathrm{T}}_{f_{1n}}\psi_{f_{1n}}\tilde{\theta}^{\mathrm{T}}_{f_{1d}}\psi_{f_{1d}}}{\hat{\theta}^{\mathrm{T}}_{f_{1d}}\psi_{f_{1d}}\theta^{\mathrm{T}}_{f_{1d}}\psi_{f_{1d}}} \le \frac{1}{k}\frac{\tilde{\theta}^{\mathrm{T}}_{f_{1d}}\tilde{\theta}_{f_{1d}}}{(\theta^{\mathrm{T}}_{f_{1d}}\psi_{f_{1d}})^2}$$

$$\begin{aligned}\sigma_1(\hat{W}_{a,1} - W^*_{a,1})^{\mathrm{T}}(\hat{W}_{a,1} - W^0_{a,1}) &= \frac{1}{2}\sigma_1\|\hat{W}_{a,1} - W^*_{a,1}\|^2 \\ &\quad + \frac{1}{2}\sigma_1\|\hat{W}_{a,1} - W^0_{a,1}\|^2 - \frac{1}{2}\sigma_1\|W^*_{a,1} - W^0_{a,1}\|^2\end{aligned}$$

and using the following nice property with regard to function $\tanh(\cdot)$ [20]

$$0 \le |x| - x\tanh\left(\frac{x}{\epsilon}\right) \le 0.2785\epsilon, \quad \text{for } \epsilon > 0, x \in R \tag{24}$$

and noting $g_1 z_1 z_2 \le (1/4)z_1^2 + (g_1 z_2)^2$, (23) can be further

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written as

$$\dot{V}_1 \le -k_{10}z_1^2 + g_1 N(\zeta_1)\dot{\zeta}_1 + \dot{\zeta}_1 + g_1^2 z_2^2 - \frac{1}{2}\sigma_1 \|\hat{W}_{a,1} - W_{a,1}^*\|^2 + \sum_{j=1}^{l_1+1} |W_{a,1j}^*| 0.2785\epsilon + \frac{1}{2}\sigma_1 \|W_{a,1}^* - W_{a,1}^0\|^2 + \frac{1}{k}\frac{\tilde{\theta}_{f_{1n}}^{\mathrm{T}}\tilde{\theta}_{f_{1n}}}{(\theta_{f_{1d}}^{\mathrm{T}}\psi_{f_{1d}})^2} + \frac{1}{k}\frac{\tilde{\theta}_{f_{1d}}^{\mathrm{T}}\tilde{\theta}_{f_{1d}}}{(\theta_{f_{1d}}^{\mathrm{T}}\psi_{f_{1d}})^2} \tag{25}$$

with constant $k_{10} = k_1 - (1/4) > 0$. This yields

$$\dot{V}_1 \le -C_{11}V_1 + C_{12} + g_1 N(\zeta_1)\dot{\zeta}_1 + \dot{\zeta}_1 + g_1^2 z_2^2 + \frac{1}{k}\frac{\tilde{\theta}_{f_{1n}}^{\mathrm{T}}\tilde{\theta}_{f_{1n}}}{(\theta_{f_{1d}}^{\mathrm{T}}\psi_{f_{1d}})^2} + \frac{1}{k}\frac{\tilde{\theta}_{f_{1d}}^{\mathrm{T}}\tilde{\theta}_{f_{1d}}}{(\theta_{f_{1d}}^{\mathrm{T}}\psi_{f_{1d}})^2} \tag{26}$$

where

$$C_{11} \triangleq \min\left\{2k_{10}, \frac{\sigma_1}{\lambda_{\max}(\Gamma_1^{-1})}\right\}, \quad C_{12} \triangleq \sum_{j=1}^{l_1+1} |W_{a,1j}^*| 0.2785\epsilon_1 + \frac{1}{2}\sigma_1 \|W_{a,1}^* - W_{a,1}^0\|^2 \tag{27}$$

It follows from the integration of (26) over $[0, t]$ that

$$0 \le V_1(t) \le \frac{C_{12}}{C_{11}} + V_1(0)e^{-C_{11}t} + e^{-C_{11}t}\int_0^t (g_1 N(\zeta_1) + 1)\dot{\zeta}_1 e^{C_{11}\tau}\mathrm{d}\tau + \int_0^t \left[\frac{1}{k}\frac{\tilde{\theta}_{f_{1n}}^{\mathrm{T}}\tilde{\theta}_{f_{1n}}}{(\theta_{f_{1d}}^{\mathrm{T}}\psi_{f_{1d}})^2} + \frac{1}{k}\frac{\tilde{\theta}_{f_{1d}}^{\mathrm{T}}\tilde{\theta}_{f_{1d}}}{(\theta_{f_{1d}}^{\mathrm{T}}\psi_{f_{1d}})^2}\right] e^{-C_{11}(t-\tau)}\mathrm{d}\tau + \int_0^t g_1^2 z_2^2 e^{-C_{11}(t-\tau)}\mathrm{d}\tau \tag{28}$$

Remark 11: Due to the presence of

$$\int_0^t \left[\frac{1}{k}\frac{\tilde{\theta}_{f_{1n}}^{\mathrm{T}}\tilde{\theta}_{f_{1n}}}{(\theta_{f_{1d}}^{\mathrm{T}}\psi_{f_{1d}})^2} + \frac{1}{k}\frac{\tilde{\theta}_{f_{1d}}^{\mathrm{T}}\tilde{\theta}_{f_{1d}}}{(\theta_{f_{1d}}^{\mathrm{T}}\psi_{f_{1d}})^2}\right] e^{-C_{11}(t-\tau)}\mathrm{d}\tau$$

and $\int_0^t g_1^2 z_2^2 e^{-C_{11}(t-\tau)}\mathrm{d}\tau$ in (28), Lemma 1 cannot be applied directly. It follows from the adaptation law (13) that $\|\hat{\theta}_{f_{1n}}(t)\| \le r_1 + \epsilon$. Because $\|\theta_{f_{1n}}\| \le r_1$, we obtain $\|\tilde{\theta}_{f_{1n}}(t)\| \le 2r_1 + \epsilon$. On the other hand, the boundedness of $\tilde{\theta}_{f_{1d}}$ is apparent due to the choice of bounded $\hat{\theta}_{f_{1d}}$ and boundedness of $\theta_{f_{1d}}$, say, $\|\tilde{\theta}_{f_{1d}}\| \le \varepsilon_{11}$ for some constant $\varepsilon_{11} > 0$. Together with the assumption of $\underline{c}_{11} \le \theta_{f_{1d}}^{\mathrm{T}}\psi_{f_{1d}}$, we have

$$0 \le \int_0^t \left[\frac{1}{k}\frac{\tilde{\theta}_{f_{1n}}^{\mathrm{T}}\tilde{\theta}_{f_{1n}}}{(\theta_{f_{1d}}^{\mathrm{T}}\psi_{f_{1d}})^2} + \frac{1}{k}\frac{\tilde{\theta}_{f_{1d}}^{\mathrm{T}}\tilde{\theta}_{f_{1d}}}{(\theta_{f_{1d}}^{\mathrm{T}}\psi_{f_{1d}})^2}\right] \times e^{-C_{11}(t-\tau)}\mathrm{d}\tau \le \frac{1}{kC_{11}}\frac{(2r_1+\epsilon)^2 + \varepsilon_{11}^2}{\underline{c}_{11}^2}$$

which is bounded. Thus, if z_2 can be regulated as bounded such that $\int_0^t g_1^2 z_2^2 e^{-C_{11}(t-\tau)}\mathrm{d}\tau$ is bounded, then, according to Lemma 1, no finite-time escape phenomenon may happen for $t_f \to \infty$, and the uniformly ultimate boundedness of $z_1(t)$ can be guaranteed. The effect of $\int_0^t g_1^2 z_2^2 e^{-C_{11}(t-\tau)}\mathrm{d}\tau$ will be dealt with at the following steps.

