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A control-design-based solution to robotic ecology: Autonomy of achieving cooperative behavior from a high-level astronaut command

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Abstract In this paper, we propose a cooperative control strategy for a group of robotic vehicles to achieve the specified task issued from a high-level astronaut command. The problem is mathematically formulated as designing the cooperative control for a general class of multiple-input-multiple-output (MIMO) dynamical systems in canonical form with arbitrary but finite relative degrees such that the outputs of the overall system converge to the *explicitly* given steady state. The proposed cooperative control for individual vehicle only need to use the sensed and communicated outputs information from its local neighboring vehicles. No fixed leader and time-invariant communication networks are assumed among vehicles. Particularly, a set of less-restrictive conditions on the connectivity of the sensor/communication networks are established, under which it is rigorously proven by using the newly found nice properties of the convergence of sequences of row stochastic matrices that the cooperative objective of the overall system can be achieved. Simulation results for a group of vehicles achieving a target and surrounding a specified object in formation are provided to support the proposed approach in this paper.

Keywords Cooperative control · Consensus problem · Multiple dynamical systems · Time-varying communication networks · Stability analysis

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1. Introduction

Cooperative control design for a group of robotic vehicles has been an active research topic recently due to its wide applicability in accomplishing complex and vital tasks such as exploring the hazardous environment. In particular, an interesting question in future space applications is how the robotic vehicles can cooperatively work together to achieve the command issued remotely from an astronaut or an operator on the earth. Mathematically, this kind of task can be formulated as the problem of designing the cooperative control through the exchanged communication information among vehicles such that the outputs of all the individual systems converge to the same steady state, that is, the so-called agreement or cooperative consensus problem.

The early studies on cooperative control have been focused on the formation control problem for a group of agents using either the classical feedback control techniques (Wang, 1989; Desai et al., 1998; Kang et al., 2000; Leonard and Fiorelli, 2001; Olfati and Murray, 2002; Swaroop and Hedrick, 1996) or the artificial intelligent based methods (Parker, 1998; Fox et al., 2000; Balch and Arkin, 1098). For instance, the stability of line formation was studied in (Swaroop and Hedrick, 1996). In (Desai et al., 1998), using graph representation, the system dynamics were converted into a set of equations corresponding to the relative position and relative angle between robots, and then formation control was designed using feedback linearization technique to stabilize the relative distances of the robots in the formation. In (Leonard and E. Fiorelli, 2001; Olfati and Murray, 2002), virtual leaders and artificial potential method were used for a group of agents maintaining the group geometry, and the closed-loop stability was proven by using the system kinetic energy and the artificial potential energy as a Lyapunov function. However, the aforementioned formation control designs generally require the assumption that the group members maintain a fixed sensor/communication network among vehicles. To deal with the more realistically dynamically changing sensor/communication network in practice, recent efforts have been paid to seek the conditions on the time-varying sensor/communication networks under which the cooperative behaviors can be achieved (Vicsek et al., 1995; Jadbabaie et al., 2003; Lin et al., 2004; Moreau 2003; Qu et al., 2004; Qu et al., 2005). Motivated by animals' flocking behavior via local interaction, a computer simulation model was first built in (Reynolds, 1987) to animate the cohesion, separation and alignment rules inherently followed by animals in group. Subsequently, a simple first-order mathematical model for heading updating according to the neighboring alignment rule was proposed in (Vicsek et al., 1995) for a group of planar particles moving with the same speed and it was verified through experimental results that finally all particles will move with the same direction. A theoretical breakthrough for the proof of this result was later obtained in (Jadbabaie et al., 2003) with the aid of graph theory and matrix analysis, and it is shown that the cooperative behavior can be achieved provided that the *undirected* agents's sensor graph is periodically connected. For the same kind of simple first-order integrator model, strongly connectivity condition for ensuring the cooperative behavior has been obtained for the case of communication being represented by a *directed* graph (Lin et al., 2004; Moreau, 2003). To deal with the more complicated dynamical systems, in our recent works (Qu et al., 2004; Qu et al., 2005), we proposed a cooperative control design for a general class of multiple-input-multiple-output (MIMO) dynamical systems with arbitrary but finite relative degrees. The connectivity requirements among vehicles are further relaxed and less-restrictive conditions on the choice of cooperative feedback matrices have been found. The obtained results show that neither strongly connectivity among vehicles nor fixed leader are required. It is worth pointing out that a different cooperative control method has been given for a class of linear systems with relative-degree one in a recent work (Feddema et al., 2002), where large-scale decentralized control techniques have been employed to first establish the related input/ouput reachability, structural observability and controllability conditions based on the communication paths available between vehicles and the information transmitted and received, and consequently the cooperative control design and stability analysis can be carried out.

In this paper, as a natural continuation of the work in (Qu et al., 2004; Qu et al., 2005), we propose a cooperative control strategy for a group of dynamical systems in the canonical form while the control objective being specified to achieve the *explicitly* given steady state. By sophisticatedly modelling the given command as an additional *virtual* vehicle, new and less-restrictive conditions on the design of cooperative control and connectivity of the sensor/communication network are established to guarantee the convergence of overall system to the desired objective. The significance of the proposed cooperative control lies in the following aspects: (a) the proposed cooperative control can deal with high-order dynamical systems with arbitrary relative degrees; (b) the proposed cooperative control only need to use the outputs feedback information, which has the advantage of saving the load of wireless communication resources; (c) no fixed or strongly connected sensor/communication networks are required. Specifically, it reveals that the cooperative behavior can be achieved if there exists at least one vehicle in the leader group can periodically receive information from the high-level command system; (d) the vehicles in the group are allowed to be heterogeneous with the nonlinear models as long as they are input-output feedback linearizable and with stable internal dynamics; (e) the proposed design is not only theoretically sound with rigorous proof, but also has appealing potentiality for practical application as shown by the agreement problem and cooperative formation control problem in the simulation examples.

The paper is organized as follows. The cooperative control design problem is formulated in section 2. Section 3 gives the proposed cooperative control and the conditions on system convergence as well as its proof. Extensive simulation results on cooperative consensus and formation control are provided in section 4, and section 5 concludes the paper. For the completeness of paper and the ease of reference, some mathematical preliminaries on row stochastic matrices, irreducibility and sequence convergence of row stochastic matrices are collected in the appendix.

2. Problem formulation

In this paper, we consider a general problem important to the development of a self-sustaining system that follows the commands from an operator. In particular, we consider a group of robotic vehicles that operate individually by themselves most of the time, communicate intermittently among their teammates within their neighboring groups, and receive high-level commands from a human operator. Such a setting will be typical in future space applications, in which commands are issued remotely from an astronaut or an operator on the earth, the commands may reach some of the vehicles (as vehicles are in general heterogeneous and of different functions and capabilities), and vehicles have limited sensing range.

Mathematically, the control design problem for achieving autonomy in robotic ecology is formulated as follows. Consider a group of q robotic vehicles described by dynamic equations (after appropriate state and control transformations): for $i = 1, \cdots, q$,

$$\dot{x}_i = A_i x_i + B_i u_i, \quad y_i = C_i x_i, \quad \dot{\eta}_i = g_i(\eta_i, x_i), \tag{1}$$

where $y_i(t) \in \Re^m$ is the output of the ith robot, $l_i \geq 1$ is an integer (representing the relative degree of the ith vehicle's dynamics), $x_i \in \Re^{l_i m}$ is the state of ith vehicle, $\eta_i \in \Re^{n_i - l_i m}$ is the substate of internal dynamics, $I_{m \times m}$ is the m-dimensional identity matrix, J_k is the kth order Jordan canonical form given by

$$J_k = \begin{bmatrix} -1 & 1 & 0 & \cdots & 0 & 0 \\ 0 & -1 & 1 & \ddots & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & -1 & 1 & 0 \\ 0 & 0 & 0 & \cdots & -1 & 1 \\ 0 & 0 & 0 & \cdots & 0 & -1 \end{bmatrix} \in \Re^{k \times k},$$

$$A_i = J_{l_i} \otimes I_{m \times m} \in \Re^{(l_i m) \times (l_i m)},$$

$$B_i = \begin{bmatrix} 0 \\ I_{m \times m} \end{bmatrix} \in \Re^{(l_i m) \times m},$$

$$C_i = \begin{bmatrix} I_{m \times m} & 0 \end{bmatrix} \in \Re^{m \times (l_i m)},$$

u_i is the control to be designed, and subsystem $\dot{\eta}_i = g_i(\eta_i, x_i)$ is input-to-state stable (Khalil, 2003).

