an indicator of performance deterioration. Yet in general this might not be true. For example, consider the problem of the minimization of

$$\left\| \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \left(\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} Q(s) \right) \mathrm{e}^{-sh} \right\|_{\hat{\mathcal{A}}}$$

by a stable $Q(s)$, where the first term plays a role of $\Delta(s)$. As $Q(s)$ does not affect the second output, the norm cannot be made smaller than $\|\mathrm{e}^{-sh}\|_{\hat{\mathcal{A}}} = 1$ and this level is achieved with $Q(s) = -1$. Yet this performance level is delay independent and it would be the optimal performance even if $h = 0$. Hence, in this case $\|\Delta(s)\|_{\hat{\mathcal{A}}} = 1$ is not a good indicator of the cost of delay.

IV. Conclusion

It has been shown that the standard L^1 optimization for dead-time systems can be reduced to a delay-free L^1 optimization problem by the use of the modified Smith predictor. This reduction, which implies that the (modified) Smith predictor is, in a sense, L^1-optimal, confirms an important role played by dead-time compensation in the control of dead-time systems.

Although only the continuous-time problem has been presented in the paper, all the arguments are straightforwardly extendible to the discrete-time case (ℓ^1 optimization).

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Globally Stabilizing Adaptive Control Design for Nonlinearly-Parameterized Systems

Zhihua Qu, Richard A. Hull, and Jing Wang

***Abstract*—In this note, a new adaptive control design is proposed for nonlinear systems that are possibly nonaffine and contain nonlinearly parameterized unknowns. The proposed control is not based on certainty equivalence principle which forms the foundation of existing and standard adaptive control designs. Instead, a biasing vector function is introduced into parameter estimate; it links the system dynamics to estimation error dynamics, and its choice leads to a new Lyapunov-based design so that affine or nonaffine systems with nonlinearly parameterized unknowns can be controlled by adaptive estimation. Explicit conditions are found for achieving global asymptotic stability of the state, and the convergence condition for parameter estimation is also found. The conditions are illustrated by several examples and classes of systems. Besides global stability and estimation convergence, the proposed adaptive control has the unique feature that it does not contains any robust control part which typically overpowers unknown dynamics, may be conservative, and also interferes with parameter estimation.**

***Index Terms*—Adaptive control, certainty equivalence principle, Lyapunov-based design, nonlinear parameterization, parameter estimation.**

I. Introduction

In most control applications, it is inevitable that various unknowns and/or uncertainties exist in the plant or its environment. This is due to the facts that modeling is less than perfect and that control systems are now required to perform autonomously and intelligently under a variety of operating conditions. In terms of unknowns and uncertainties, one can separate them into two categories: Constant unknown parameters, and state-dependent/time-dependent/dynamic uncertainties. To maintain stability and performance, a successful design must deal with whatever unknowns exist.

If there are state-dependent/time-dependent/dynamic uncertainties the plant belongs to the class of so-called uncertain systems and, as an effective control design methodology, robust control theory for nonlinear uncertain systems have been a focus of research in recent years [9], [11], [12]. In a typical robust control design, the uncertainties in the plant dynamics are required to be bounded in some norm by a known scalar function of the state. Upon having the bounding function, structural property of the uncertainties in terms of their functional dependence and locations in system dynamics need to be studied, classes of stabilizable uncertain systems have been found, and several robust control design procedures have been proposed (see [12] and the references therein).

For systems whose dynamics are known except for a number of constant parameters, it is natural and effective to use the adaptive control methodology which is to achieve stability and performance by estimating the parameters online. The most popular method of designing adaptive control is summarized by the certainty equivalence principle. That is, first assume the parameters be known and design a perfect-knowledge control; then, the adaptive control is the same except that the unknown parameters are replaced by their corresponding estimates,

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and finally adaptation laws are synthesized to generate the estimates in such a way that the closed-loop stability and performance are guaranteed. In addition, estimation convergence is pursued whenever possible. Standard adaptive control results in [9] are based on the certainty equivalence principle and are shown to be effective for the class of systems whose unknown parameters appear linearly in their dynamics.

While adaptive control has been extensively studied and widely applied, there are only a few results reported so far on adaptive control of nonlinearly parameterized systems, i.e., systems whose unknown parameters appear nonlinearly in their dynamics. For a scalar system with fractional parameterization, its parameters appear linearly in both numerator and denominator of its scalar fractional dynamics. Earlier results on adaptive control of nonlinearly parameterized systems focused upon such scalar fractionally parameterized systems as fermentation process [4], biochemical process [5], and friction dynamics [2]. It is shown in [4] that, since parameters in the numerator are linearly parameters, dealing with the parameters in the denominator is the key and that, by using estimate projection and a proper Lyapunov function, stable adaptive control can be successfully designed. Since the denominator in a scalar fractional parameterization cannot be zero and thus has a fixed sign, it can be embedded into an integral Lyapunov function such that the time derivative of Lyapunov function along the system trajectory no longer contains any denominator. It is based on this simple observation that an adaptive control is designed in [17] for scalar systems with fractional parameterization. Because of the method used, denominator parameters are not estimated in the regulation problem.

Clearly, the simple design methods in [5] and [17] cannot handle multivariable systems with fractional parameterization. A new design method proposed in [15] solves the problem by embedding two layers of estimation into adaptive control: All parameters in the fractional parameterization are estimated using adaptation laws, a robust observer is used to estimate the impact of the whole fractional dynamics, and the observer state is used in the adaptation laws. The result in [15] can be extended in principle to systems with general nonlinear parameterization but its shortcoming is that the resulting stability is semiglobal as systems dynamics are nonlinear and a nonlinear observer along the line of [1], [3], and [8] is used. It is worth noting that the certainty equivalence principle is used in the results of [5], [15], and [17], and that those results offer little on parameter convergence.