Step i $(2 \le i \le n-1)$: A similar procedure is employed recursively for each step $i = 2, \ldots, n-1$. The derivative of $(1/2)z_i^2$ is

$$z_i\dot{z}_i = z_i\left[g_i x_{i+1} + \frac{\theta_{f_{in}}^{\mathrm{T}}\psi_{f_{in}}}{\theta_{f_{id}}^{\mathrm{T}}\psi_{f_{id}}} + \Delta_i - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j} \times \left(\frac{\theta_{g_{jn}}^{\mathrm{T}}\psi_{g_{jn}}}{\theta_{g_{jd}}^{\mathrm{T}}\psi_{g_{jd}}}x_{j+1} + \frac{\theta_{f_{jn}}^{\mathrm{T}}\psi_{f_{jn}}}{\theta_{f_{jd}}^{\mathrm{T}}\psi_{f_{jd}}} + \Delta_j\right) + \beta_i\right] \tag{29}$$

where

$$\beta_i = -\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}_{f_{jn}}}\dot{\hat{\theta}}_{f_{jn}} - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}_{g_{jn}}}\dot{\hat{\theta}}_{g_{jn}} - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_d^{(j)}}y_d^{(j+1)} - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{W}_{a,j}}\dot{\hat{W}}_{a,j} - \frac{\partial\alpha_{i-1}}{\partial t}$$

In view of Assumptions 3 and 4, we have

$$z_i\left(\Delta_i(t,x) - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\Delta_j(t,x)\right) \le |z_i|\left[W_i^{*\mathrm{T}}\psi_i + \sum_{j=1}^{i-1}W_j^{*\mathrm{T}}\left|\frac{\partial\alpha_{i-1}}{\partial x_j}\right|\psi_j + \delta_i^* + \sum_{j=1}^{i-1}\left|\frac{\partial\alpha_{i-1}}{\partial x_j}\right|\delta_j^*\right] \le |z_i| W_{a,i}^{*\mathrm{T}}\overline{\psi}_{a,i}(\overline{x}_i) \tag{30}$$

where by the abuse of notation, let $|W_j^*| \triangleq [|W_{j1}^*|, \ldots, |W_{jl_j}^*|]^{\mathrm{T}}$

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with W_{jk}^* being the kth element of W_j^* for $j = 1, \ldots, i$, and

$$W_{a,i}^* \triangleq \left[|W_i^*|^{\mathrm{T}}, |W_1^*|^{\mathrm{T}}, \ldots, |W_{i-1}^*|^{\mathrm{T}}, \delta_i^*, \delta_1^*, \ldots, \delta_{i-1}^* \right]^{\mathrm{T}}$$

$$\overline{\psi}_{a,i} \triangleq \left[\psi_i^{\mathrm{T}}, \left(\frac{1}{4}\left(\frac{\partial \alpha_{i-1}}{\partial x_1}\right)^2 + 1\right)\psi_1^{\mathrm{T}}, \ldots, \left(\frac{1}{4}\left(\frac{\partial \alpha_{i-1}}{\partial x_{i-1}}\right)^2 + 1\right) \times \psi_{i-1}^{\mathrm{T}}, 1, \left(\frac{1}{4}\left(\frac{\partial \alpha_{i-1}}{\partial x_1}\right)^2 + 1\right), \ldots, \left(\frac{1}{4}\left(\frac{\partial \alpha_{i-1}}{\partial x_{i-1}}\right)^2 + 1\right) \right]^{\mathrm{T}}$$

Thus, it follows from (29) to (30) that

$$z_i \dot{z}_i \le z_i \left[g_i x_{i+1} + \frac{\theta_{f_{in}}^{\mathrm{T}} \psi_{f_{in}}}{\theta_{f_{id}}^{\mathrm{T}} \psi_{f_{id}}} - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} \left(\frac{\theta_{g_{jn}}^{\mathrm{T}} \psi_{g_{jn}}}{\theta_{g_{jd}}^{\mathrm{T}} \psi_{g_{jd}}} x_{j+1} + \frac{\theta_{f_{jn}}^{\mathrm{T}} \psi_{f_{jn}}}{\theta_{f_{jd}}^{\mathrm{T}} \psi_{f_{jd}}} \right) + \beta_i \right] + |z_i| W_{a,i}^{*\mathrm{T}} \overline{\psi}_{a,i}(\overline{x}_i) \tag{31}$$

Consider the Lyapunov function candidate V_i

$$V_i = \frac{1}{2} z_i^2 + \frac{1}{2} (\hat{W}_{a,i} - W_{a,i}^*)^{\mathrm{T}} \Gamma_i^{-1} (\hat{W}_{a,i} - W_{a,i}^*) \tag{32}$$

where $\Gamma_i = \Gamma_i^{\mathrm{T}} > 0$. It then follows that the time derivative of V_i is

$$\dot{V}_i \le z_i \left[g_i x_{i+1} + \frac{\theta_{f_{in}}^{\mathrm{T}} \psi_{f_{in}}}{\theta_{f_{id}}^{\mathrm{T}} \psi_{f_{id}}} - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} \left(\frac{\theta_{g_{jn}}^{\mathrm{T}} \psi_{g_{jn}}}{\theta_{g_{jd}}^{\mathrm{T}} \psi_{g_{jd}}} x_{j+1} + \frac{\theta_{f_{jn}}^{\mathrm{T}} \psi_{f_{jn}}}{\theta_{f_{jd}}^{\mathrm{T}} \psi_{f_{jd}}} \right) + \beta_i \right] + |z_i| W_{a,i}^{*\mathrm{T}} \overline{\psi}_{a,i}(\overline{x}_i) + (\hat{W}_{a,i} - W_{a,i}^*)^{\mathrm{T}} \Gamma_i^{-1} \dot{\hat{W}}_{a,i} \tag{33}$$

Be choosing the intermediate control α_i and parameters adaptation law as

$$\alpha_i = N(\zeta_i) \left[k_i z_i + \frac{\hat{\theta}_{f_{in}}^{\mathrm{T}} \psi_{f_{in}}}{\hat{\theta}_{f_{id}}^{\mathrm{T}} \psi_{f_{id}}} - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} \left(\frac{\hat{\theta}_{g_{jn}}^{\mathrm{T}} \psi_{g_{jn}}}{\hat{\theta}_{g_{jd}}^{\mathrm{T}} \psi_{g_{jd}}} x_{j+1} + \frac{\hat{\theta}_{f_{jn}}^{\mathrm{T}} \psi_{f_{jn}}}{\hat{\theta}_{f_{jd}}^{\mathrm{T}} \psi_{f_{jd}}} \right) + \beta_i + \hat{W}_{a,i}^{\mathrm{T}} \operatorname{Tanh}\left(\frac{z_i \overline{\psi}_{a,i}}{\epsilon_i} \right) \overline{\psi}_{a,i} - v_i \right] \tag{34}$$