Remark 1 *In this paper, for the purpose of generality in dealing with heterogenous vehicles in the group, the cooperative control design problem is formulated according to the dynamical systems in canonical form given by (1). In point of fact, the above formulation includes at least two classes of systems as special cases: input-output linearizable systems with stable internal dynamics (Qu, 1998), and tracking dynamics of nonholonomic systems in the chained form (Qu et al., 2004), which fit into most of the dynamics of the practical robotic vehicles (Qu and Dawson, 1996). Examples will be given in the simulation section for illustration purpose.*

The design objective of cooperative control in this paper is to find a cooperative control law $u_i = U_i(y_1, \cdots, y_q, r_{ss})$ such that, for any given high-level command vector $r_{ss} \in \Re^m$, the overall system is globally asymptotically stable in the sense that

$$\lim_{t \to \infty} y_i(t) = r_{ss}, \quad i = 1, \cdots, q. \tag{2}$$

To account for human presence, our approach is to model the human command as the 0th vehicle, a *virtual vehicle* described by

$$\dot{x}_0 = -x_0 + u_0, \quad y_0 = x_0, \tag{3}$$

where $x_0 \in \Re^m$ is the state. By setting initial condition $x_0(t_0) = r_{ss}$ and by choosing virtual control $u_0(t) = x_0(t)$, equation (3) has the unique solution $x_0(t) = r_{ss}$. Thus, the overall system consists of (1) and (3), and the control objective is to make

$$\lim_{t \to \infty} y_i(t) = x_0(t_0) = r_{ss}, \quad i = 1, \cdots, q. \tag{4}$$

In next section, we first address the general cooperative control design problem according to the time-varying sensor/communications (possibly unpredictable) among vehicles, and then establish the less-restrictive conditions on the connectivity requirements for sensor/communication networks so as to achieve the control objective (4).

3. Proposed cooperative control

To make a group of individual vehicles achieve an object collaboratively, a necessary condition is that the vehicles in the group are capable of exchanging information through sensor/communication networks. For instance, consider a scenario that a group of vehicles implementing the target searching task. The vehicles move randomly at beginning and then the possible location of the target is sensed by some vehicles or issued by the high-level commander only to some (maybe the fast-reached) vehicles, thus through the information exchange among vehicles, it is possible that all the group members can finally reach the target together. In such a scenario, the vehicles in the group operate by themselves most of the time with or without the target location in minds, and exchange of output information among the vehicles occurs only intermittently and locally. The questions naturally arisen are thus: (a) how to design the cooperative control law according to the shared outputs information among vehicles? (b) what is the less-restrictive condition imposed on the connectivity of the sensor/communication networks to make sure that the cooperative behavior can be reached? In this section, we explicitly address these two problems and give the ascertained answers.

3.1. General class of cooperative controls

The proposed cooperative control reacts to the sensor/communications among vehicles. Consider that the vehicles in the group have the limited sensor view, and each vehicle can only acquire the information from other vehicles in a relative direction and distance of itself. To capture this nature of information flow, let us define the following sensing/communication matrix and its corresponding time sequence $\{t_k^s : k = 0, 1, \cdots\}$ as:

$$S(t) = \begin{bmatrix} S_1(t) \\ S_2(t) \\ \vdots \\ S_q(t) \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12}(t) & \cdots & s_{1q}(t) \\ s_{21}(t) & s_{22} & \cdots & s_{2q}(t) \\ \vdots & \vdots & \vdots & \vdots \\ s_{q1}(t) & s_{q2}(t) & \cdots & s_{qq} \end{bmatrix} \in \Re^{q\times q},$$
$$\begin{cases} S(t) = S(t_k^s), \quad \forall t \in [t_k^s, t_{k+1}^s) \\ S(k) \triangleq S(t_k^s), \end{cases} \tag{5}$$

where $s_{ii} \equiv 1$; $s_{ij}(t) = 1$ if the jth vehicle is in the sensor range of the ith vehicle at time t, and $s_{ij} = 0$ if otherwise; and $t_0^s \triangleq t_0$. Time sequence $\{t_k^s\}$ and the corresponding changes in the row $S_i(t)$ of matrix $S(t)$ are detectable instantaneously by and locally at the ith vehicle. Assume without loss of any generality[1] that $0 < \underline{c}_t \le t_{k+1}^s - t_k^s \le \overline{c}_t < \infty$, Figure 1 illustrates an example of sensor graph. In the graph, the line with arrow indicates the directed communication channel between vehicles. If the ith vehicle receives information from the jth vehicle, then there is a line from the jth vehicle pointing at the ith vehicle. The corresponding sensing/communication matrix is

$$S = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}. \tag{6}$$

Recall that to accomplish the cooperative control objective (4), the virtual vehicle (3) is introduced. Correspondingly, communication from the virtual vehicle to the physical vehicles is also intermittent and local, thus we can introduce the following augmented sensor/communication matrix as:

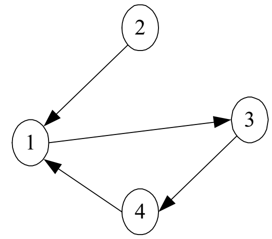

Fig. 1 A sensor/communication network for 4 vehicles

$$\overline{S}(t) = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ s_{10} & & & \\ \vdots & & S(t) & \\ s_{q0} & & & \end{bmatrix} \in \Re^{(q+1)\times(q+1)},$$
$$\begin{cases} \overline{S}(t) = \overline{S}(t_k^s), \quad \forall t \in [\overline{t}_k^s, t_{k+1}^s) \\ \overline{S}(k) \triangleq \overline{S}(t_k^s), \end{cases} \tag{7}$$

To this end, let the proposed cooperative control be: for $i = 1, \cdots, q$,

$$u_i(t) = \sum_{j=0}^{q} \frac{s_{ij}(t)}{\sum_{\eta=0}^{q} s_{i\eta}(t)} K_c y_j \triangleq \overline{G}_i(t)\overline{y}, \tag{8}$$

where $s_{ij}(t)$ are piecewise-constant entries of (7), $K_c \in \Re^{m\times m}$ is a constant, nonnegative, and row stochastic matrix (see appendix) to be designed, $\overline{y} = [y_0^T\, y_1^T \ \cdots \ y_q^T]^T$ and

$$\overline{G}_i \triangleq \left[G_{i0}\, G_{i1} \cdots G_{iq} \right], \quad G_{ij} \triangleq \frac{s_{ij}(t)}{\sum_{\eta=0}^{q} s_{i\eta}(t)} K_c. \tag{9}$$

It then follows from K_C being row stochastic and $\overline{S}(t)$ being piecewise constant that $\overline{G}_i(t)$ is also piecewise constant and row stochastic (i.e., satisfies the properties that $\overline{G}_i \mathbf{1}_{m(q+1)} = \mathbf{1}_m$ where $\mathbf{1}_m$ is defined in the appendix.) Accordingly, let the closed loop feedback gain matrix be

$$\overline{G} = \begin{bmatrix} \overline{G}_0^T & \overline{G}_1^T & \cdots & \overline{G}_q^T \end{bmatrix}^T \in \Re^{(m(q+1))\times(m(q+1))},$$

where $\overline{G}_0 = \begin{bmatrix} I_{m\times m} & 0 & \cdots & 0 \end{bmatrix}$.

It then follows from (1), (3) and (8) that

$$\dot{\overline{x}} = [\overline{A} + \overline{BG}(t)\overline{C}]\overline{x} = [-I_{N_q\times N_q} + \overline{E}(t)]\overline{x}, \tag{10}$$

where $\overline{x} = [x_0^T, x_1^T, \cdots, x_q^T]^T \in \Re^{N_q}$, $N_q = m + mL_q$, $L_q = \sum_{i=1}^q l_i$, $x_0 = [x_{01}, x_{02}, \cdots, x_{0m}]^T \in \Re^m$, $x_i = [x_{i1}^T, x_{i2}^T, \cdots, x_{il_i}^T]^T \in \Re^{ml_i}$, $x_{ij} = [x_{ij1}, x_{ij2}, \cdots, x_{ijm}]^T \in \Re^m$ with $i = 1, \cdots, q$ and $j = 1, \cdots, l_i$ and $\overline{A} = \text{diag}\{I_{m\times m}, A_1, \cdots, A_q\} \in \Re^{N_q\times N_q}$, $\overline{C} = \text{diag}\{I_{m\times m}, C_1, \cdots, C_q\} \in \Re^{((m+1)q)\times N_q}$, $\overline{B} = \text{diag}\{I_{m\times m}, B_1, \cdots, B_q\} \in \Re^{N_q\times((m+1)q)}$,

[1] If $S(t)$ becomes a constant matrix after some finite time, an infinite time sequence t_k^s can always be chosen to yields a finite $\overline{c}_t$ except that $S(t_k^s)$ remains constant. On the other hand, requirement of $\underline{c}_t$ not being too small is needed for implementation.

and $\overline{E}(t) \in \Re^{N_q \times N_q}$, $\overline{E}_{ii} \in \Re^{(l_i m)\times(l_i m)}$, $\overline{E}_{ij}(t) \in \Re^{(l_i m)\times(l_j m)}$ are defined as[2]

$$\overline{E}(t) \triangleq \begin{bmatrix} \overline{E}_{00}(t) & \overline{E}_{01}(t) & \cdots & \overline{E}_{0q}(t) \\ \overline{E}_{10}(t) & \overline{E}_{11}(t) & \cdots & \overline{E}_{1q}(t) \\ \vdots & \vdots & \vdots & \vdots \\ \overline{E}_{q0}(t) & \overline{E}_{q1}(t) & \cdots & \overline{E}_{qq}(t) \end{bmatrix}$$

$$= \begin{bmatrix} I_{m\times m} & 0 & \cdots & 0 \\ \overline{E}_{10}(t) & \overline{E}_{11}(t) & \cdots & \overline{E}_{1q}(t) \\ \vdots & \vdots & \vdots & \vdots \\ \overline{E}_{q0}(t) & \overline{E}_{q1}(t) & \cdots & \overline{E}_{qq}(t) \end{bmatrix},$$

with

$$\bar{E}_{ii} = \begin{bmatrix} 0 & I_{(l_i-1)\times(l_i-1)} \otimes I_{m\times m} \\ G_{ii} & 0 \end{bmatrix}, \quad i = 1, \cdots, q,$$

and

$$\bar{E}_{ij} = \begin{bmatrix} 0 & 0 \\ G_{ij} & 0 \end{bmatrix}, \quad i=1,\cdots,q, \quad j=0,1,\cdots,q, \quad i \neq j.$$

It is obvious that matrix $\overline{E}(t)$ is also piecewise constant and row stochastic at any given time instant t.