Since constant parameters are bounded and, hence, can be treated as a special case of bounded uncertainties, robust control designs can directly be used to handle nonlinearly parameterized dynamics, and no parameter estimation is needed. Adaptation laws of parameter estimation can be embedded into the robust control frame (and thus called robust adaptive controls), either by such robustification tools as leakage law [12], or by imposing convex/concave conditions on parameters [2], or by projecting estimates into a compact set [6], or by introducing linearly parameterized bounding function on nonlinear parameterization [10]. In all these cases, the robust control part is dominating and, hence, little can be said about convergence property of the estimates. In this note, nonlinearly parameterized dynamics are compensated for solely by adaptive control, which is technically a much harder problem. It is worth noting that, if the bounding function on uncertainties contains unknown but linearly parameterized parameters, robust control laws can be embedded into the adaptive control framework to render adaptive robust controls [12]–[14].

The above synopsis of existing results points to the need of developing a systematic methodology to effectively estimate unknown parameters and to design adaptive control for systems that are nonlinearly parameterized and possibly nonaffine. The objective of this paper is to provide an innovative design technique toward fulfilling the need. Specifically, both affine and nonaffine systems with general nonlinear parameterization are considered, the new design modifies the certainty equivalence principle by injecting a state biasing vector function as an additive term to parameter estimate vector in the adaptive control, and the biasing function links the system dynamics to estimation error dynamics. The biasing function leads to a new family of Lyapunov functions and, under certain conditions, its choice makes it possible to generate either negative definite or negative semidefinite terms associated with (a biased) estimation error. A straightforward Lyapunov argument produces explicit conditions on global asymptotic stability of the state and on convergence of parameter estimation. The conditions are illustrated by several examples and classes of nonlinear systems. Compared with the existing results, the proposed method and results have several unique features. First, both affine and nonaffine systems can be studied. Second, general nonlinear parameterization can be handled. Third, the proposed adaptive control does not contains any robust control part. Fourth, the proposed adaptive control deviates from those synthesized from the certainty equivalence principle by introducing the biasing function. The use of biasing function is instrumental to handle nonlinear parameterization, to generate Lyapunov functions, and to render a simple condition on convergence of parameter estimation for general nonlinear systems.

II. Problem Formulation

Consider the following class of nonaffine and nonlinearly parameterized systems:

$$\dot{x} = F(t, x, \phi, u) \tag{1}$$

where $F(\cdot)$ has a known function expression, $x \in \Re^n$ is the state, $u \in \Re^m$ is the control, and $\phi \in \Re^l$ is the vector of unknown constant parameters. Our objective is to synthesize an adaptive control and a corresponding adaptation law of form $u = u(t, x, \hat{\phi})$ and $\dot{\hat{\phi}} = \Psi(t, x, \hat{\phi})$ such that the closed-loop system is globally asymptotically stable. To this end, the following technical assumptions are introduced.

Assumption 1: The value of ϕ is unknown but belongs to known compact set $\Omega_\phi = \{\phi : \|\phi\| \le c_\phi\}$.

Assumption 2: If vector ϕ were known or measured, system (1) could be globally and asymptotically stabilized for all $\phi \in \Re^l$ under the ideal control $u = U^*(t, x, \phi) \triangleq U_n^*(t, x) + U_p^*(t, x, \phi)$, where $U_n^*(\cdot)$ is a nonlinear nominal control of known expression, and $U_p^*(\cdot)$ is a perfect-knowledge cancellation control of known expression and with $U_p^*(t, x, 0) = 0$. Specifically, there exist known function $F_n(t, x)$ and Lyapunov function $V_n(t, x)$ such that

$$\begin{aligned} F\left(t, x, \phi, U_n^*(t, x) + U_p^*(t, x, \phi)\right) &= F_n(t, x) \\ \gamma_1(\|x\|) \le V_n(t, x) &\le \gamma_2(\|x\|) \\ \left\| \frac{\partial V_n(t, x)}{\partial x} \right\| &\le c_n \gamma_3^{\alpha_1}(\|x\|) \end{aligned} \tag{2}$$

and

$$\frac{\partial V_n(t, x)}{\partial t} + \left[\frac{\partial V_n(t, x)}{\partial x}\right]^T F_n(t, x) \le -\gamma_3(\|x\|) \tag{3}$$

where $\gamma_i : \Re^+ \to \Re^+$ are class $\mathcal{K}_\infty$ functions, and $c_n > 0$ and $0 < \alpha_1 < 1$ are constants.

Assumption 3: Functions $F(\cdot)$ and $U^*(\cdot)$ are differentiable, uniformly bounded with respect to t, and locally uniformly bounded with respect to x and ϕ.

Remark 2.1: It should be noted that existence of $U^*(t, x, \phi)$ in Assumption 2 is the key and that the rest of the conditions in the above assumptions are standard. In this paper, the problem of nonlinearly parameterized systems are studied using *solely* the adaptive control method, i.e., a robust control part or any other robustification method is not applied for the purpose of parameter identification. Hence, if the parameters can be adequately estimated, the corresponding control (after parameters converge) ensures asymptotical stability. In light of this, existence of $U^*(t,x,\phi)$ is not only sufficient but also necessary. In the case that $U^*(t,x,\phi)$ is known to exist but does not have a known expression, approximation approaches such as neural network could be applied (in which case linear overparameterization might be used to simplify the adaptive control design by trading off convergence of parameter estimation). $\diamond$

For notational simplicity throughout the note, nonlinear scalar and vector functions are expressed into products of a matrix and the state. Such matrix representations can always be done for any differentiable nonlinear vector function of x and for any scalar function of second order or higher in x. In what follows, three sets of matrix representations are introduced. First, matrix $A_n(\cdot)$ of the closed-loop nominal system is defined as

$$\begin{aligned} F_n(t,x) &= \int_0^1 \frac{\partial F_n(t,\beta x)}{\partial \beta} d\beta \\ &= \left[\int_0^1 \left.\frac{\partial F_n(t,\zeta)}{\partial \zeta}\right|_{\zeta=\beta x} d\beta\right] x \triangleq A_n(t,x)x. \end{aligned} \tag{4}$$

Second, matrices $P(\cdot)$ and $P'(\cdot)$ are introduced to express partial derivatives of Lyapunov function, for given constant c_1 ($c_1 \geq 0$ can be freely chosen in most cases), as

$$\begin{aligned} V_n^{c_1}(t,x)\frac{\partial V_n(t,x)}{\partial x} &\triangleq P^T(t,x)x \\ V_n^{c_1}(t,x)\frac{\partial V_n(t,x)}{\partial t} &\triangleq x^T P'(t,x)x. \end{aligned} \tag{5}$$