with

$$\dot{\zeta}_i = k_i z_i^2 + z_i \frac{\hat{\theta}_{f_{in}}^{\mathrm{T}} \psi_{f_{in}}}{\hat{\theta}_{f_{id}}^{\mathrm{T}} \psi_{f_{id}}} - z_i \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} \left(\frac{\hat{\theta}_{g_{jn}}^{\mathrm{T}} \psi_{g_{jn}}}{\hat{\theta}_{g_{jd}}^{\mathrm{T}} \psi_{g_{jd}}} x_{j+1} + \frac{\hat{\theta}_{f_{jn}}^{\mathrm{T}} \psi_{f_{jn}}}{\hat{\theta}_{f_{jd}}^{\mathrm{T}} \psi_{f_{jd}}} \right) + z_i \beta_i + z_i \hat{W}_{a,i}^{\mathrm{T}} \operatorname{Tanh}\left(\frac{z_i \overline{\psi}_{a,i}}{\epsilon_i} \right) \overline{\psi}_{a,i} - z_i v_i \tag{35}$$

$$v_i = -\frac{1}{4} k z_i \psi_{f_{in}}^{\mathrm{T}} \psi_{f_{in}} - \frac{1}{4} k z_i \psi_{f_{id}}^{\mathrm{T}} \psi_{f_{id}} \left(\frac{\hat{\theta}_{f_{in}}^{\mathrm{T}} \psi_{f_{in}}}{\hat{\theta}_{f_{id}}^{\mathrm{T}} \psi_{f_{id}}} \right)^2 - \frac{1}{4} k z_i \sum_{j=1}^{i-1} \left(\frac{\partial \alpha_{i-1}}{\partial x_j} x_{j+1} \right)^2 \psi_{g_{jn}}^{\mathrm{T}} \psi_{g_{jn}} - \frac{1}{4} k z_i \sum_{j=1}^{i-1} \psi_{g_{jd}}^{\mathrm{T}} \psi_{g_{jd}} \times \left(\frac{\partial \alpha_{i-1}}{\partial x_j} x_{j+1} \right)^2 \left(\frac{\hat{\theta}_{g_{jn}}^{\mathrm{T}} \psi_{g_{jn}}}{\hat{\theta}_{g_{jd}}^{\mathrm{T}} \psi_{g_{jd}}} \right)^2 - \frac{1}{4} k z_i \sum_{j=1}^{i-1} \left(\frac{\partial \alpha_{i-1}}{\partial x_j} \right)^2 \psi_{f_{jn}}^{\mathrm{T}} \psi_{f_{jn}} - \frac{1}{4} k z_i \sum_{j-1}^{i-1} \psi_{f_{jd}}^{\mathrm{T}} \psi_{f_{jd}} \left(\frac{\partial \alpha_{i-1}}{\partial x_j} \right)^2 \left(\frac{\hat{\theta}_{f_{jn}}^{\mathrm{T}} \psi_{f_{jn}}}{\hat{\theta}_{f_{jd}}^{\mathrm{T}} \psi_{f_{jd}}} \right)^2 \tag{36}$$

and

$$\dot{\hat{W}}_{a,i} = \Gamma_i z_i \operatorname{Tanh}\left(\frac{z_i \overline{\psi}_{a,i}}{\epsilon_i} \right) \overline{\psi}_{a,i} - \Gamma_i \sigma_i (\hat{W}_{a,i} - W_{a,i}^0) \tag{37}$$

where constant $k_i > 1/4, \epsilon_i$ is a small positive constant, and $\sigma_1 > 0$ and $W_{a,1}^0$ are design constants. Using the same techniques as did in step 1, we obtain

$$\dot{V}_i \le -k_{i0} z_i^2 + g_i N(\zeta_i) \dot{\zeta}_i + \dot{\zeta}_i + g_i^2 z_{i+1}^2 - \frac{1}{2} \sigma_i \| \hat{W}_{a,i} - W_{a,i}^* \|^2 + \sum_{j=1} |W_{a,ij}^*| 0.2785 \epsilon_i + \frac{1}{2} \sigma_i \| W_{a,i}^* - W_{a,i}^0 \|^2 + \sum_{j=1}^{i} \frac{\tilde{\theta}_{f_{jn}}^{\mathrm{T}} \tilde{\theta}_{f_{jn}} + \tilde{\theta}_{f_{jd}}^{\mathrm{T}} \tilde{\theta}_{f_{jd}}}{k(\theta_{f_{jd}}^{\mathrm{T}} \psi_{f_{jd}})^2} + \sum_{j=1}^{i-1} \frac{\tilde{\theta}_{g_{jn}}^{\mathrm{T}} \tilde{\theta}_{g_{jn}} + \tilde{\theta}_{g_{jd}}^{\mathrm{T}} \tilde{\theta}_{g_{jd}}}{k(\theta_{g_{jd}}^{\mathrm{T}} \psi_{g_{jd}})^2} \tag{38}$$

with constant $k_{i0} = k_i - (1/4) > 0$. Similarly, this yields

$$0 \le V_i(t) \le \frac{C_{i2}}{C_{i1}} + V_i(0) e^{-C_{i1} t} + e^{-C_{i1} t} \int_0^t (g_i N(\zeta_i) + 1) \dot{\zeta}_i e^{C_{i1} \tau} \mathrm{d}\tau + \int_0^t \left[\sum_{j=1}^{i} \frac{\tilde{\theta}_{f_{jn}}^{\mathrm{T}} \tilde{\theta}_{f_{jn}} + \tilde{\theta}_{f_{jd}}^{\mathrm{T}} \tilde{\theta}_{f_{jd}}}{k(\theta_{f_{jd}}^{\mathrm{T}} \psi_{f_{jd}})^2} + \sum_{j=1}^{i-1} \frac{\tilde{\theta}_{g_{jn}}^{\mathrm{T}} \tilde{\theta}_{g_{jn}} + \tilde{\theta}_{g_{jd}}^{\mathrm{T}} \tilde{\theta}_{g_{jd}}}{k(\theta_{g_{jd}}^{\mathrm{T}} \psi_{g_{jd}})^2} \right] e^{-C_{i1}(t-\tau)} \mathrm{d}\tau + \int_0^t g_i^2 z_{i+1}^2 e^{-C_{i1}(t-\tau)} \mathrm{d}\tau \tag{39}$$

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where

$$C_{i1} \triangleq \min\left\{2k_{i0}, \frac{\sigma_i}{\lambda_{\max}(\Gamma_i^{-1})}\right\},$$
$$C_{i2} \triangleq \sum_{j=1} |W_{a,ij}^*| 0.2785 \epsilon_i + \frac{1}{2}\sigma_i \| W_{a,i}^* - W_{a,i}^0 \|^2 \tag{40}$$

Remark 12: Similarly, adaptation laws (13) can ensure the boundedness of

$$\int_0^t \sum_{j=1}^{i} \frac{\tilde{\theta}_{f_{jn}}^{\mathrm{T}} \tilde{\theta}_{f_{jn}} + \tilde{\theta}_{f_{jd}}^{\mathrm{T}} \tilde{\theta}_{f_{jd}}}{k(\theta_{f_{jd}}^{\mathrm{T}} \psi_{f_{jd}})^2} e^{-C_{11}(t-\tau)} \mathrm{d}\tau + \int_0^t \sum_{j=1}^{i-1} \frac{\tilde{\theta}_{g_{jn}}^{\mathrm{T}} \tilde{\theta}_{g_{jn}} + \tilde{\theta}_{g_{jd}}^{\mathrm{T}} \tilde{\theta}_{g_{jd}}}{k(\theta_{g_{jd}}^{\mathrm{T}} \psi_{g_{jd}})^2} e^{-C_{11}(t-\tau)} \mathrm{d}\tau$$

Thus, if z_{i+1} can be regulated as bounded such that $\int_0^t g_i^2 z_{i+1}^2 e^{-c_i(t-\tau)} \mathrm{d}\tau$ is bounded at the following steps, then, according to Lemma 1, the boundedness of $z_i(t)$ can be guaranteed.