According to (10), the cooperative control objective (4) can be rewritten as

$$\lim_{t\to\infty} x_{\mu 1}(t) = r_{ss} = x_0(0), \quad \text{or} \quad \lim_{t\to\infty} x_{\mu 1 i}(t) = x_{0i}(0), \tag{11}$$

where $\mu = 1, \cdots, q$, and $i = 1, \cdots, m$. Obviously, the cooperative control design is successful if

$$\lim_{t \geq t_k^s,\, k\to\infty} \bar{x}(t) = \bar{x}_{ss}, \tag{12}$$

with $\bar{x}_{ss} = \mathbf{1}_{L_q+1} \otimes r_{ss}$. In next section, the choice of feedback gain matrix K_c and a set of less-restrictive conditions on sensor/communications are given to ensure convergence of (12).

3.2. Conditions and convergence analysis

It is clear from the design of cooperative control $u_i(t)$ in (8) that the convergence of (12) relies on the connectivity of the sensing/communication matrices $S(t)$ in (5) and $\overline{S}(t)$ in (7). When the control objective is only focused on the convergence of all systems' states instead of pursuing the specified convergence task such as defined in (4), that is, the final achieved common value is unpredictable while depending initial conditions of the states and connectivity of sensor/communication networks, some remarkable results have been reported very recently (Lin et al., 2004; Qu et al., 2004; Qu et al., 2005). In particular, if there exists a sequence $\{S(t_k^s)\}$ for which the sensor/communication matrix $S(t_k^s)$ is strongly connected (irreducible), the convergence result was obtained in (Lin et al., 2004) for linear system with single integrator model. It was later extended to a general class of dynamical systems in (Qu et al., 2004). Obviously, strong connectivity of sensor/communication network is a stringent condition which is costly to maintain and also sensitive to communication failures. Therefore, to enhance the robustness and improve the performance, it is preferred that in the situation of ensuring the success of cooperative control, the exchange of information among vehicles should be kept as sparse as possible. A recent work in (Qu et al., 2005) revealed that the matrices $S(t_k^s))$ do not need to be strongly connected (that is, $S(t_k^s)$ are reducible) for the convergence although the final convergence value is uncertain. In this subsection, we further extend the works in (Qu et al., 2005)(Qu et al., 2004), and explore the analytical conditions on $\bar{S}(t)$ under which the specified new control objective (4) can be achieved.

To search for the less-restrictive conditions on $S(t)$ and $\bar{S}(t)$, let us consider the general case of $S(t)$ being reducible. It is shown in (Minc, 1988) that, if $S(t)$ is reducible, then there exist an integer $1 < p \leq q$ and permutation matrix $T_1(k) \in \Re^{q\times q}$ such that $S_{T_1}(k) = T_1^T(k) S(t_k^s) T_1(k)$, where

$$S_{T_1}(k) = \begin{bmatrix} S_{T_1,11}(k) & 0 & \cdots & 0 \\ S_{T_1,21}(k) & S_{T_1,22}(k) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ S_{T_1,p1}(k) & S_{T_1,p2}(k) & \cdots & S_{T_1,pp}(k) \end{bmatrix}, \tag{13}$$

with $S_{T_1,ii}(k) \in \Re^{q_i\times q_i}$ being irreducible, $\sum_{i=1}^k q_i = q$. Correspondingly, by employing the augmented permutation matrices

$$\overline{T}_1(k) = \text{diag}\{1, T_1(k)\},$$
$$T_2 = \text{diag}\{I_{m\times m}, T_1 \times \otimes I_{m\times m}\} \in \Re^{(q+1)m\times(q+1)m},$$

we have

$$\overline{S}_{T_1}(k) = \overline{T}_1^T(k)\overline{S}(k)\overline{T}_1(k)$$
$$= \begin{bmatrix} S_{T_1,00}(k) & 0 & 0 & \cdots & 0 \\ S_{T_1,10}(k) & S_{T_1,11}(k) & 0 & \cdots & 0 \\ S_{T_1,20}(k) & S_{T_1,21}(k) & S_{T_1,22}(k) & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ S_{T_1,p0}(k) & S_{T_1,p1}(k) & S_{T_1,p2}(k) & \cdots & S_{T_1,pp}(k) \end{bmatrix},$$

[2] Whenever $l_i - 1 = 0$, the corresponding rows and columns of $I_{(l_i-1)\times(l_i-1)} \otimes I_{m\times m}$ are empty, i.e., removed from $\overline{E}$.

and

$$\overline{G}_{T_2}(k)=T_2^T(k)\overline{G}(k)T_2(k)$$

$$=\begin{bmatrix} G_{T_2,00}(k) & 0 & 0 & \cdots & 0 \\ G_{T_2,10}(k) & G_{T_2,11}(k) & 0 & \cdots & 0 \\ G_{T_2,20}(k) & G_{T_2,21}(k) & G_{T_2,22}(k) & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ G_{T_2,p0}(k) & G_{T_2,p1}(k) & G_{T_2,p2}(k) & \cdots & G_{T_2,pp}(k) \end{bmatrix},$$

where $S_{T_1,00}(k)=1$ and $S_{T_1,i0}\in\Re^{q_i\times 1}$ for $i=1,\cdots,p$, and $G_{T_2,00}(k)=I_{m\times m}G_{T_2,ii}\in\Re^{q_im\times q_im}$, $i\neq 0$.

Now we are ready to present the conditions on sensor/communication matrix $\overline{S}(k)$ to achieve the control objective (4).

Assumption 1 *Suppose that the sensor/communication matrices S(k) are reducible for all k, and there exists a* sub-sequence $\{s_v, v=0,1,\cdots,\infty\}$ of $\{0,1,2,\cdots,\infty\}$ *and* $\lim_{v\to\infty} s_v=+\infty$, *such that for all time instants* $t^s_{s_v}$ *permutation matrices* $T_1(t^s_{s_v})$ *are same. Moreover, the sensor/communication matrix sub-sequence* $\{\overline{S}_{T_1}(t^s_{s_v})\}$ *satisfies the conditions for* $i=1,\cdots,p$*: (a)* $S_{T_1,ii}(t^s_{s_v})$ *is irreducible; (b)* $S_{T_1,10}(t^s_{s_v})\succ 0$*; (c) for every* $i>1$*, there is at least one* j *such that* $S_{T_1,ij}(t^s_{s_v})\succ 0$, $j\in[1,i-1]$.

Remark 2 *Assumption 1 is less restrictive in the sense that we allow that the directed sensor graph to be always not strongly connected. Moreover, the vehicles in the group doesn't need to maintain the fixed communication topology. We only need the existence of sub-sequence of* $\{s_v, v=0,1,\cdots,\infty\}$ *in which the vehicles keep the same communication topologies. This is possible since the number of the vehicle communication topologies is finite given the group size. The condition (a) always holds according to transformation (13). The condition (b) is to make sure that there will at lease one vehicle in the leader group can receive the command from the virtual vehicle. The condition (c) is imposed to maintain the information exchange among vehicles for convergence.* □

Theorem 1. *Consider dynamical systems in (1) and (3) under assumption1. Given cooperative control (8) with the choice of* $K_c=I_{m\times m}$*, the control objective (4) can be achieved.*

Proof: The proof consists of two steps. In step 1), we first introduce two coordinates transformations, by which the analytical solution of the diagonal subsystem $z_i(t)$ can be explicitly obtained. Then, in step 2), the convergence of $z_i(t)$ is proven for all $i=1,\cdots,m$, which is equivalent to the proof of (12). □

Step 1): Note that under the choice of $K_c=I_{m\times m}$, the nonzero sub-blocks $G_{ij}(t)$ of $\overline{G}(t)$ is of the diagonal form, that is, $G_{ij}(t)=\mathrm{diag}\{G_{ij,ss}(t)\}$, $s=1,\cdots,m$. Thus, to facilitate the proof, let us first introduce the following state transformation $z=T\overline{x}$ to diagonalize the system dynamics:

$$z=[z_1^T,\cdots,z_m^T]^T,$$

$$z_i=[z_{i0},z_{i1}^T,z_{i2}^T,\cdots,z_{iq}^T]^T\in\Re^{1+L_q},\ z_{i0}=x_{0i},\ z_{ij}=[z_{ij1},\cdots,z_{ijl_j}]^T\in\Re^{l_j},\ i=1,\cdots,m,\ j=1,\cdots,q,$$

$$\begin{array}{llllllll} z_{i1}: & z_{i11}=x_{11i}, & & z_{ij}: & z_{ij1}=x_{j1i}, & & z_{iq}: & z_{iq1}=x_{q1i} \\ & z_{i12}=x_{12i} & & & z_{ij2}=x_{j2i}, & & & z_{iq2}=x_{q2i}, \\ & \vdots & \cdots & & \vdots & \cdots & & \vdots \\ & z_{i1l}=x_{1l_1i} & & & z_{ijl}=x_{jl_ji}, & & & z_{iql}=x_{ql_qi}. \end{array} \tag{14}$$