Similarly, matrix $Q_n(t,x)$ can be found using

$$V_n^{c_1}(t,x)\left\{\frac{\partial V_n(t,x)}{\partial t} + \left[\frac{\partial V_n(t,x)}{\partial x}\right]^T F_n(t,x)\right\} \triangleq x^T Q_n(t,x)x.$$

Thus, it follows from (3), (4), and (5) that matrix

$$Q_n(t,x) \triangleq -\left[P' + \frac{1}{2}PA_n + \frac{1}{2}A_n^T P^T\right] \tag{6}$$

is both positive definite and decrescent. Third, let us denote

$$B(t,x,\phi,u) \triangleq \frac{\partial F(t,x,\phi,u)}{\partial u} \quad K_\phi(t,x,\phi) \triangleq \frac{\partial U_p^*(t,x,\phi)}{\partial \phi} \tag{7}$$

as input matrix and "gain" matrix (associated with parameter vector ϕ), respectively. Hence, it follows from (7) that, for any argument $\beta \in [0,1]$ and for any pair of vectors z_1 and z_2 of proper dimensions

$$\begin{aligned} M&(t,z_1,\phi,z_2)z_2 \\ &\triangleq \left[-\int_0^1 B\left(t,z_1,\phi,U^*(t,z_1,\phi-\beta z_2)\right)\right. \\ &\qquad \left.\times K_\phi(t,z_1,\phi-\beta z_2)d\beta\right] z_2 \qquad (8) \\ &= \left[\int_0^1 \frac{\partial F\left(t,z_1,\phi,U^*(t,z_1,\phi-\beta z_2)\right)}{\partial \beta z_2} d\beta\right] z_2 \qquad (9) \\ &= F\left(t,z_1,\phi,U^*(t,z_1,\phi-z_2)\right) \\ &\quad - F\left(t,z_1,\phi,U^*(t,z_1,\phi)\right) \qquad (10) \end{aligned}$$

in which design matrix $M(\cdot)$ (to be invoked later in the proposed design) can be computed from (8)–(10).

III. New Nonlinear Adaptive Control Design

The proposed new design is motivated by reviewing the standard adaptive control design based on the certainty equivalence principle.

A. Standard Adaptive Control

Consider first the class of affine systems with stable nominal dynamics: $\dot{x} = F_n(t,x) + B(t,x)[u - U_p^*(t,x,\phi)]$. This class is a subset of systems in (1). In the case of linear parameterization, $U_p^*(t,x,\phi) = W(t,x)\phi$ holds for some matrix $W(t,x)$. In this case, adaptive control can be designed using the certainty equivalence principle. Specifically, let

$$u = U_p^*(t,x,\hat{\phi}) = W(t,x)\hat{\phi} \quad \dot{\hat{\phi}} = -N(t,x)P^T(t,x)x \tag{11}$$

where matrix $N(\cdot)$ is chosen through a Lyapunov-based stability argument. To be specific, let us choose Lyapunov function to be

$$L_a(t,x,\tilde{\phi}) = \frac{1}{1+c_1}V_n^{1+c_1}(t,x) + \frac{1}{2}\|\tilde{\phi}\|^2$$

where $c_1 > 0$ is a constant, and $\tilde{\phi} = \phi - \hat{\phi}$ is the estimation error. It follows from (5) that, by setting $N(t,x) = W^T(t,x)B^T(t,x)$

$$\begin{aligned} \dot{L}_a = &- x^T Q_n x - x^T P(t,x)B(t,x)W(t,x)\tilde{\phi} \\ &+ \tilde{\phi}^T N(t,x)P^T(t,x)x \\ = &- x^T Q_n x \end{aligned}$$

from which global asymptotic stability of state x can be concluded. Mathematically, the standard design takes full advantage of linear parameterization by combining all the terms related to ϕ and $\hat{\phi}$ and in both system dynamics and $\dot{L}_a$ into terms of $\tilde{\phi}$ only (so that ϕ and $\hat{\phi}$ do not exist separately) and then by cancelling each other among themselves through the choice of $N(t,x)$. As a result, the resulting upper bound on $\dot{L}_a$ is made to be a negative definite function of x only.

If $U^*(t,x,\phi)$ is nonlinearly parameterized (or if the system is not affine), the above design process of direct cancellation in a Lyapunov argument inherently fails due to the fact that $U^*(t,x,\phi) - U^*(t,x,\hat{\phi}) \neq W(t,x)\tilde{\phi}$. Instead, $U^*(t,x,\phi) - U^*(t,x,\hat{\phi}) = W'(t,x,\phi,\hat{\phi})\tilde{\phi}$, the expression of matrix $W'(\cdot)$ could be very complicated, and, more importantly, matrix $W'(\cdot)$ contains unknowns. To handle this difficulty of nonlinearity, alternative designs must be pursued.

In most of the work up to now, the prevailing method is to use $u = U^*(t,x,\hat{\phi}) + u_R$, where u_R is a robust control term [12]. In such a design, adaptation law must be modified (by projections or by such robustification as leakage law) to ensure $\hat{\phi}$ being bounded, and robust control u_R compensates for the difference $U^*(t,x,\phi) - U^*(t,x,\hat{\phi})$ by domination. As a result, u_R tends to be both conservative and dominating (over adaptive portion $U^*(t,x,\hat{\phi})$), u_R may become discontinuous if asymptotic stability of x is required, and parameter estimation has little convergence property. These are the main drawbacks of this line of robust adaptive designs. In fact, it would be easier in many cases to simply fix $\hat{\phi}$ and design a robust control. One exception is the adaptive design for a class of nonlinearly parameterized systems [15], in which a robust observer is used to estimate the total impact of $U^*(t,x,\phi)$.