Step n: In this final step, the actual control u appears. Similarly, we have

$$\begin{aligned} z_n \dot{z}_n &= z_n \left[g_n u + \frac{\theta_{f_{nn}}^{\mathrm{T}} \psi_{f_{nn}}}{\theta_{f_{nd}}^{\mathrm{T}} \psi_{f_{nd}}} + \Delta_n - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} \right. \\ &\quad \left. \times \left(\frac{\theta_{g_{jn}}^{\mathrm{T}} \psi_{g_{jn}}}{\theta_{g_{jd}}^{\mathrm{T}} \psi_{g_{jd}}} x_{j+1} + \frac{\theta_{f_{jn}}^{\mathrm{T}} \psi_{f_{jn}}}{\theta_{f_{jd}}^{\mathrm{T}} \psi_{f_{jd}}} + \Delta_j \right) + \beta_n \right] \\ &\le z_n \left[g_n u + \frac{\theta_{f_{nn}}^{\mathrm{T}} \psi_{f_{nn}}}{\theta_{f_{nd}}^{\mathrm{T}} \psi_{f_{nd}}} - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} \right. \\ &\quad \left. \times \left(\frac{\theta_{g_{jn}}^{\mathrm{T}} \psi_{g_{jn}}}{\theta_{g_{jd}}^{\mathrm{T}} \psi_{g_{jd}}} x_{j+1} + \frac{\theta_{f_{jn}}^{\mathrm{T}} \psi_{f_{jn}}}{\theta_{f_{jd}}^{\mathrm{T}} \psi_{f_{jd}}} \right) + \beta_n \right] + |z_n| W_{a,n}^{*\mathrm{T}} \overline{\psi}_{a,n}(x) \end{aligned} \tag{41}$$

where

$$\begin{aligned} \beta_n &= -\sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}_{f_{jn}}} \dot{\hat{\theta}}_{f_{jn}} - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}_{g_{jn}}} \dot{\hat{\theta}}_{g_{jn}} - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial y_d^{(j)}} y_d^{(j+1)} \\ &\quad - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \hat{W}_{a,j}} \dot{\hat{W}}_{a,j} - \frac{\partial \alpha_{n-1}}{\partial t} \end{aligned} \tag{42}$$

$$W_{a,n}^* \triangleq \left[W_n^{*\mathrm{T}}, W_1^{*\mathrm{T}}, \ldots, W_{n-1}^{*\mathrm{T}}, \delta_n^*, \delta_1^*, \ldots, \delta_{n-1}^* \right]^{\mathrm{T}} \tag{43}$$

$$\begin{aligned} \overline{\psi}_{a,n} &\triangleq \left[\psi_n^{\mathrm{T}}, \left| \frac{\partial \alpha_{n-1}}{\partial x_1} \right| \psi_1^{\mathrm{T}}, \ldots, \left| \frac{\partial \alpha_{n-1}}{\partial x_{n-1}} \right| \psi_{n-1}^{\mathrm{T}}, 1, \right. \\ &\quad \left. \left| \frac{\partial \alpha_{n-1}}{\partial x_1} \right|, \ldots, \left| \frac{\partial \alpha_{n-1}}{\partial x_{n-1}} \right| \right]^{\mathrm{T}} \end{aligned} \tag{44}$$

To this end, let the control input be designed as

$$\begin{aligned} u &= N(\zeta_n) \left[k_n z_n + \frac{\hat{\theta}_{f_{nn}}^{\mathrm{T}} \psi_{f_{nn}}}{\hat{\theta}_{f_{nd}}^{\mathrm{T}} \psi_{f_{nd}}} - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} \right. \\ &\quad \times \left(\frac{\hat{\theta}_{g_{jn}}^{\mathrm{T}} \psi_{g_{jn}}}{\hat{\theta}_{g_{jd}}^{\mathrm{T}} \psi_{g_{jd}}} x_{j+1} + \frac{\hat{\theta}_{f_{jn}}^{\mathrm{T}} \psi_{f_{jn}}}{\hat{\theta}_{f_{jd}}^{\mathrm{T}} \psi_{f_{jd}}} \right) \\ &\quad \left. + \beta_n + \hat{W}_{a,n}^{\mathrm{T}} \mathrm{Tanh}\left(\frac{z_n \overline{\psi}_{a,n}}{\epsilon_n} \right) \overline{\psi}_{a,n} - v_n \right] \end{aligned} \tag{45}$$

with

$$\begin{aligned} \dot{\zeta}_n &= k_n z_n^2 + z_n \frac{\hat{\theta}_{f_{nn}}^{\mathrm{T}} \psi_{f_{nn}}}{\hat{\theta}_{f_{nd}}^{\mathrm{T}} \psi_{f_{nd}}} \\ &\quad - z_n \sum_{j=1}^{i-1} \frac{\partial \alpha_{n-1}}{\partial x_j} \left(\frac{\hat{\theta}_{g_{jn}}^{\mathrm{T}} \psi_{g_{jn}}}{\hat{\theta}_{g_{jd}}^{\mathrm{T}} \psi_{g_{jd}}} x_{j+1} + \frac{\hat{\theta}_{f_{jn}}^{\mathrm{T}} \psi_{f_{jn}}}{\hat{\theta}_{f_{jd}}^{\mathrm{T}} \psi_{f_{jd}}} \right) \\ &\quad + z_n \beta_n + z_n \hat{W}_{a,n}^{\mathrm{T}} \mathrm{Tanh}\left(\frac{z_n \overline{\psi}_{a,n}}{\epsilon_n} \right) \overline{\psi}_{a,n} - z_n v_n \end{aligned} \tag{46}$$

$$\begin{aligned} v_n &= -\frac{1}{4} k z_n \psi_{f_{nn}}^{\mathrm{T}} \psi_{f_{nn}} - \frac{1}{4} k z_n \psi_{f_{nd}}^{\mathrm{T}} \psi_{f_{nd}} \left(\frac{\hat{\theta}_{f_{nn}}^{\mathrm{T}} \psi_{f_{nn}}}{\hat{\theta}_{f_{nd}}^{\mathrm{T}} \psi_{f_{nd}}} \right)^2 \\ &\quad - \frac{1}{4} k z_n \sum_{j=1}^{n-1} \left(\frac{\partial \alpha_{n-1}}{\partial x_j} x_{j+1} \right)^2 \psi_{g_{jn}}^{\mathrm{T}} \psi_{g_{jn}} \\ &\quad - \frac{1}{4} k z_n \sum_{j=1}^{n-1} \psi_{g_{jd}}^{\mathrm{T}} \psi_{g_{jd}} \left(\frac{\partial \alpha_{n-1}}{\partial x_j} x_{j+1} \right)^2 \left(\frac{\hat{\theta}_{g_{jn}}^{\mathrm{T}} \psi_{g_{jn}}}{\hat{\theta}_{g_{jd}}^{\mathrm{T}} \psi_{g_{jd}}} \right)^2 \\ &\quad - \frac{1}{4} k z_n \sum_{j=1}^{n-1} \left(\frac{\partial \alpha_{n-1}}{\partial x_j} \right)^2 \psi_{f_{jn}}^{\mathrm{T}} \psi_{f_{jn}} \\ &\quad - \frac{1}{4} k z_n \sum_{j-1}^{n-1} \psi_{f_{jd}}^{\mathrm{T}} \psi_{f_{jd}} \left(\frac{\partial \alpha_{n-1}}{\partial x_j} \right)^2 \left(\frac{\hat{\theta}_{f_{jn}}^{\mathrm{T}} \psi_{f_{jn}}}{\hat{\theta}_{f_{jd}}^{\mathrm{T}} \psi_{f_{jd}}} \right)^2 \end{aligned} \tag{47}$$

and let adaptation law be

$$\dot{\hat{W}}_{a,n} = \Gamma_n z_n \mathrm{Tanh}\left(\frac{z_n \overline{\psi}_{a,n}}{\epsilon_n} \right) \overline{\psi}_{a,n} - \Gamma_n \sigma_n (\hat{W}_{a,n} - W_{a,n}^0) \tag{48}$$

where constant $k_n > 0$, ϵ_n is a small positive constant, and $\sigma_n > 0$ and $W_{a,n}^0$ are design constants.