It is easy to see that T is a permutation matrix, which permutes the rows of $\overline{x}$ to obtain z. It follows from (10) and (14) that

$$\dot{z}=[-I_{N_q\times N_q}+\tilde{E}(t)]z, \tag{15}$$

where $\tilde{E}\stackrel{\triangle}{=}T\overline{E}T^{-1}=\mathrm{diag}\{\tilde{E}_{11},\cdots,\tilde{E}_{mm}\}$ with $\tilde{E}_{ii}\in\Re^{(1+L_q)\times(1+L_q)}$, $i=1,\cdots,m$ given by

$$\tilde{E}_{ii}=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & I_{(l_1-1)\times(l_1-1)} & 0 & 0 & \cdots & 0 & 0 \\ G_{10,ii} & G_{11,ii} & 0 & G_{12,ii} & 0 & \cdots & G_{1q,ii} & 0 \\ 0 & 0 & 0 & 0 & I_{(l_2-1)\times(l_2-1)} & \cdots & 0 & 0 \\ G_{20,ii} & G_{21,ii} & 0 & G_{22,ii} & 0 & \cdots & G_{2q,ii} & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & 0 & I_{(l_q-1)\times(l_q-1)} \\ G_{q0,ii} & G_{q1,ii} & 0 & G_{q2,ii} & 0 & \cdots & G_{qq,ii} & 0 \end{bmatrix}.$$

On the other hand, note the fact of $S(k)$ being reducible in assumption 1 and that there exists $T_1(k)$ to make it into the tri-angular form in (13), we have the corresponding augmented permutation matrix

$$T_3 = \mathrm{diag}\{T_{31}, \cdots, T_{3m}\} \in \Re^{N_q \times N_q},$$

such that

$$\tilde{E}_{T_3} = T_3^T \tilde{E} T_3 = \mathrm{diag}\left\{T_{31}^T \tilde{E}_{11} T_{31}, \cdots, T_{3m}^T \tilde{E}_{mm} T_{3m}\right\},$$

with

$$\tilde{E}_{T_{3i}} \triangleq T_{3i}^T \tilde{E}_{ii} T_{3i} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ \tilde{E}_{T_{3i},10}(t) & \tilde{E}_{T_{3i},11}(t) & 0 & \cdots & 0 \\ \tilde{E}_{T_{3i},20}(t) & \tilde{E}_{T_{3i},21}(t) & \tilde{E}_{T_{3i},22}(t) & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \tilde{E}_{T_{3i},p0}(t) & \tilde{E}_{T_{3i},p1}(t) & \tilde{E}_{T_{3i},p2}(t) & \cdots & \tilde{E}_{T_{3i},pp}(t) \end{bmatrix}, \quad (16)$$

where $i = 1, \cdots, m$, $T_{3i} = \mathrm{diag}\{1, T'_{3i}\} \in \Re^{(1+L_q)}$, and T'_{3i} is given from the augmentation of T_1.[3]

To this end, we further define the state transformation $\xi = T_3^T z$, and the system dynamics (15) becomes

$$\dot{\xi} = -(I_{N_q \times N_q} - T_3^T \tilde{E} T_3)\xi. \quad (17)$$

It follows that the solution of (17) is given by

$$\xi(t_{k+1}^s) = \prod_{\eta=1}^{k} Q(\eta)\xi(t_0^s) \triangleq Q(k)Q(k-1)\cdots Q(1)\xi(t_0^s), \quad (18)$$

where $Q(\eta) = e^{[-I+T_3^T(\eta-1)\tilde{E}(t_{\eta-1}^s)T_3(\eta-1)](t_\eta^s - t_{\eta-1}^s)}$. It then follows from (18) and $z(k) = T_3(k)\xi(k)$ that

$$z(t_{k+1}^s) = \prod_{\eta=1}^{k} T_3(\eta)Q(\eta)T_3(\eta)^T z(t_0^s). \quad (19)$$

Since $T_3(\eta) = \mathrm{diag}\{T_{31}(\eta), \cdots, T_{3m}(\eta)\}$ and $\tilde{E}(\eta) = \mathrm{diag}\{\tilde{E}_{11}, \cdots, \tilde{E}_{mm}\}$ are in the diagonal structure, we know from (18) that $Q(\eta) = \mathrm{diag}\{Q_{11}, \cdots, Q_{mm}\}$ are also in the diagonal structure and

$$z_i(t_{k+1}^s) = \prod_{\eta=1}^{k} T_{3i}(\eta)Q_{ii}(\eta)T_{3i}(\eta)^T z_i(t_0^s), \quad i = 1, \cdots, m, \quad (20)$$

where $Q_{ii}(\eta) = e^{[-I+T_{3i}^T(\eta)\tilde{E}_{ii}(\eta)T_{3i}(\eta)](t_\eta^s - t_{\eta-1}^s)}$. It is then obvious from (16) that $Q_{ii}(\eta)$ is in the low-triangular structure given by

$$Q_{ii}(\eta) = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ Q_{ii,10}(\eta) & Q_{ii,11}(\eta) & 0 & \cdots & 0 \\ Q_{ii,20}(\eta) & Q_{ii,21}(\eta) & Q_{ii,22}(\eta) & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ Q_{ii,p0}(\eta) & Q_{ii,p1}(\eta) & Q_{ii,p2}(\eta) & \cdots & Q_{ii,pp}(\eta) \end{bmatrix}, \quad (21)$$

Moreover, $Q_{\mathrm{ii}}(\eta)$ is row stochastic and its diagonal elements are lower-bounded by a positive value (Freedman, 1983).

Step 2): It follows from the transformation $z = T\overline{x}$ in (14) that the proof of (12) is equivalent to show that

$$\lim_{t\to\infty} z_i(t) = \mathbf{1}_{(1+L_q)} z_{i0}(0) = \mathbf{1}_{(1+L_q)} x_{0i}(0), \quad \forall i = 1, \cdots, m. \quad (22)$$

To prove (22), it follows from (20) that it suffices to prove

$$\lim_{k\to\infty} \prod_{\eta=1}^{k} T_{3i}(\eta)Q_{ii}(\eta)T_{3i}(\eta)^T = 1_{(1+L_q)} \otimes c_s, \quad (23)$$

where $c_s = [1, 0, \cdots, 0] \in \Re^{1\times(1+L_q)}$.

It follows from the condition (a) of assumption 1 that $S_{T_1,\mu}(t_{s_v}^s)$ is irreducible, for $1 \le \mu \le p$. By the definition of $G(t_{s_v}^s)$ in (9), we know that $G_{T_2,\mu\mu}(t_{s_v}^s)$ is irreducible. Thus, according to lemma 1 and the definition of $\tilde{E}_{T_{3i}}(t_{s_v}^s)$ in (16), we know that $\tilde{E}_{T_{3i},\mu\mu}(t_{s_v}^s)$ is irreducible, and $Q_{ii,\mu\mu}(s_v) > 0$ (Qu et al., 2004). On the other hand, it follows from $S_{T_1,\mu\nu}(t_{s_v}^s) \succ 0$ and (9) that $G_{T_2,\mu\nu}(t_{s_v}^s) \succ 0$ which leads to $\tilde{E}_{T_{3i},\mu\nu}(t_{s_v}^s) \succ 0$ and $Q_{ii,\mu\nu}(s_v) \succ 0$. It then follows from the condition (b) of assumption 1 and corollary 1 that

$$\lim_{v\to\infty} Q_{ii}(s_v)Q_{ii}(s_{v-1})\cdots Q_{ii}(s_0) = 1_{(1+L_q)} \otimes c_s. \quad (24)$$

Define $Q'_{ii}(s_v) = T_{3i}(s_v)^T T_{3i}(s_v-1)P_{ii}(s_v-1)T_{3i}^T(s_v-1)\cdots T_{3i}(s_{v-1}+1)P_{ii}(s_{v-1}+1)T_{3i}^T(s_{v-1}+1)T_{3i}(s_{v-1})$. Note that $Q'_{ii}(s_v)$ has positive diagonal elements, (23) follows from (24) by using corollary 2.

[3] The augmentation rule is: if in the μth row of permutation matrix T_1, its element $T_{1,\mu\nu} = 1$, then the $\mu\nu$th sub-block of T'_{3i} is given by: $T'_{3i,\mu\nu} = I_{l_\nu \times l_\nu}$ otherwise, $T'_{3i,\mu\nu} = 0 \in \Re^{l_\nu \times l_\nu}$.