B. Basic Design of the Proposed New Adaptive Control

The proposed adaptive control is of form

$$u = U^*\left(t,x,\hat{\phi} - h(t,x) + h(t,0)\right) \tag{12}$$

where function $h(\cdot) : \Re^+ \times \Re^n \rightarrow \Re^l$ is a function to be chosen by the designer (as long as $h(t,x)$ is well defined in terms of x and is uniformly bounded with respect to t), and $\hat{\phi}$ is the vector of parameter estimates and is updated by

$$\dot{\hat{\phi}} = -N(t,x)P^T(t,x)x + \frac{\partial h(t,x)}{\partial x}F_n(t,x) + \frac{\partial h(t,x)}{\partial t} - \frac{\partial h(t,0)}{\partial t}. \quad (13)$$

Clearly, adaptive control (12) is different from typical adaptive controls that are of form $u = U^*(t,x,\hat{\phi})$ and are based on the certainty equivalence principle. In particular, the design of adaptive control (12) and its adaptation law (13) has two degrees of freedom: Choices of $h(t,x)$ and $N(t,x)$. Function $h(t,x)$ in (12) can be viewed as a *time-varying state biasing function* injected into parameter estimation, and its presence introduces the additional degree of freedom necessary to handle nonlinearly parameterized dynamics. It will be shown in Section III-D that, for linear parameterized and affine systems, the proposed design includes the standard adaptive control design as a special case.

Let us define feedback injection gain matrix $C(\cdot)$ and scaling gain $\rho(\cdot)$ as follows:

$$C(t,x) \triangleq -\frac{\partial h(t,x)}{\partial x} \quad \rho(\xi) \triangleq \frac{dV(\xi)}{d\xi} \quad (14)$$

where $V(\cdot)$ is a differentiable $\mathcal{K}_\infty$ function to be chosen as a component of the proposed design to ensure stability. The following theorem provides the basic result on the proposed adaptive control design and its stability analysis.

Theorem 1: Consider system (1) satisfying Assumptions 1–3 and under adaptive control (12) and adaptation law (13). Then, the closed-loop system is globally asymptotically stable if $N(t,z_1)$, $C(t,z_1)$, and $\rho(\|z_2\|^2)$ can be chosen such that matrix $C(\cdot)$ is integrable (in the sense of (14)) and that matrix

$$\overline{Q}(t,z) \triangleq \begin{bmatrix} 2(1-\beta_n)Q_n & -P(N^T\rho + M) \\ -(N\rho + M^T)P^T & (CM + M^TC^T)\rho \end{bmatrix} \quad (15)$$

is positive semidefinite with respect to $z = [z_1^T \; z_2^T]^T$, where $0 < \beta_n < 1$ is any constant freely chosen $z_1 = x$, $z_2 = \tilde{\phi} + h(t,z_1) - h(t,0)$, and matrices $Q_n(t,z_1)$, $P(t,z_1)$, and $M(t,z_1,\phi,z_2)$ are those defined from (6), (5), and (8), respectively. Furthermore, if matrix $\overline{Q}(t,z)$ is made positive definite with respect to z, $\tilde{\phi}(t)$ converges to zero as time approaches infinity.

Proof: It follows from (2) that, under adaptive control (12) and adaptation law (13), the closed-loop system has the following dynamics: For any choice of $N(\cdot)$

$$\begin{aligned}\dot{x} &= F_n(t,x) \\ &\quad - [F(t,x,\phi,U^*(t,x,\phi)) \\ &\quad\quad -F\left(t,x,\phi,U^*\left(t,x,\hat{\phi} - h(t,x) + h(t,0)\right)\right)\Big] \\ \dot{\tilde{\phi}} &= N(t,x)P^T(t,x)x - \frac{\partial h(t,x)}{\partial x}F_n(t,x) \\ &\quad - \frac{\partial h(t,x)}{\partial t} + \frac{\partial h(t,0)}{\partial t}.\end{aligned}$$

Using (4), (8), and (14), we can rewrite the above equations of closed-loop dynamics as

$$\dot{z}_1 = A_n(t,z_1)z_1 + M(t,z_1,\phi,z_2)z_2 \quad (16)$$
$$\dot{z}_2 = N(t,z_1)P^T(t,z_1)z_1 - C(t,z_1)M(t,z_1,\phi,z_2)z_2. \quad (17)$$

Now, choose the overall Lyapunov function to be

$$L(t,z) = \frac{1}{1+c_1}V_n^{1+c_1}(t,z_1) + \frac{1}{2}V\left(\|z_2\|^2\right)$$

where $V_n(\cdot)$ is that in Assumption 2, and $V(\cdot)$ is the sub-Lyapunov function in (14). It follows from (16) and (17) that

$$\dot{L} = -\beta_n x^T Q_n(t,x)x - \frac{1}{2}z^T\overline{Q}(t,z)z \quad (18)$$

where $0 < \beta_n < 1$ is a scaling factor, and matrix $\overline{Q}(\cdot)$ is given by (15). Stability of x becomes obvious by invoking the property of matrix $\overline{Q}(t,z)$ in (15) and applying it to (18). In the case that $\overline{Q}(t,z)$ is positive definite, both z_1 and z_2 converge to zero. It follows from the definition of z_2 that $\tilde{\phi}$ converges to zero. □

It is straightforward to show that matrix $\overline{Q}(\cdot)$ in (15) is positive semidefinite or positive definite if the following inequalities hold for all z, for all t, and for all $\phi \in \Omega_\phi$

$$\begin{aligned}&2(1-\beta_n)\rho\left(\|z_2\|^2\right)\lambda_{\min}(CM + M^TC^T)\lambda_{\min}(Q_n) \\ &\quad > \left\|N(t,z_1)\rho\left(\|z_2\|^2\right) + M^T(t,z_1,\phi,z_2)\right\|^2 \cdot \|P(t,z_1)\|^2 \quad (19)\end{aligned}$$

where $0 < \beta_n < 1$, and $\lambda_{\min}(\cdot)$ denotes the operation of finding the minimum eigenvalue. Furthermore, it follows from (3) that inequality (19) holds if

$$\begin{aligned}&\rho\left(\|z_2\|^2\right)\lambda_{\min}(CM + M^TC^T)\gamma_1^{c_1}(\|z_1\|)\gamma_3^{1-2\alpha_1}(\|z_1\|) \\ &\quad > \frac{1}{2(1-\beta_n)}\left\|N(t,z_1)\rho\left(\|z_2\|^2\right) + M^T(t,z_1,\phi,z_2)\right\|^2 \\ &\quad\quad \times c_n^2\gamma_2^{2c_1}(\|z_1\|).\end{aligned}$$

In order to establish the above inequality or (19) or (15) for most nonlinearly parameterized systems, the designer needs to invoke Assumption 1 and find bounds on all the terms associated with ϕ. In certain cases, the initial condition of $\hat{\phi}$ can be judiciously selected to yield tighter bounds.