Consider the Lyapunov function candidate

$$V_n = \frac{1}{2} z_n^2 + \frac{1}{2} (\hat{W}_{a,n} - W_{a,n}^*)^{\mathrm{T}} \Gamma_n^{-1} (\hat{W}_{a,n} - W_{a,n}^*)$$

Its time derivative satisfies

$$\dot{V}_n \leq -k_n z_n^2 + g_n N(\zeta_n)\dot{\zeta}_n + \dot{\zeta}_n - \frac{1}{2}\sigma_n \|\hat{W}_{a,n} - W_{a,n}^*\|^2 + \sum_{j=1} |W_{a,nj}^*| 0.2785\epsilon_n + \frac{1}{2}\sigma_n \|W_{a,n}^* - W_{a,n}^0\|^2 + \sum_{j=1}^{n} \frac{\tilde{\theta}_{fjn}^{\mathrm{T}}\tilde{\theta}_{fjn} + \tilde{\theta}_{fjd}^{\mathrm{T}}\tilde{\theta}_{fjd}}{k(\theta_{fjd}^{\mathrm{T}}\psi_{fjd})^2} + \sum_{j=1}^{n-1} \frac{\tilde{\theta}_{gjn}^{\mathrm{T}}\tilde{\theta}_{gjn} + \tilde{\theta}_{gjd}^{\mathrm{T}}\tilde{\theta}_{gjd}}{k(\theta_{gjd}^{\mathrm{T}}\psi_{gjd})^2} \quad (49)$$

which yields

$$0 \leq V_n(t) \leq \frac{C_{n2}}{C_{n1}} + V_n(0)e^{-C_{n1}t} + e^{-C_{n1}t}\int_0^t (g_n N(\zeta_n) + 1)\dot{\zeta}_n e^{C_{n1}\tau}\mathrm{d}\tau + \int_0^t \left[\sum_{j=1}^{n} \frac{\tilde{\theta}_{fjn}^{\mathrm{T}}\tilde{\theta}_{fjn} + \tilde{\theta}_{fjd}^{\mathrm{T}}\tilde{\theta}_{fjd}}{k(\theta_{fjd}^{\mathrm{T}}\psi_{fjd})^2} + \sum_{j=1}^{n-1} \frac{\tilde{\theta}_{gjn}^{\mathrm{T}}\tilde{\theta}_{gjn} + \tilde{\theta}_{gjd}^{\mathrm{T}}\tilde{\theta}_{gjd}}{k(\theta_{gjd}^{\mathrm{T}}\psi_{gjd})^2}\right] e^{-C_{n1}(t-\tau)}\mathrm{d}\tau \quad (50)$$

where

$$C_{n1} \triangleq \min\left\{2k_n, \frac{\sigma_n}{\lambda_{\max}(\Gamma_n^{-1})}\right\},$$
$$C_{n2} \triangleq \sum_{j=1} |W_{a,nj}^*| 0.2785\epsilon_n + \frac{1}{2}\sigma_i \|W_{a,n}^* - W_{a,n}^0\|^2 \quad (51)$$

The main results of the paper is stated as follows.

Theorem 1 (Stability): Consider uncertain nonlinear system (1) with unknown functions $f_i(\cdot)$ and unknown virtual control coefficients $g_i(\cdot)$ under Assumptions 1, 2 and 3. Given any bounded smooth reference trajectory $y_d(t)$ with bounded time derivative $y_d^{(1)}, \ldots, y_d^{(n)}$ and for any bounded initial condition $x(0)$, $\hat{\theta}_{f_i n}(0) \in \Omega_{1i}$, $\hat{\theta}_{g_i n}(0) \in \Omega_{2i}$, $\hat{\theta}_{f_i d}(0) \in \Omega_{3i}$, and $\hat{\theta}_{g_i d}(0) \in \Omega_{4i}$, if we apply the controller (45) with the parameters updating laws (13) and the estimates $\hat{\theta}_{fid}$ and $\hat{\theta}_{gid}$ are arbitrarily chosen as long as inequalities $\hat{\theta}_{fid}^{\mathrm{T}}\psi_{fid} > 0$ and $\hat{\theta}_{gid}^{\mathrm{T}}\psi_{gid} > 0$ are satisfied, then

i. The solutions of the resulting closed-loop system are uniformly ultimately bounded.

ii. Given any constant $r_z > \sum_{i=1}^{n} 2[(C_{i2}/C_{i1}) + (C_{i3}/C_{i1})]$, there exists T such that

$$\|z(t)\| \in \Omega_z \triangleq \{z \in \Re^n : \|z(t)\| \leq r_z\}, \quad \forall t \geq \mathrm{T} \quad (52)$$

where $z \triangleq [z_1, z_2, \ldots, z_n]^{\mathrm{T}}$, C_{i1} and C_{i2} for $i = 1, \ldots, n$ are given by (27), (40) and (51), C_{i3}, $i = 1, \ldots, n-1$ is the upper bound of term

$$[g_i N(\zeta_i) + 1]\dot{\zeta}_i + \left[\sum_{j=1}^{i} \frac{\tilde{\theta}_{fjn}^{\mathrm{T}}\tilde{\theta}_{fjn} + \tilde{\theta}_{fjd}^{\mathrm{T}}\tilde{\theta}_{fjd}}{k(\theta_{fjd}^{\mathrm{T}}\psi_{fjd})^2} + \sum_{j=1}^{i-1} \frac{\tilde{\theta}_{gjn}^{\mathrm{T}}\tilde{\theta}_{gjn} + \tilde{\theta}_{gjd}^{\mathrm{T}}\tilde{\theta}_{gjd}}{k(\theta_{gjd}^{\mathrm{T}}\psi_{gjd})^2}\right] + g_i^2 z_{i+1}^2$$

and C_{n3} is the upper bound of term

$$[g_n N(\zeta_n) + 1]\dot{\zeta}_n + \left[\sum_{j=1}^{n} \frac{\tilde{\theta}_{fjn}^{\mathrm{T}}\tilde{\theta}_{fjn} + \tilde{\theta}_{fjd}^{\mathrm{T}}\tilde{\theta}_{fjd}}{k(\theta_{fjd}^{\mathrm{T}}\psi_{fjd})^2} + \sum_{j=1}^{n-1} \frac{\tilde{\theta}_{gjn}^{\mathrm{T}}\tilde{\theta}_{gjn} + \tilde{\theta}_{gjd}^{\mathrm{T}}\tilde{\theta}_{gjd}}{k(\theta_{gjd}^{\mathrm{T}}\psi_{gjd})^2}\right]$$

In addition, the size of compact set Ω_z can be reduced by an appropriate choice of the design parameters k, k_i, σ_i, ϵ_i and Γ_i

iii. The output tracking error satisfies the following property

$$|z_1(t)| \leq \sqrt{2\left(\frac{C_{12}}{C_{11}} + \frac{C_{13}}{C_{11}}\right) + 2V_1(0)e^{-C_{11}t}} \quad (53)$$