Remark 3 *It is worth emphasizing that the proof of theorem 1 is based on first finding the solution of system (15) and then studying the convergence of the pre-multiplying row stochastic matrix products (23). The transformations introduced, such as T, T_1, T_2 and T_3, are all for the analysis purpose rather than for design. Actually, the proposed cooperative control law (8) is quite simple and convenient for real-time implementation and practical application. On the other hand, the conditions introduced in assumption 1 expose to be the less-restrictive requirements on the connectivity of sensor/communications among vehicles compared to the known results in the recent cooperative control literatures (Jadbabaie et al., 2003; Lin et al., 2004; Ren and Beard, 2004; Qu et al., 2005). More importantly, those conditions can be taken as a guideline to devise the large-scale communication networks for the success of cooperative control while be robust to the possible drop-out of some communication links during certain time intervals. To illustrate the applicability of the proposed cooperative control, extensive simulation studies on consensus problem and formation control problem are presented in next section.*

In what follows, we give a simple example for better understanding of the reducible sensor/communication matrix and the corresponding transformations.

Example 1 *Consider a group of 4 robots in (1) with $m = 2$ and $l_i = 1, i = 1, \cdots, 4$. Let the human command be modelled as the 0th robot. Then we have $N_q = m + \sum_{j=1}^{4} l_j m = 10$, and $\overline{x} = [x_0^T, x_1^T, x_2^T, x_3^T, x_4^T]^T$ with $x_i = [x_{i1}, x_{i2}]^T$, $i = 0, 1, \cdots, 4$. Suppose that at the current time instant, the sensor/communication graph is given by figure 1. Obviously, the corresponding sensor/communication matrix S in (6) is reducible. Let the permutation matrix T_1 be*

$$T_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Then we have $S_{T_1} = \begin{bmatrix} S_{T_1,11} & 0 \\ S_{T_1,21} & S_{T_1,22} \end{bmatrix}$, *with*

$$S_{T_1,11} = 1, \quad S_{T_1,21} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad S_{T_1,22} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix},$$

where $S_{T_1,11}$ and $S_{T_1,22}$ are irreducible. To this end, four robots is reformulated into two subgroups, that is, group 1 $= \{2\}$ and group 2 $= \{1, 3, 4\}$. Assume that the group 1 can receive command from the 0th robot. Then the augmented sensor/communication matrix is

$$\overline{S}_{T_1} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & S_{T_{1,11}} & 0 \\ S_{T_{1,20}} & S_{T_{1,21}} & S_{T_{1,22}} \end{bmatrix},$$

where $S_{T_1,20} = [0, 0, 0]^T$. It follows from (9) that

$$\overline{G} = \begin{bmatrix} I_{2\times2} & 0 & 0 & 0 & 0 \\ 0 & G_{11} & G_{12} & 0 & G_{14} \\ G_{20} & 0 & G_{22} & 0 & 0 \\ 0 & G_{31} & 0 & G_{33} & 0 \\ 0 & 0 & 0 & G_{43} & G_{44} \end{bmatrix}. \tag{25}$$

The corresponding permutation matrix T_2 is $\begin{bmatrix} I_{2\times2} & 0 \\ 0 & T_1 \otimes I_{2\times2} \end{bmatrix}$, *and*

$$\overline{G}_{T_2} = \begin{bmatrix} I_{2\times2} & 0 & 0 \\ G_{T_2,10} & G_{T_2,11} & 0 \\ 0 & G_{T_2,21} & G_{T_2,22} \end{bmatrix},$$

with $G_{T_2,10} = G_{20}$, $G_{T_2,11} = G_{22}$ and

$$G_{T_2,21} = \begin{bmatrix} G_{12} \\ 0 \\ 0 \end{bmatrix}, \quad G_{T_2,22} = \begin{bmatrix} G_{11} & 0 & G_{14} \\ G_{31} & G_{33} & 0 \\ 0 & G_{43} & G_{44} \end{bmatrix}.$$

On the other hand, it follows from (25) and $l_i = 1$ for $i = 1, \cdots, 4$, that $\overline{E} = \overline{G}$. Define the following coordinate transformation matrix $T \in \Re^{10\times10}$:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

we have $z = T\overline{x} = [z_1^T, z_2^T]^T$ *with* $z_1 = [x_{01}, x_{11}, x_{21}, x_{31}, x_{41}]^T$ *and* $z_1 = [x_{02}, x_{12}, x_{22}, x_{32}, x_{42}]^T$. *Thus, it follows from* $\overline{E}$ *that*

$$\tilde{E} = T\overline{E}T^{-1} = \mathit{diag}\ \{\tilde{E}_{11}, \tilde{E}_{22}\},$$

with

$$\tilde{E}_{ss} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & G_{11,ss} & G_{12,ss} & 0 & G_{14,ss} \\ G_{20,ss} & 0 & G_{22,ss} & 0 & 0 \\ 0 & G_{31,ss} & 0 & G_{33,ss} & 0 \\ 0 & 0 & 0 & G_{43,ss} & G_{44,ss} \end{bmatrix},$$

$s = 1, 2.$

Noting the structure of T_1, *the corresponding permutation matrix* T_3 *is*

$$T_3 = diag\{T_{31}, T_{32}\},$$

with

$$T_{3s} = \begin{bmatrix} 1 & 0 \\ 0 & T_1 \end{bmatrix}, \quad s = 1, 2.$$

To this end, we have

$$\tilde{E}_{T_3} = T_3^T \tilde{E} T_3 = \begin{bmatrix} T_{31}^T \tilde{E}_{11} T_{31} & 0 \\ 0 & T_{32}^T \tilde{E}_{22} T_{32} \end{bmatrix}$$

where

$$T_{3s}^T \tilde{E}_{ss} T_{3s} = \begin{bmatrix} 1 & 0 & 0 \\ \tilde{E}_{T_{3s},10} & \tilde{E}_{T_{3s},11} & 0 \\ 0 & \tilde{E}_{T_{3s},21} & \tilde{E}_{T_{3s},22} \end{bmatrix},$$

with $\tilde{E}_{T_{3s},10} = G_{20,ss}$, $\tilde{E}_{T_{3s},11} = G_{22,ss}$, *and*

$$\tilde{E}_{T_{3s},21} = \begin{bmatrix} G_{12,ss} \\ 0 \\ 0 \end{bmatrix},$$

$$\tilde{E}_{T_{3s},22} = \begin{bmatrix} G_{11,ss} & 0 & G_{14,ss} \\ G_{31,ss} & G_{33,ss} & 0 \\ 0 & G_{43,ss} & G_{44,ss} \end{bmatrix}.$$

□

4. Simulation

In this section, simulation results for a group of vehicles approaching a given target and surrounding a specified object in formation are provided, respectively.

4.1. Vehicle platforms

In general, the vehicle described by (1) are heterogeneous, i.e, the vehicles can be any combination of vehicle dynamics which are input-output feedback linearizable with stable internal dynamics, such as point-mass agent (Vicsek et al., 1995), unmanned aerial vehicle (Menon and Sweriduk, 1999) and nonholonomic chained systems (Qu et al., 2004). In what follows, two examples of platforms are given, and to simplify the notation, the subscript $_i$ denoting the ith vehicle is omitted.

Example 2 *A point-mass agent whose equation of motion is:*

$$\dot{\phi}_1 = \phi_2, \quad \dot{\phi}_2 = v, \quad \psi = \phi_1, \tag{26}$$

where $\phi_1 \in \Re^m$ *is the position of the agent,* $\phi_2 \in \Re^m$ *is the velocity,* ψ *is the output, and* $v \in \Re^m$ *is the control. Then, under the state and input transformation of*

$$x_1 = \phi_1, \quad x_2 = x_1 + \phi_2, \quad v = -2x_2 + x_1 + u,$$

dynamical system of (26) can be transformed into (1) with

$$A = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix} \otimes I_{m\times m}, \quad B = \begin{bmatrix} 0 \\ I_{m\times m} \end{bmatrix}, \quad C = \begin{bmatrix} I_{m\times m} & 0 \end{bmatrix}.$$

Example 3 *A 4-wheel differential driven mobile robot model is given by*

$$\begin{aligned} &\dot{r}_x = r_v \cos(r_\theta), \quad \dot{r}_y = r_v \sin(r_\theta), \\ &\dot{r}_\theta = r_\omega, \quad \dot{r}_v = F/m, \quad \dot{r}_\omega = \tau/J, \end{aligned} \tag{27}$$

where (r_x, r_y) *is the inertial position of the robot,* r_θ *is the orientation,* r_v *is the linear speed,* r_ω *is the angular speed,* τ *is the applied torque,* F *is the applied force,* m *is the mass, and* J *is the moment of inertia. By taking the robot "hand" position as the guide point (which is a point located a distance* L *from* (r_x, r_y) *along the line that is perpendicular to the wheel axis), the robot model in (27) can be feedback linearized to*

$$\dot{\phi}_1 = \phi_2, \quad \dot{\phi}_2 = v,$$

with a stable internal dynamics

$$\dot{r}_\theta = \begin{bmatrix} -\frac{1}{2L}\sin(r_\theta) & \frac{1}{2L}\cos(r_\theta) \end{bmatrix} \phi_2,$$

where $v = [v_1, v_2]^T \in \Re^2$, $\phi_1 = \begin{bmatrix} r_x + L\cos(r_\theta) \\ r_y + L\sin(r_\theta) \end{bmatrix} \in \Re^2$ *is the position of "hand" point,* $\phi_2 \in \Re^2$*, and*

$$\begin{bmatrix} F \\ \tau \end{bmatrix} = \begin{bmatrix} \frac{1}{m}\cos(r_\theta) & -\frac{L}{J}\sin(r_\theta) \\ \frac{1}{m}\sin(r_\theta) & \frac{L}{J}\cos(r_\theta) \end{bmatrix}^{-1} \times \left(v - \begin{bmatrix} -r_v r_\omega \sin(r_\theta) - L^2 r_\omega^2 \cos(r_\theta) \\ r_v r_\omega \cos(r_\theta) - L^2 r_\omega^2 \sin(r_\theta) \end{bmatrix} \right).$$

□

4.2. Cooperative consensus

In this subsection, the proposed cooperative control is simulated for a group of 3 vehicles described by (1) with $m = 2$ and $l_i = 2$. The control objective is that 3 vehicles move cooperatively towards the specified target position. For illustration purpose, we assume that the sensor/communications change randomly among the following given patterns:

$$S_1 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad S_2 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix},$$

$$S_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}. \tag{28}$$

Moreover, assume that vehicle 1 can *intermittently* receive the information from the human command, which is defined as the virtual vehicle with the model given by (3). It is straightforward to verify that all the communication patterns in (28) are reducible, and that the sequence consisting of S_1 and S_3 satisfying the assumption 1. Thus, by using the control feedback matrices given in (9) with $K_c = I_{2\times 2}$, we can guarantee that the control objective can be achieved.