In the subsequent subsections, we will study how to make design selections of $N(t,z_1)$, $C(t,z_1)$, and $\rho(\|z_2\|^2)$ to establish global stability and convergence using the theorem, and to identify the types or classes of nonlinearly parameterized systems to which the proposed method is applicable.

C. Case Studies of the New Adaptive Control Design

In this subsection, in order to illustrate the theorem or condition (19), adaptive control design is carried out for the following examples which contain some of typical nonlinear parameterizations.

Example 1: Consider the following square system:

$$\dot{x} = -x - D(t,x)\mathrm{vec}\left\{e^{E_i(t,x)\phi_i}\right\} + u$$

where $x,\ u,\ \phi \in \Re^n$, $E_i(t,x) \in \Re$ for $i = 1,\cdots,n$, $D(t,x) \in \Re^{n\times n}$, and vec$\{\cdot\}$ denote the column vector formed by the elements. It follows that $F_n(t,x) = -x$, $U_n^* = 0$, and $U_p^*(t,x,\phi) = D(t,x)\mathrm{vec}\{e^{E_i(t,x)\phi_i}\}$. Thus, it follows from (8) that letting $E(t,x) = \mathrm{diag}\{E_i(t,x)\}$

$$M(t,x,\phi,z_2) = -\int_0^1 D(t,x)E(t,x)\mathrm{diag}\left\{e^{E_i(t,x)[\phi_i - \beta z_{2i}]}\right\}d\beta.$$

Let us choose $V_n(t,x) = 0.5\|x\|^2$, $c_1 = 1$, $N(t,z_1) = 0$, and $\beta_n = 0.5$. It follows from (4) and (5) that $P(t,x) = Q_n(t,x) = I$. Hence, matrix $\overline{Q}(t,x)$ is positive semidefinite if

$$\rho\left(\|z_2\|^2\right)\left[C(t,x)M(t,x,\phi,z_2) + M^T(t,x,\phi,z_2)C^T(t,x)\right] \geq M^T(t,x,\phi,z_2)M(t,x,\phi,z_2).$$

Letting $C(t,x) = -[D(t,x)E(t,x)]^{-1}\rho_c(t,x)$ for some scalar function $\rho_c(\cdot)$, we can rewrite the aforementioned as

$$2\rho\left(\|z_2\|^2\right)\rho_c(t,x)\int_0^1 \operatorname{diag}\left\{e^{E_i(t,x)[\phi_i-\beta z_{2i}]}\right\}d\beta \geq \int_0^1 \operatorname{diag}\left\{e^{E_i(t,x)[\phi_i-\beta z_{2i}]}\right\}d\beta E^T(t,x)D^T(t,x)D(t,x) \times E(t,x)\int_0^1 \operatorname{diag}\left\{e^{E_i(t,x)[\phi_i-\beta z_{2i}]}\right\}d\beta$$

which is guaranteed by

$$2\rho\left(\|z_2\|^2\right)\rho_c(t,x)\int_0^1 \operatorname{diag}\left\{e^{E_i(t,x)[\phi_i-\beta z_{2i}]}\right\}d\beta \geq \|D(t,x)E(t,x)\|^2\int_0^1 \operatorname{diag}\left\{e^{2E_i(t,x)[\phi_i-\beta z_{2i}]}\right\}d\beta.$$

Therefore, functions $\rho(\|z_2\|^2)$ and $\rho_c(t,x)$ should be chosen such that for all $i \in \{1,\cdots,n\}$

$$2\rho\left(\|z_2\|^2\right)\rho_c(t,x) \geq \|D(t,x)E(t,x)\|^2 e^{E_i(t,x)[\phi_i-\beta z_{2i}]}.$$

Many choices exist; for instance, given $|\phi_i| \leq c'_\phi$, $\rho_c(t,x) = \|D(t,x)E(t,x)\|^2 e^{\max_i[|E_i(t,x)|c'_\phi + E_i^2(t,x)]}$ and $\rho(\xi) = e^\xi$ (which implies $V(\xi) = e^\xi - 1$). Thus, for the system, the proposed adaptive control and adaptation law are given by (12) and (13), respectively, provided that the resulting choice $C(t,x) = -[D(t,x)E(t,x)]^{-1}\rho_c(t,x)$ is integrable as specified by (14). ⋄

In Example 1, the choice of $N(\cdot) = 0$ is made, and an additional property can be concluded using the following corollary.

Corollary 1: Consider system (1) under adaptive control (12) and adaptation law (13). If inequality (19) holds with the choice of $N(t,x) = 0$, the theorem holds and biased estimation error $z_2 = \hat{\phi}(t) + h(t,x) - h(t,0)$ is monotonously decreasing in magnitude.

Proof: It follows from (17) that given $N(t,x) = 0$

$$\frac{d\|z_2\|^2}{dt} = -z_2^T[CM + M^TC^T]z_2.$$

According to (19), the right hand side of the aforementioned is seminegative definite, and, hence, monotone property of $\|z_2\|$ with respect to time can be concluded. □

The following example shows importance of the monotone property in avoiding potential singularity of an adaptive control designed for nonlinearly parameterized systems. It is shown that the proposed adaptive design does not need any projection (which would require additional and prior knowledge of a positive lower on ϕ) commonly used in existing adaptive design for systems with linear parameterization.