Proof: To show item (i), let us consider (50). Due to the utilisation of adaptation laws (13) and the choices of $\hat{\theta}_{f_{nd}}$ and $\hat{\theta}_{g_{nd}}$, the boundedness of $\hat{\theta}_{f_{nn}}$ and $\hat{\theta}_{g_{nn}}$ and that of $\tilde{\theta}_{f_{nn}}$, $\tilde{\theta}_{f_{nd}}$, $\tilde{\theta}_{g_{nn}}$, $\tilde{\theta}_{g_{nd}}$ can be guaranteed. Together with the assumption of $\underline{c}_{1j} \leq \theta_{fjd}^{\mathrm{T}}\psi_{fjd}$ and $\underline{c}_{2j} \leq \theta_{gjd}^{\mathrm{T}}\psi_{gjd}$, the boundedness of the following term

$$\int_0^t \frac{1}{k}\left[\sum_{j=1}^{n} \frac{\tilde{\theta}_{fjn}^{\mathrm{T}}\tilde{\theta}_{fjn} + \tilde{\theta}_{fjd}^{\mathrm{T}}\tilde{\theta}_{fjd}}{(\theta_{fjd}^{\mathrm{T}}\psi_{fjd})^2} + \sum_{j=1}^{n-1} \frac{\tilde{\theta}_{gjn}^{\mathrm{T}}\tilde{\theta}_{gjn} + \tilde{\theta}_{gjd}^{\mathrm{T}}\tilde{\theta}_{gjd}}{(\theta_{gjd}^{\mathrm{T}}\psi_{gjd})^2}\right] e^{-c_n(t-\tau)}\mathrm{d}\tau$$

can be guaranteed. Thus, noting (50), and using Lemma 1, we can conclude that $\zeta_n(t)$ and $V_n(t)$, hence $z_n(t)$ and $\hat{W}_{a,n}$ are uniformly ultimately bounded. From the boundedness of $z_n(t)$, the boundedness of the extra term $\int_0^t g_{n-1}^2 z_n^2 e^{-c_{n-1}(t-\tau)}\mathrm{d}\tau$ at Step $n-1$ is readily obtained. Applying Lemma 1 $n-1$ times, it can be seen from the above design procedures that $\zeta_i(t), V_i(t), z_i(t)$, and hence $x_i(t)$ and $\hat{W}_{a,i}$ are uniformly ultimately bounded.

For item (ii), it follows that

$$\|z(t)\|^2 \leq \sum_{i=1}^{n} 2V_i(t) \quad (54)$$

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On the other hand, it follows from the boundedness of all closed-loop signals that the nonlinear terms

$$[g_i N(\zeta_i) + 1]\dot{\zeta}_i + \left[\sum_{j=1}^{i} \frac{\tilde{\theta}_{f_{jn}}^{\mathrm{T}} \tilde{\theta}_{f_{jn}} + \tilde{\theta}_{f_{jd}}^{\mathrm{T}} \tilde{\theta}_{f_{jd}}}{k(\theta_{f_{jd}}^{\mathrm{T}} \psi_{f_{jd}})^2} + \sum_{j=1}^{i-1} \frac{\tilde{\theta}_{g_{jn}}^{\mathrm{T}} \tilde{\theta}_{g_{jn}} + \tilde{\theta}_{g_{jd}}^{\mathrm{T}} \tilde{\theta}_{g_{jd}}}{k(\theta_{g_{jd}}^{\mathrm{T}} \psi_{g_{jd}})^2} \right] + g_i^2 z_{i+1}^2$$

and

$$[g_n N(\zeta_n) + 1]\dot{\zeta}_n + \left[\sum_{j=1}^{n} \frac{\tilde{\theta}_{f_{jn}}^{\mathrm{T}} \tilde{\theta}_{f_{jn}} + \tilde{\theta}_{f_{jd}}^{\mathrm{T}} \tilde{\theta}_{f_{jd}}}{k(\theta_{f_{jd}}^{\mathrm{T}} \psi_{f_{jd}})^2} + \sum_{j=1}^{n-1} \frac{\tilde{\theta}_{g_{jn}}^{\mathrm{T}} \tilde{\theta}_{g_{jn}} + \tilde{\theta}_{g_{jd}}^{\mathrm{T}} \tilde{\theta}_{g_{jd}}}{k(\theta_{g_{jd}}^{\mathrm{T}} \psi_{g_{jd}})^2} \right]$$

are upper bounded by certain constants, say C_{i3} and C_{n3}, respectively. Thus, we have

$$\int_0^t \left\{ (g_i N(\zeta_i) + 1)\dot{\zeta}_i + \left[\sum_{j=1}^{i} \frac{\tilde{\theta}_{f_{jn}}^{\mathrm{T}} \tilde{\theta}_{f_{jn}} + \tilde{\theta}_{f_{jd}}^{\mathrm{T}} \tilde{\theta}_{f_{jd}}}{k(\theta_{f_{jd}}^{\mathrm{T}} \psi_{f_{jd}})^2} + \sum_{j=1}^{i-1} \frac{\tilde{\theta}_{g_{jn}}^{\mathrm{T}} \tilde{\theta}_{g_{jn}} + \tilde{\theta}_{g_{jd}}^{\mathrm{T}} \tilde{\theta}_{g_{jd}}}{k(\theta_{g_{jd}}^{\mathrm{T}} \psi_{g_{jd}})^2} \right] + g_i^2 z_{i+1}^2 \right\} e^{-C_{i1}(t-\tau)} \mathrm{d}\tau \leq \int_0^t C_{i3} e^{-C_{i1}(t-\tau)} \mathrm{d}\tau \leq \frac{C_{i3}}{C_{i1}} \tag{55}$$

and

$$\int_0^t \left\{ (g_n N(\zeta_n) + 1)\dot{\zeta}_n + \left[\sum_{j=1}^{n} \frac{\tilde{\theta}_{f_{jn}}^{\mathrm{T}} \tilde{\theta}_{f_{jn}} + \tilde{\theta}_{f_{jd}}^{\mathrm{T}} \tilde{\theta}_{f_{jd}}}{k(\theta_{f_{jd}}^{\mathrm{T}} \psi_{f_{jd}})^2} + \sum_{j=1}^{n-1} \frac{\tilde{\theta}_{g_{jn}}^{\mathrm{T}} \tilde{\theta}_{g_{jn}} + \tilde{\theta}_{g_{jd}}^{\mathrm{T}} \tilde{\theta}_{g_{jd}}}{k(\theta_{g_{jd}}^{\mathrm{T}} \psi_{g_{jd}})^2} \right] \right\} e^{-C_{n1}(t-\tau)} \mathrm{d}\tau \leq \int_0^t C_{n3} e^{-C_{n1}(t-\tau)} \mathrm{d}\tau \leq \frac{C_{n3}}{C_{n1}} \tag{56}$$

It then follows from (28), (39), (50), (54), (55) and (56) that (52) holds.

Item (iii) follows from the definition of V_1 and (28).

Remark 13: In the proof of item (i) of theorem 1, we show that all the closed-loop system signals are uniformly ultimately bounded. That is, there exists a sufficiently large compact set $\overline{\Omega}_i$ such that $\bar{x}_i(t) \in \overline{\Omega}_i$ for all t. Although the actual size of $\overline{\Omega}_i$ is not known in advance, they can really be made as large as deemed necessary for any given bounded initial conditions. Accordingly, we can guarantee the hold of (8) by choosing neural network large enough to cover $\overline{\Omega}_i$ for bounded initial conditions. Theoretically speaking, the obtained result is semi-global in the sense that bounded initial conditions guarantee the boundedness of all the signals in the closed-loop system provided the neural network is chosen to cover a compact set of sufficiently large size.