In the simulation, let the initial positions of the robots are $[6, 3]^T$, $[2, 5]^T$ and $[4, 1]^T$, respectively. The target position is $[2.5, 2]^T$. Figure 2 illustrates the convergence of the robots's positions, which verifies the effectiveness of the proposed cooperative control in this paper. The cooperative control inputs for vehicle 1 to vehicle 3 are shown by Figure 3a to Figure 3c, respectively. It is observed from figure 2 that each vehicle does not need to take the shortest path to the target. This phenomena is attributed to the nature of cooperative control problem. In essence, different from the conventional control methods, cooperative control is according to the information exchange and sharing among group members to *collaboratively* accomplish the given task, such as approaching the target. Specifically, in the simulation, it is assumed that only vehicle 1 is able to intermittently receive the information about target position, and the cooperative control design for vehicles 2 and 3 are based on the position information of the vehicles within their sensor/communication networks, and the corresponding trajectories are dependent on the properties of the communication networks. Nonetheless, if all the vehicle in the group have the target information, then the decoupled tracking control can be *individually* designed according to the classical control approaches for optimal trajectory, such as that in (Qu et al., 2004).

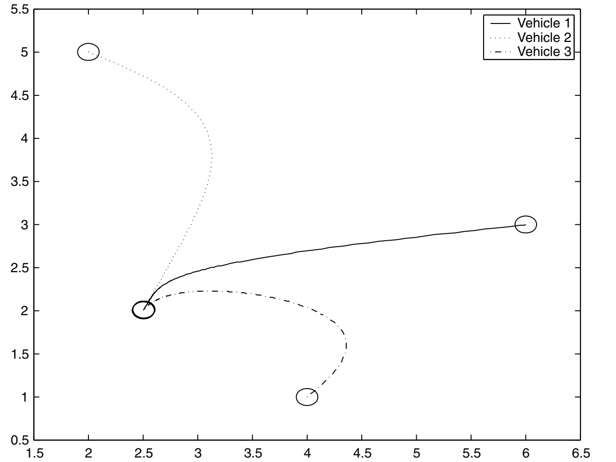

Fig. 2 Convergence to the specified target under cooperative controls

To further verify the performance of the proposed cooperative control design, a random disturbance with 0 mean and 0.05 variance is added into the output measurements. Figures 4 to 5 are the simulation results which indicate that the proposed cooperative control is robust and cooperative convergence can still be achieved even in the presence of random disturbance although the control signals become more chattering. This is understandable because that the convergence of the cooperative control system inherently relies on the connectivity of sensor/communication networks.

4.3. Surrounding a target

To see broad applicability of the proposed cooperative control methodology, we show here how to design a formation control using the proposed method. As an example, let us start with the double integrator model given by (26) with $m = 2$, that is, for the ith vehicle, we have

$$\dot{\phi}_{i1} = \phi_{i2}, \quad \dot{\phi}_{i2} = v_i, \quad i = 1, \cdots, q, \tag{29}$$

where $\phi_{i1} = [\phi_{i11}, \phi_{i12}] \in \Re^2$ is the planar position of the ith vehicle, $\phi_{i2} = [\phi_{i21}, \phi_{i22}] \in \Re^2$ its velocity, and $v_i = [v_{i1}, v_{i2}]T$ the control input.

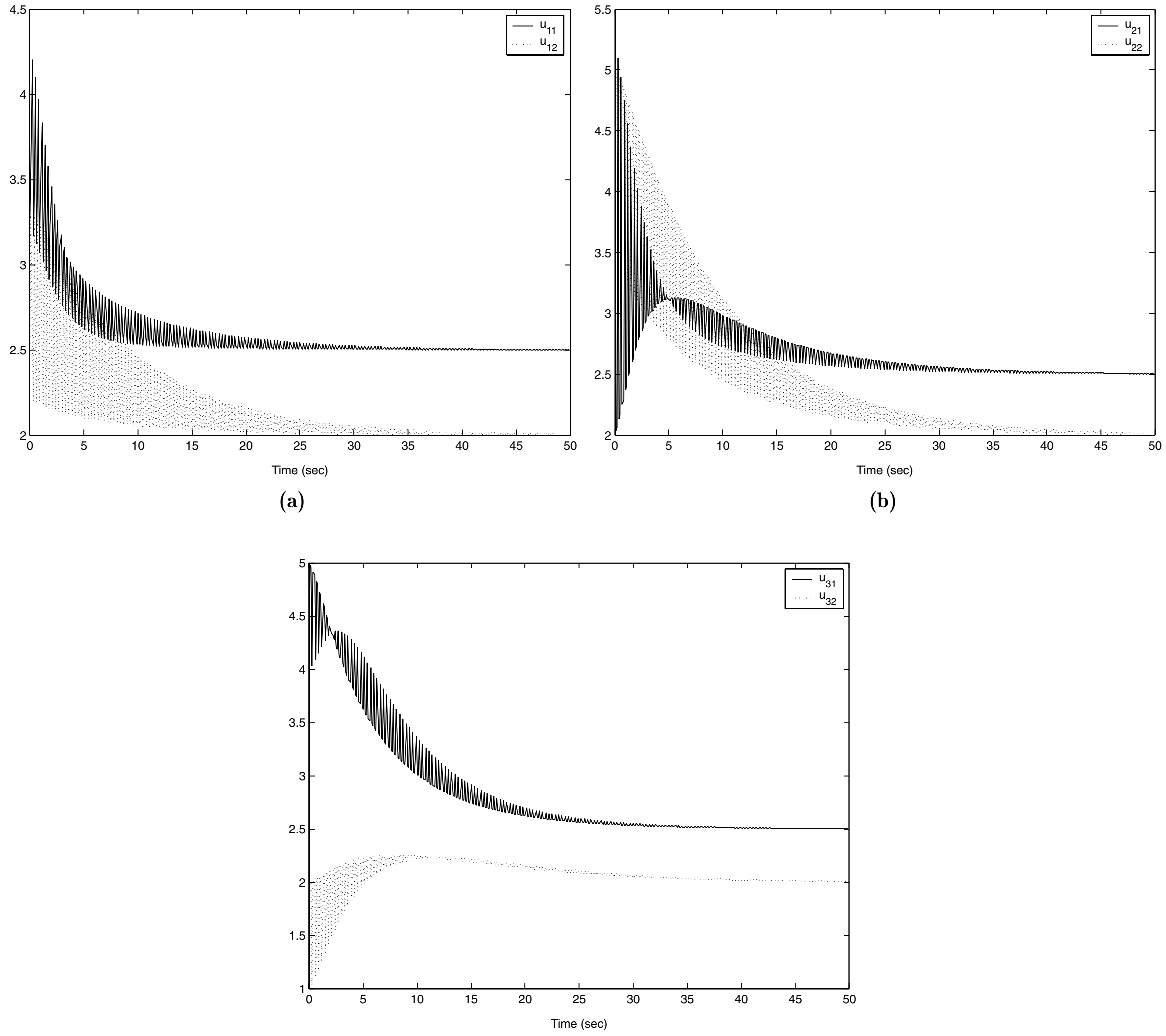

Fig. 3 Cooperative controls

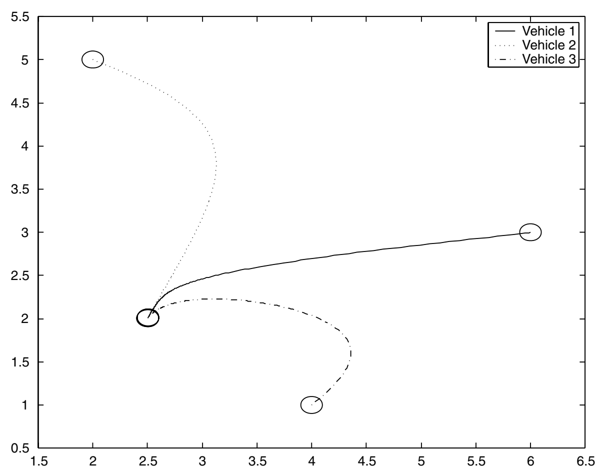

Fig. 4 Convergence in the presence of random disturbance

A formation is defined in a coordinate frame, which moves with the desired trajectory. Let $e_1(t) \in \Re^2$ and $e_2(t) \in \Re^2$ be the orthonormal vectors which forms the moving frame $F(t)$. Let $\phi_d(t) = [\phi_{d1}(t), \phi_{d2}(t)] \in \Re^2$ be any desired trajectory of the origin of the moving frame. A formation consists of q points in $F(t)$, denoted by $\{P_1, \ldots s, P_{\mathrm{q}}\}$, where

$$P_i = d_{i1}(t)e_1(t) + d_{i2}(t)e_2(t), \quad i = 1, \cdots, q, \tag{30}$$

with $d_i(t) = [d_{i1}(t), d_{i2}(t)] \in \Re^2$ being the desired relative position for the ith vehicle in the formation. It is obvious that $d_{\mathrm{i}}(t)$ being constant refers to the rigid formation. The desired position for the ith vehicle is then

$$\phi_i^d(t) = \phi_d(t) + d_{i1}(t)e_1(t) + d_{i2}(t)e_2(t). \tag{31}$$

Figure 6 illustrates a formation setup for 3 agents.