Example 2: Consider the following scalar system:

$$\dot{x} = -x - \frac{1}{1+\phi\,(2+\sin(t)+x^2)} + u^3, \qquad \phi \geq 0.$$

It follows that $F_n(t,x) = -x$, $U_n^* = 0$, and $U_p^*(t,x,\phi) = 1/[1+\phi(2+\sin(t)+x^2)]^{1/3}$. It follows from (10) that $M = M(t,x,\phi,z_2)$ and

$$M = \frac{2+\sin(t)+x^2}{[1+(\phi-z_2)\,(2+\sin(t)+x^2)]\,[1+\phi\,(2+\sin(t)+x^2)]}.$$

It is apparent that $0 < M(t,x,\phi,z_2) < 1/\phi$ so long as $\phi - z_2 \geq 0$.

For the system, Lyapunov function is chosen to be $V_n(t,x) = 0.5x^2$. Hence, $P = 1$ and $Q_n = 1$ with $c_1 = 1$. Choosing $N(t,z_1) = 0$ and $\beta_n = 0.5$, we can rewrite condition (19) as $2(1-\beta_n)\rho(\|z_2\|^2)C(t,z_1) > M(t,x,\phi,z_2)$, which holds under the choices of $C(t,z_1) = 2+\sin(t)$ and $\rho(\|z_2\|^2) = 1/\phi$ provided that $\phi - z_2 \geq 0$ is maintained.

According to (14), the choice of $C(\cdot)$ renders $h(t,x) = -2x - x\sin(t)$. Thus, the corresponding adaptive control and adaptation law given by (12) and (13) are $u = 1/[1+(\hat{\phi}+2x+x\sin(t))(2+\sin(t)+x^2)]^{1/3}$ and $\dot{\hat{\phi}} = 2x + x\sin(t) - x\cos(t)$, respectively. It follows from Corollary 1 that, by choosing $\hat{\phi}(t_0)$ to be any value satisfying $\hat{\phi}(t_0) \geq -2x(t_0) - x(t_0)\sin(t_0)$, inequality $\phi - z_2(t) = \hat{\phi}(t) + 2x(t) + x\sin(t) \geq 0$ holds for all $t \geq t_0$. Thus, all the conditions are satisfied, and the proposed control is well defined and singularity-free.⋄

Next, we consider a system to which standard adaptive control design could be applied through overparameterization. However, overparameterization generally leads to the loss of convergence of parameter estimation.

Example 3: Consider the following scalar system:

$$\dot{x} = -x - \left[(1+x^2)\phi - x\phi^2 + \frac{1}{3}\phi^3\right] + u.$$

It is apparent that $F_n(t,x) = -x$, $U_n^* = 0$, and $U_p^*(t,x,\phi) = (1+x^2)\phi - x\phi^2 + \phi^3/3$. It follows from (8) that the design "matrix" is defined by $M(t,x,\phi,z_2) = -\int_0^1[1+x^2-2x(\phi-\beta z_2)+(\phi-\beta z_2)^2]d\beta$ and, hence, $M(\cdot)$ satisfies the inequality $1 < -M(t,x,\phi,z_2) \leq 1+2x^2+4\phi^2+4z_2^2/3$.

Again, let us choose $V_n(t,x) = 0.5x^2$, $c_1 = 1$, $N(t,z_1) = 0$ and $\beta_n = 0.5$. Equation (19) can be expressed as $\rho(\|z_2\|^2)C(t,z_1)M(t,x,\phi,z_2) > M^2(t,x,\phi,z_2)$, and it holds under the simple choices of $C(t,z_1) = -(1+2x^2)$ and $\rho(\|z_2\|^2) = (1+4\phi^2)(1+4z_2^2/3)$. Clearly, both $C(t,z_1)$ and $\rho(\|z_2\|^2)$ are integrable, and $h(t,x) = x + 2x^3/3$. Thus, with the expressions of $U^*(\cdot)$ and $h(\cdot)$, the proposed adaptive control and adaptation law are readily given by (12) and (13), respectively. ⋄

D. Application to Affine Systems With Separable Nonlinear Parameterization

Next, consider the class of affine systems whose unknown parameters are separated into several groups. That is

$$\begin{aligned}\dot{x} &= F(t,x,\phi,u)\\ &= F_n(t,x) + B(t,x)\\ &\quad\times\left[u - W(t,x)\phi_1 - \sum_{j=2}^{\eta} f_i(t,x,\phi_i)\right]\end{aligned} \tag{20}$$

where $\eta \geq 1$ is an integer, $\phi_i \in \Re^{l_i}$ with $\sum_{i=1}^{\eta} l_i = l$, $\phi_1 \in \Re^{l_1}$ consists of linearly parameterized unknowns, and $\phi_i \in \Re^{l_i}$ with $i \geq 2$

are nonlinearly parameterized, unseparable unknowns. In this case, the proposed adaptive control becomes

$$u = W(t,x)\left[\hat{\phi}_1 - h_1(t,x) + h_1(t,0)\right] + \sum_{j=2}^{\eta} f_j\left(t, x, \hat{\phi}_j - h_j(t,x) + h_j(t,0)\right) \quad (21)$$

and

$$\dot{\hat{\phi}}_i = -N_i(t,x)P^T(t,x)x + \frac{\partial h_i(t,x)}{\partial x}F_n(t,x) + \frac{\partial h_i(t,x)}{\partial t} - \frac{\partial h_i(t,0)}{\partial t}, \qquad 1 \le i \le \eta. \quad (22)$$

Then, under the choices of (21) and (22), error dynamics are

$$\begin{aligned}\dot{z}_1 &= A_n(t,z_1)z_1 + \sum_{j=1}^{\eta} M_j(t,z_1,\phi,z_{2j})z_{2j}\\ \dot{z}_{2i} &= N_i(t,z_1)P^T(t,z_1)z_1 - C_i(t,z_1)\\ &\quad \times \sum_{j=1}^{\eta} M_j(t,z_1,\phi_j,z_{2j})z_{2j}\end{aligned}$$

where $z_1 \triangleq x$, $z_{2i} \triangleq \tilde{\phi}_i + h_i(t,z_1) - h_i(t,0)$ for $1 \le i \le \eta$

$$M_1(t,z_1) \triangleq -B(t,z_1)W(t,z_1)$$

$$M_j(t,z_1,\phi_j,z_{2j})z_{2j} \triangleq B(t,z_1)\int_0^1 \frac{\partial f_j(t,x,\phi_j - \beta z_{2j})}{\partial \beta z_{2j}} d\beta$$

for $2 \le j \le \eta$, and

$$C_i(t,z_1) \triangleq -\frac{\partial h_i(t,z_1)}{\partial z_1}, \qquad 1 \le i \le \eta.$$