Remark 14: The control performance can be improved in the sense of reducing the size of Ω_z by an appropriate choice of the design parameters k, k_i, σ_i, ϵ_i and Γ_i. In particular, by increasing k_i and Γ_i, the value of C_{i1} is increased that leads to reduction of the size of Ω_z. However, large k_i and Γ_i might result in a high-gain control that is not desirable practically. Therefore control parameters should be carefully adjusted for achieving suitable transient performance and control action.

4 Simulation

In this section, the proposed robust adaptive neural control is simulated for the following second-order nonlinear system

$$\begin{aligned} \dot{x}_1 &= \frac{\theta_{g_{1n}}(t)\psi_{g_{1n}}(x_1, t)}{\theta_{g_{1d}}(t)\psi_{g_{1d}}(x_1, t)} x_2 + \frac{\theta_{f_{1n}}(t)\psi_{f_{1n}}(x_1, t)}{\theta_{f_{1d}}(t)\psi_{f_{1d}}(x_1, t)} + \Delta_1(t, x) \\ \dot{x}_2 &= [1.5 + 0.5\sin(t)]u + \Delta_2(t, x) \\ y &= x_1 \end{aligned} \tag{57}$$

The system (57) is in the form of (1) with

$$f_1(x_1, t) = \frac{\theta_{f_{1n}}(t)\psi_{f_{1n}}(x_1, t)}{\theta_{f_{1d}}(t)\psi_{f_{1d}}(x_1, t)}, \quad g_1(x_1, t) = \frac{\theta_{g_{1n}}(t)\psi_{g_{1n}}(x_1, t)}{\theta_{g_{1d}}(t)\psi_{g_{1d}}(x_1, t)}$$

and

$$f_2(x, t) = 0, \quad g_2(x, t) = 1.5 + 0.5\sin(t)$$

For the simulation purpose, we assume that

$$\begin{aligned} &\theta_{g_{1n}}(t) = 1.5 + 0.5\sin(t), \quad \theta_{g_{1d}}(t) = 1, \\ &\theta_{f_{1n}}(t) = 0.2 + 0.1\sin(t), \quad \theta_{f_{1d}}(t) = 1 \\ &\psi_{g_{1n}}(x_1, t) = 1, \quad \psi_{g_{1d}}(x_1, t) = 1 + x_1^2, \quad \psi_{f_{1n}}(x_1, t) = x_1^2, \\ &\psi_{f_{1d}}(x_1, t) = 2 + \sin(t) \quad \Delta_1(t, x) = 0.6\sin(x_2), \quad \text{and,} \\ &\Delta_2(t, x) = 0.5(x_1^2 + x_2^2)\sin^3 t \end{aligned}$$

and let the reference signal be $y_d(t) = 0.5[\sin(t) + \sin(0.5t)]$.

Apparently, the bounding functions for $\Delta_1(t, x)$ and $\Delta_2(t, x)$ are $\phi_1 = 0.6$ and $\phi_2 = 0.5(x_1^2 + x_2^2)$, respectively. In the simulation, we apply neural networks to approximate unknown functions ϕ_1 and ϕ_2 and use adaptation laws (13) to estimate unknown time-varying parameters $\theta_{g_{1n}}(t)$ and $\theta_{f_{1n}}(t)$. For the estimates of unknown time-varying parameters $\theta_{g_{1d}}(t)$ and $\theta_{f_{1d}}(t)$, by studying the structure properties of known functions $\psi_{g_{1d}} = 1 + x_1^2$ and $\psi_{f_{1d}} = 2 + \sin(t)$, we can simply let $\hat{\theta}_{g_{1d}} = 1$ and $\hat{\theta}_{f_{1d}} = 2$ to satisfy the inequalities $\hat{\theta}_{g_{1d}} \psi_{g_{1d}} > 0$ and $\hat{\theta}_{f_{1d}} \psi_{f_{1d}} > 0$.

According to the design procedures presented in Section 3, we have the corresponding control and adaptation laws as follows

$$\alpha_1 = N(\zeta_1)\left[k_1 z_1 + \frac{\hat{\theta}_{f_{1n}} x_1^2}{\hat{\theta}_{f_{1d}}[2 + \sin(t)]} + \hat{W}_{a,1}^{\mathrm{T}} \,\mathrm{Tanh}\left(\frac{z_1 \overline{\psi}_{a,1}}{\epsilon_1}\right) \overline{\psi}_{a,1}(x_1) - \dot{y}_d - v_1 \right]$$

$$\dot{\zeta}_1 = k_1 z_1^2 + z_1 \frac{\hat{\theta}_{f_{1n}} x_1^2}{\hat{\theta}_{f_{1d}}[2 + \sin(t)]} + z_1 \hat{W}_{a,1}^{\mathrm{T}} \,\mathrm{Tanh}\left(\frac{z_1 \overline{\psi}_{a,1}}{\epsilon_1}\right) \overline{\psi}_{a,1}(x_1) - z_1 \dot{y}_d - z_1 v_1$$

$$v_1 = -\frac{1}{4} k z_1 x_1^4 - \frac{1}{4} k z_1 [2 + \sin(t)]^2 \left[\frac{\hat{\theta}_{f_{1n}} x_1^2}{\hat{\theta}_{f_{1d}}(2 + \sin(t))} \right]^2$$

$$\dot{\hat{W}}_{a,1} = \Gamma_1 z_1 \,\mathrm{Tanh}\left(\frac{z_1 \overline{\psi}_{a,1}}{\epsilon_1}\right) \overline{\psi}_{a,1} - \Gamma_1 \sigma_1 (\hat{W}_{a,1} - W_{a,1}^0)$$

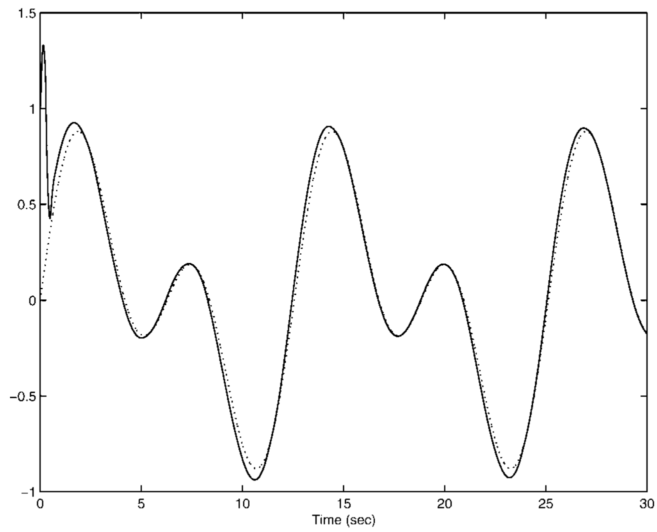
$$u = N(\zeta_2)\left[k_2 z_2 - \frac{\partial \alpha_1}{\partial x_1}\left(\hat{\theta}_{g_{1n}} x_2 + \frac{\hat{\theta}_{f_{1n}} x_1^2}{\hat{\theta}_{f_{1d}}[2 + \sin(t)]} \right) + \beta_2 + \hat{W}_{a,2}^{\mathrm{T}} \,\mathrm{Tanh}\left(\frac{z_2 \overline{\psi}_{a,2}}{\epsilon_2}\right) \overline{\psi}_{a,2} - v_2 \right]$$

$$\dot{\zeta}_2 = k_2 z_2^2 - z_2 \frac{\partial \alpha_1}{\partial x_1}\left(\hat{\theta}_{g_{1n}} x_2 + \frac{\hat{\theta}_{f_{1n}} x_1^2}{\hat{\theta}_{f_{1d}}[2 + \sin(t)]} \right) + z_2 \beta_2 + z_2 \hat{W}_{a,2}^{\mathrm{T}} \,\mathrm{Tanh}\frac{z_2 \overline{\psi}_{a,2}}{\epsilon_2} \overline{\psi}_{a,2} - z_2 v_2$$