In what follows, we show that, through state and input transformations, the formation control problem for (29) can be recast as the cooperative control design problem for (1). Let the input transformation be

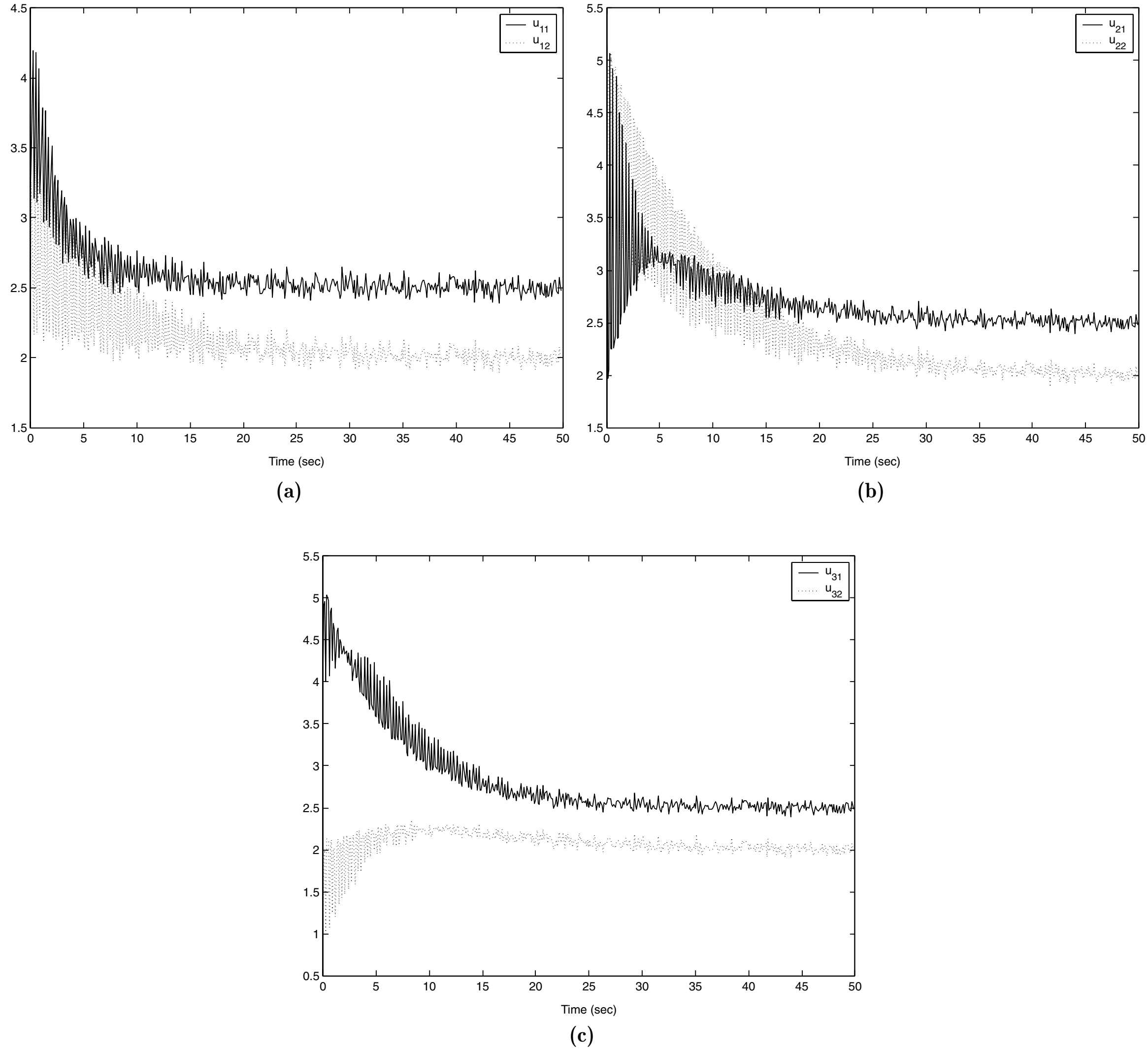

Fig. 5 Cooperative controls in the presence of random disturbance

$$x_{i1} = \phi_{i1} - \phi_i^d(t), \quad x_{i2} = \phi_{i2} + x_{i1} - \dot{\phi}_i^d(t), \tag{32}$$

and the decentralized control be

$$v_i(t) = -2x_{i2} + x_{i1} + u_i. \tag{33}$$

Substituting (32) and (33) into (29), we obtain the canonical model (1) with $x_i = [x_{i1}^T, x_{i2}^T]^T \in \Re^4$, $u_i \in \Re^2$, and $y_i = x_{i1}^T \in \Re^2$. To this end, if we can design the cooperative control u_{i} such that states $x_{\mathrm{i}1}$ and $x_{\mathrm{i}2}$ for all i converge to the same steady state x_{ss}, then it follows from (32) that

$$\phi_{i1} \to x_{ss} + \phi_i^d(t), \quad \phi_{i2} \to \dot{\phi}_i^d(t),$$

from which it can be seen that the desired formation is achieved for the whole group while vehicles moving with the desired trajectory shape.

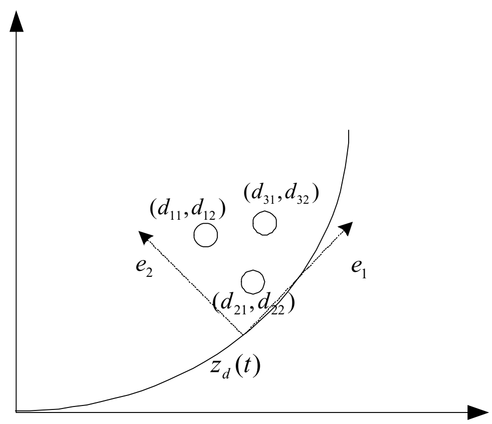

Fig. 6 Formation Setup

In the simulation, assume that in the group there are 3 vehicles given by (29). Similarly, assume that the sensor/communications change randomly among the

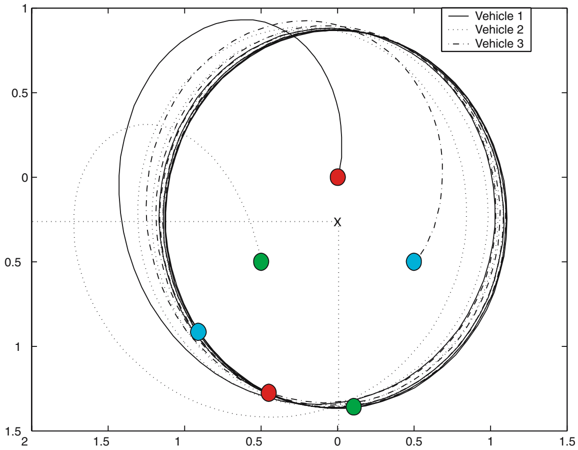

Fig. 7 Circle motion under cooperative control

given patterns (28), and the vehicle 1 can acquire the human command. The formation control objective is to make the vehicles circle around a target located at $[0,\ -0.25]^T$, that is, the virtual vehicle in (3) stays at $x_0(t_0) = [0,\ -0.25]^T$. All vehicles try to keep the same radius and speed, and relative angle offset. Let

$$\phi^d(t) = [\cos(t),\ \sin(t)]^T.$$

The moving frame $F(t)$ is defined as

$$e_1(t) = \begin{bmatrix} -\sin(t) \\ \cos(t) \end{bmatrix}, \quad e_2(t) = \begin{bmatrix} \cos(t) \\ \sin(t) \end{bmatrix}.$$

The formation is defined by the three points:

$$P_1 = \text{the origin of } F(t), \quad P_2 = d_1 e_1 + d_2 e_2,$$

$$P_3 = -d_1 e_1 + d_2 e_2,$$

where $d_1 = 0.5$ and $d_2 = -0.1340$. The initial positions for three vehicles are given by $[0, 0]^T$, $[-0.5, -0.5]^T$, and $[0.5, -0.5]^T$, respectively. Figures 7 shows that the circle motion around the given target is achieved while maintaining the formation among vehicles.