To see the effect of decomposition, let us consider Lyapunov function

$$L_h(t,z) = \frac{1}{1+c_1}V_n^{1+c_1}(t,z_1) + \frac{1}{2}\sum_{i=1}^{\eta} V_i\left(\|z_2\|^2\right) \quad (23)$$

where $z_2 = [z_{21}^T \ \cdots \ z_{2\eta}^T]^T$. Taking the time derivative yields

$$\dot{L}_h(t,z) = -\beta_n x^T Q_n(t,x)x - \frac{1}{2}z^T\overline{Q}_h(t,z)z$$

where $0 < \beta_n < 1$ is a constant, and $\overline{Q}_h(t,z)$ is given in (24), as shown at the bottom of the page (in which $\rho_i(\xi) = dV_i(\xi)/d\xi$, $\varrho = \operatorname{diag}\{\rho_1,\cdots,\rho_\eta\}$, $C = [C_1^T C_2^T \ \cdots \ C_\eta^T]^T$, and $M = [M_1 \ M_2 \ \cdots \ M_\eta]$). Comparing $\overline{Q}_h(\cdot)$ in (24) with matrix $\overline{Q}(\cdot)$ in (15), we know that the decomposition in (20) makes little difference in yielding a successful Lyapunov design of adaptive control should Lyapunov function is chosen to be of form (23).

Nonetheless, it is apparent that, if the system only has linear parameterization (i.e., $l_1 = l$ and $\eta = 1$), the proposed control (21) and (22) with the choices of $h_1(t,x) = 0$, $\rho_1 = 1$, and $N_1(t,x) = W^T(t,x)B^T(t,x)$ reduces to the standard adaptive control in (11). In the general case that $l > l_1$ or $\eta > 1$, we can choose $N_1(t,x) = W^T(t,x)B^T(t,x)$ but not $N_i = -M_i^T$ (for $2 \le i \le \eta$) as matrix M_i is a function of not only t and x but also z_2. Hence, the apparent choices are $\rho_1 = 1, N_1(t,x) = W^T(t,x)B^T(t,x), N_2(t,x) = \cdots = N_\eta(t,x) = 0$, and $C_1(t,x) \neq 0$ under which the design objective is to choose $C(t,x)$ and $\varrho(\cdot)$ such that the resulting matrix of $\overline{Q}_h(t,z)$ is positive definite or semidefinite.

E. Extension to Nonlinearly Parameterized Systems in the Feedback Form

In system (20), nonlinear parameterization is decomposed horizontally into separable groups. Separation can also be explored vertically in system dynamics. To motivate this, consider the following class of affine systems:

$$\dot{x} = F_n(t,x) + B\left[-U^*(t,x,\phi) + u\right] \quad (25)$$

where $B = [0 \ \cdots \ 0 \ 1]^T$. It follows from (8) that

$$\begin{aligned}C(t,z_1)M(t,z_1,\phi,z_2) &= -C(t,z_1)B\int_0^1 K_\phi(t,z_1,\phi-\beta z_2)d\beta\\ &= \frac{\partial h(t,x)}{\partial x_n}\int_0^1 K_\phi(t,z_1,\phi-\beta z_2)d\beta.\end{aligned}$$

If $\partial h(t,x)/\partial x_n$ together with $N(t,x)$ and $\rho(\cdot)$ can be chosen to make $\overline{Q}$ positive semidefinite or definite; $\partial h(t,x)/\partial x_n$ is always integrable to solve for biasing function $h(t,x)$. Now, consider the class of nonlinearly parameterized systems in the feedback form

$$\begin{cases}\dot{x}_1 = x_2 - U_1^*(t,x_1,\phi_1)\\ \dot{x}_2 = x_3 - U_2^*(t,x_1,x_2,\phi_2)\\ \vdots\\ \dot{x}_n = u - U_n^*(t,x_1,\cdots,x_n,\phi_n).\end{cases} \quad (26)$$

In this case, the proposed nonlinear adaptive design can be carried out with aid of the backstepping technique [9], [12]. At each step of the backstepping design, the corresponding B matrix is simply 1 and, hence, integrability is always guaranteed. The following corollary illustrates the combined design for $n = 2$, and adaptive control of system (26) can be developed accordingly for any $n > 2$.

Corollary 2: Suppose that, for the two fictitious systems $\dot{x}_1 = v_1 - U_1^*(t,x_1,\phi_1)$ and $\dot{x}_2 = v_2 - U_2^*(t,x_1,x_2,\phi_2)$, adaptive controls $v_1 = -x_1 + U_1^*(t,x_1,\hat{\phi}_1 - h_1(t,x_1) + h_1(t,0))$ and $v_2 = -x_2 +$

$$\overline{Q}_h(t,z) = \left[\begin{array}{c|c} 2(1-\beta_n)Q_n & -P\left(N_1^T\rho_1 + M_1\right) \ -P\left(N_2^T\rho_2 + M_2\right) \ \cdots \ -P\left(N_\eta^T\rho_\eta + M_\eta\right) \\ \hline -\left(N_1\rho_1 + M_1^T\right)P^T & \\ -\left(N_2\rho_2 + M_2^T\right)P^T & \\ \vdots & \varrho CM + M^TC^T\varrho \\ -\left(N_\eta\rho_\eta + M_\eta^T\right)P^T & \end{array}\right] \quad (24)$$

$U_2^*(t, x_1, x_2, \hat{\phi}_2 - h_2(t, x_2) + h_2(t, 0))$ can be designed by properly choosing $\dot{\hat{\phi}}_i$ and $h_i(\cdot)$ according to the theorem. Then, adaptive control u can be found for the second-order system $\dot{x}_1 = x_2 - U_1^*(t, x_1, \phi_1)$ and $\dot{x}_2 = u - U_2^*(t, x_1, x_2, \phi_2)$.