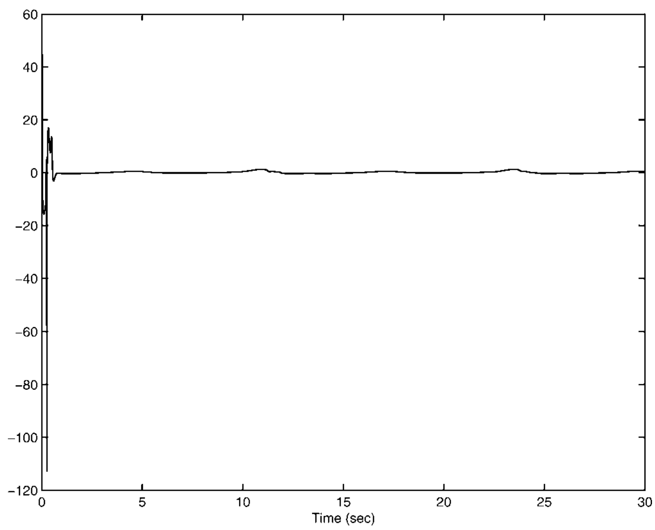
$$v_2 = -\frac{1}{4} k z_2 \left(\frac{\partial \alpha_1}{\partial x_1} x_2 \right)^2 - \frac{1}{4} k z_2 \left(\frac{\partial \alpha_1}{\partial x_1} \right)^2 x_1^4 - \frac{1}{4} k z_2 [2 + \sin(t)]^2 \left(\frac{\partial \alpha_1}{\partial x_1} \right)^2 \left(\frac{\hat{\theta}_{f_{jn}} x_1^2}{\hat{\theta}_{f_{jd}}[2 + \sin(t)]} \right)^2$$

$$\dot{\hat{W}}_{a,2} = \Gamma_2 z_2 \,\mathrm{Tanh}\left(\frac{z_2 \overline{\psi}_{a,2}}{\epsilon_2}\right) \overline{\psi}_{a,2} - \Gamma_2 \sigma_2 (\hat{W}_{a,2} - W_{a,2}^0)$$

In the simulation, RBF neural networks are applied and we select the centres and widths as: Neural network $W_1^{*\mathrm{T}} \psi_1(x_1)$ contains nine nodes, with centres μ_l $(l = 1, \ldots, 9)$ evenly spaced in $[-5, 5]$, and widths $\eta_l = 1$ $(l = 1, \ldots, 9)$. Neural network $W_2^{*\mathrm{T}} \psi_2(x)$ contains 63 nodes, with centres μ_l $(l = 1, \ldots, 63)$ evenly spaced in $[-5, 5] \times [-7.5, 7.5]$, and widths $\eta_l = 1$ $(l = 1, \ldots, 63)$. The following initial conditions and controller design parameters are adopted: $x(0) = [1, 0]^{\mathrm{T}}$, $\hat{W}_{a,1}(0) = 0$, $\hat{W}_{a,2}(0) = 0$, $\hat{\theta}_{f1n}(0) = 0$, $\hat{\theta}_{g1n}(0) = 0$, $\hat{\theta}_{f1d} = 0.1$, and $\Gamma_1 = \Gamma_2 = 0.1, k = k_1 = k_2 = 4, \sigma_1 = \sigma_2 = 0.01, \epsilon_1 = \epsilon_2 = 0.1$ and $W_{a,1}^0 = W_{a,2}^0 = 0$. Simulation results are provided in Figs. 1 and 2. Fig. 1*a* shows that the output of the system converges to a small neighbourhood of the desired trajectory. The boundedness of control input and the parameter estimates is illustrated in Figs. 1*b* and 2.



a



b

Figure 1 *System response*

a Output tracking (x_1: 'solid line' and y_d: 'dotted line')
b Control inputs

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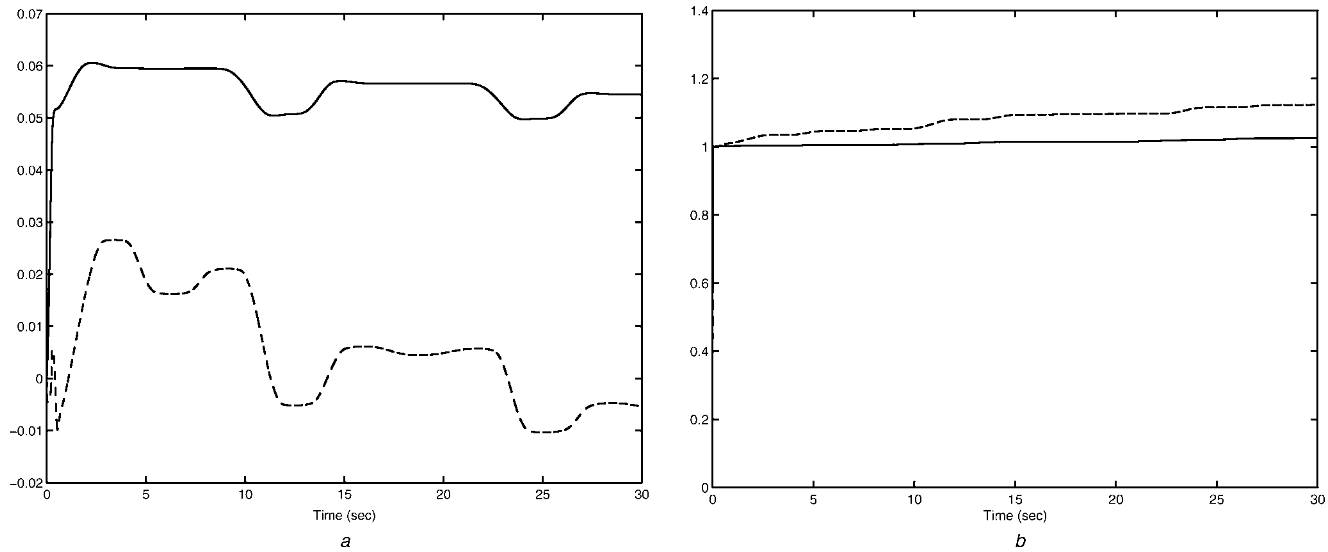

Figure 2 *Parameters estimates*

a Boundedness of parameters estimates ($\hat{\theta}_{f1n}$: 'solid line'; $\hat{\theta}_{g1n}$: 'dashed line')
b Boundedness of weights ($\|\hat{W}_{a,1}\|$: 'solid line'; $\|\hat{W}_{a,2}\|$: 'dashed line')

5 Conclusion

In this paper, a new robust adaptive neural control has been presented for a class of time-varying nonlinear uncertain systems with nonlinear fractional parameterisation. It has been proved that the proposed robust adaptive scheme can guarantee the semi-global uniform ultimate boundedness of all the closed-loop system signals. The proposed design expands the class of nonlinear systems for which robust and adaptive controls have been studied with the aid of Lyapunov direct method and neural network approximations.

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