Proof: Define that, for $i = 1,\ 2$, $Z_{i*} = [z_{i1}\ z_{i2}]^T$, and $Z_{*i} = [z_{1i}\ z_{2i}]^T$, and that $Z_{12} = [z_{12}\ z_{21}]^T$, where

$$\begin{cases} z_{11} = x_1 \\ z_{12} = \phi_1 - \hat{\phi}_1 + h_1(t, z_{11}) - h_1(t, 0) \\ z_{21} = x_2 - U_1^*\left(t, z_{11}, \hat{\phi}_1 - h_1(t, z_{11}) + h_1(t, 0)\right) + z_{11} \\ z_{22} = \phi_2 - \hat{\phi}_2 + h_2(t, z_{21}) - h_2(t, 0). \end{cases} \tag{27}$$

Select adaptation laws to be

$$\begin{aligned} \dot{\hat{\phi}}_1 = & - N_1(t, z_{11}) z_{11} - N_{12}'(t, z_{11}) z_{21} - \frac{\partial h_1(t, z_{11})}{\partial z_{11}} z_{11} \\ & + \frac{\partial h_1(t, z_{11})}{\partial t} - \frac{\partial h_1(t, 0)}{\partial t} \\ \dot{\hat{\phi}}_2 = & - N_2(t, z_{21}) z_{21} - \frac{\partial h_2(t, z_{21})}{\partial z_{21}} z_{21} \\ & + \frac{\partial h_2(t, z_{21})}{\partial t} - \frac{\partial h_2(t, 0)}{\partial t} \end{aligned}$$

which are of the same form as (13) except for the extra term $N_{12}'(\cdot)$. It follows that, under the above adaptation laws and under adaptive control

$$\begin{aligned} u = & - z_{21} - z_{11} - S_2^2(t, x_1, \hat{\phi}_1) z_{21} \\ & + \frac{\partial U_1^*\left(t, z_{11}, \hat{\phi}_1 - h_1(t, z_{11}) + h_1(t, 0)\right)}{\partial t} \\ & + \frac{\partial U_1^*\left(t, z_{11}, \hat{\phi}_1 - h_1(t, z_{11}) + h_1(t, 0)\right)}{\partial \hat{\phi}_1} \dot{\hat{\phi}}_1 \\ & - S_2(t, x_1, \hat{\phi}_1) x_2 \\ & + U_2^*\left(t, x_1, x_2, \hat{\phi}_2 - h_2(t, z_{21}) + h_2(t, 0)\right) \\ & + S_2(t, x_1, \hat{\phi}_1) U_1^*\left(t, z_{11}, \hat{\phi}_1 - h_1(t, z_{11}) + h_1(t, 0)\right) \end{aligned}$$

the closed-loop system associated with $\dot{x}_1 = x_2 - U_1^*(t, x_1, \phi_1)$ and $\dot{x}_2 = u - U_2^*(t, x_1, x_2, \phi_2)$ can be rewritten as

$$\begin{cases} \dot{z}_{11} = -z_{11} + z_{21} + M_1 z_{12} \\ \dot{z}_{12} = N_1 z_{11} + N_{12}' z_{21} - C_1 M_1 z_{12} \\ \dot{z}_{21} = -z_{21} - z_{11} - S_2^2 z_{21} + M_2 z_{22} + S_2 M_1 z_{12} \\ \dot{z}_{22} = N_2 z_{21} - C_2 M_2 z_{22} \end{cases}$$

where M_1 and M_2 are defined (as previously) according to $U_1^*(\cdot)$ and $U_2^*(\cdot)$, respectively, and

$$S_2(t, x_1, \hat{\phi}_1) = 1 - \frac{\partial U_1^*\left(t, x_1, \hat{\phi}_1 - h_1(t, x_1) + h_1(t, 0)\right)}{\partial x_1}.$$

Now, consider Lyapunov function $L = 0.5[z_{11}^2 + z_{21}^2 + V_1(\|z_{12}\|^2) + V_2(\|z_{22}\|^2)$. It follows that, by setting $\beta_n = 0.5$

$$\dot{L} = -\frac{1}{2}\|Z_{*1}\|^2 - \frac{1}{2} Z_{1*}^T \overline{Q}_1' Z_{1*} - \frac{1}{2} Z_{2*}^T \overline{Q}_2' Z_{2*} - 0.5 Z_{12}^T \overline{Q}_3' Z_{12}$$

where

$$\begin{aligned} \overline{Q}_1' &\triangleq \begin{bmatrix} 1 & -(N_1\rho_1 + M_1) \\ -(N_1\rho_1 + M_1) & C_1 M_2 \rho_1 \end{bmatrix} \\ \overline{Q}_2' &\triangleq \begin{bmatrix} 1 & -(N_2\rho_2 + M_2) \\ -(N_2\rho_2 + M_2) & 2 C_2 M_2 \rho_2 \end{bmatrix} \\ \overline{Q}_3' &\triangleq \begin{bmatrix} C_1 M_1 & -(N_{12}'\rho_1 + S_2 M_1) \\ -(N_{12}'\rho_1 + S_2 M_1) & 2 S_2^2 \end{bmatrix}. \end{aligned}$$

It follows from the proposition that there are choices of C_1, C_2, N_1, and N_2 under which, if the value of C_1 is doubled, both $\overline{Q}_1'$ and $\overline{Q}_2'$ are positive semidefinite. Comparing matrices $\overline{Q}_3'$ and $\overline{Q}_1'$, we know that $\overline{Q}_3'$ can also be made positive semidefinite by choosing N_{12}'. Thus, global stability can be concluded for the proposed adaptive control. □

IV. Conclusion

A new design is proposed for adaptive control of nonlinear systems with unknown parameters. The parameters can appear nonlinearly in system dynamics, and the system may not be affine. The proposed design introduces a (nonlinear) biasing vector function into parameter estimation by subtracting itself from parameter estimate in the adaptive control. The biasing function renders a new Lyapunov function and a set of conditions for achieving global asymptotic stability of the state and even global convergence of parameter estimation. The conditions are illustrated by examples, and they are shown to be valid for several nonlinear parameterizations. In addition, it is shown that the proposed design can be combined with other nonlinear control design methodologies such as the backstepping design.

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