5. Conclusion

In this paper, a cooperative control strategy has been proposed for a group of robotic vehicles to collaboratively accomplish the issued tasks. A set of less-restrictive conditions on the requirements for sensor/communication networks have been established for the convergence of the overall closed-loop system. Particularly, the proposed cooperative control method is scalable in the sense that the information of drop-out and add-in of the vehicles in the group can be totally captured by the sensor/communication matrix $S(t)$ in (5), and accordingly the cooperative control (8) can be designed. In addition, the final convergent behavior can be adaptively adjusted with the change of the specified target and simulation results have also shown that the proposed method is robust against uncertainties in sensors and detection disturbances.

In summary, the proposed control-design methodology enables us to analyze, understand and achieve cooperative behavior and autonomy for a team of robotic vehicles that are autonomous by themselves to operate in space, communicate with and/or sense each other intermittently, and receive and follow high-level commands from astronauts or operators on earth. In addition to the development of new and effective rules and controls, the framework can also be used to gain fundamental understandings about and develop analytical analysis tools for the rules and behaviors inspired by nature.

Appendix

The cooperative control design and system convergence analysis result of this paper rely on the study of the convergence of the infinite products of sequences of row stochastic matrices. In the appendix, we give some preliminaries by first introducing the related notations and definitions and then presenting some useful results on irreducible matrices as well as the convergence properties of products of row stochastic matrices.

Nonnegative matrices and row stochastic matrices

Let $\mathbf{1}_p$ be the p-dimensional column vector with all its elements being 1, and $\mathbf{J}_{r_1 \times r_2} \in \Re^{r_1 \times r_2}$ be a matrix whose elements are all 1. I_{m} is the m–dimensional identity matrix. $\otimes$ denotes the Kronecker product.

A nonnegative matrix has all entries nonnegative. A square real matrix is *row stochastic* if it is nonnegative and its row sums all equal 1.

Given a sequence of nonnegative matrix $E(k)$, the notation of $E(k) \succ 0$, $k = 0, 1, \cdots$, means that, there is a subsequence $\{l_v, v = 1, \cdots, \infty\}$ of $\{0, 1, 2, \ldots, \infty\}$ such that $\lim_{v\to\infty} l_v = +\infty$ and $E(l_v) \neq 0$.

A non-negative matrix E is said to be *reducible* if the set of its indices, $\mathcal{I} \triangleq \{1, 2, \cdots, n\}$, can be divided into two disjoint nonempty sets $\mathcal{S} \triangleq \{i_1, i_2, \cdots, i_\mu\}$ and $\mathcal{S}^c \triangleq \mathcal{I}/\mathcal{S} = \{j_1, j_2, \cdots, j_\nu\}$ (with $\mu + \nu = n$) such that $E_{i_\alpha} J_\beta$, where $\alpha = 1, \ldots, \mu$ and $\beta = 1, \ldots, \nu$. Matrix E is said to be *irreducible* if it is not reducible.

A square matrix $E \in \Re^{n\times n}$ can be used to define a directed graph with n nodes $v_1, \ldots, v_n$, and there is a directed arc from v_i to v_j if and only if $E_{ij} \neq 0$. A directed graph represented by E is *strongly connected* if between every pair of distinct nodes v_i, v_j there is a directed path of finite length that begins at v_i and ends at v_j. The fact that a directed graph represented by E is strongly connected is equivalent to that matrix E is irreducible (Minc, 1988).

Results on sequence convergence of row stochastic matrices

The following lemma provides a necessary and sufficient condition on the irreducibility of a non-negative matrix in a special structure, which has direct relation to the canonical system model discussed in this paper.

Lemma 1 *(Qu et al., 2004) Given any non-negative matrix $E \in \Re^{(qm)\times(qm)}$ with sub-blocks $E_{ij} \in \Re^{m\times m}$, let $\overline{E} = [\overline{E}_{ij}] \in \Re^{(Lm)\times(Lm)}$ with $L = l_1 + \cdots + l_q$ be defined by*

$$\overline{E}_{ii} = \begin{bmatrix} 0 & I_{(l_i-1)} \otimes I_m \\ E_{ii} & 0 \end{bmatrix}, \quad \overline{E}_{ij} = \begin{bmatrix} 0 & 0 \\ E_{ij} & 0 \end{bmatrix}$$

where $l_i \geq 1$ are positive integers for $i = 1, \cdots, q$. Then, $\overline{E}$ is irreducible if and only if E is irreducible.

The classical convergence result of the infinite products of sequences of row stochastic matrices (Wolfowitz, 1963) has been applied in the study of coordination behavior of groups of mobile autonomous agents (Jadbabaie et al., 2003; Lin et al., 2004). In our recent works (Qu et al., 2004; Qu et al., 2005), we relaxed the condition in (Wolfowitz, 1963) and found an easy-to-check condition on the convergence of a sequence of row stochastic matrices in the lower-triangular structure, and also extended it to the case of the products of lower-triangular matrices and general matrices. These new results are useful for establishing the less-restrictive conditions on the design of cooperative control and the connectivity requirements among individual systems. In what follows, we recall these two results without proof for briefness.

Lemma 2 *(Qu et al., 2004) Consider a sequence of nonnegative, row stochastic matrices $P(k) \in \Re^{R\times R}$ in the lower-triangular structure,*

$$P(k) = \begin{bmatrix} P_{11}(k) & & & \\ P_{21}(k) & P_{22}(k) & & \\ \vdots & \vdots & \ddots & \\ P_{m1}(k) & P_{m2}(k) & \cdots & P_{mm}(k) \end{bmatrix} \in \Re^{R\times R},$$

where $R = \sum_{i=1}^m r_i$, sub-blocks $P_{ii}(k)$ on the diagonal are square and of dimension $\Re^{r_i\times r_i}$, sub-blocks $P_{ij}(k)$ off diagonal are of appropriate dimensions. Suppose that $P_{ii}(k) \geq \epsilon_i \mathbf{J}_{r_i\times r_i}$ for some constant $\epsilon_i > 0$ and for all ($i = 1, \ldots s, m$), and in the ith row of $P(k)$ ($i > 1$), there is at least one j ($j < i$) such that $P_{ij} \succ 0$. Then,

$$\lim_{k\to\infty} \prod_{l=0}^{k-1} P(k-l) = \mathbf{1}_R c,$$

where constant vector $c = [c_1, 0, \cdots, 0] \in \Re^{1\times R}$ with that constant vector $c_1 \in \Re^{1\times r_1}$ exists and given by

$$c_1 = \lim_{k\to\infty} \prod_{l=0}^{k-1} P_{11}(k-l).$$

Lemma 3 *(Qu et al., 2005) Given sequences of row stochastic matrices $P(k) \in \Re^{R\times R}$ and $P'(k) \in \Re^{R\times R}$, where $P(k)$ is in the lower-triangular structure and $P'(k)$ satisfying $P'_{ii}(k) \geq \epsilon_p > 0$. Then,*

$$\lim_{k\to\infty} \prod_{l=0}^{k-1} P(k-l)P'(k-l) = \mathbf{1}_R c_1,$$

if and only if $\lim_{k\to\infty} \prod_{l=0}^{k-1} P(k-l) = \mathbf{1}_R c_2$, where c_1 and c_2 are constant vectors.

Lemmas 2 and 3 present sets of conditions on the convergence of the product of a sequence of row stochastic matrices containing lower-triangular matrices. In general, the final convergence values (say, vector c in lemma 2 and vector c_1 in lemma 3) are not determinable. However, it is of interest to note that if lower-triangular matrices have a special feature, then the final convergence value can be determined. The following corollaries summarize this observation, which can be directly proved following the same insights of lemmas 2 and 3, respectively. Corollaries 1 and 2 will be used to establish the main result of this paper.

Corollary 1 *Consider a sequence of nonnegative, row stochastic matrices $P(k) \in \Re^{R\times R}$ in the lower-triangular structure, where $R = \sum_{i=1}^m r_i$, sub-blocks $P_{ij}(k)$ $P_{ii}(k)$ on the diagonal are square and of dimension $\Re^{r_i\times r_i}$, sub-blocks off diagonal are of appropriate dimensions. Suppose that $P_{11}(k) = 1$, and $P_{ii}(k) \geq \epsilon_i \mathbf{J}_{ri\times r_i}$ for some constant $\epsilon_i > 0$ and for all ($i = 2, \ldots s, m$), and in the ith row of $P(k)$ ($i > 1$), there is at least one j ($j < i$) such that $P_{ij} \succ 0$. Then, $\lim_{k\to\infty} \prod_{l=0}^{k-1} P(k-l) = \mathbf{1}_R c$, where constant vector $c = [1, 0, \cdots, 0] \in \Re^{1\times R}$.*

Proof: The proof follows from the fact that $\lim_{k\to\infty} \prod_{l=0}^{k-1} P_{11}(k-l) = \mathbf{1}_{\mathbf{r_1}}$ and lemma 2. □

Corollary 2 *Given sequences of row stochastic matrices $P(k) \in \Re^{R\times R}$ and $P'(k) \in \Re^{R\times R}$, where $P'(k)$*

satisfy $P'_{ii}(k) \geq \epsilon_p > 0$. *Let* $\overline{P}(k) \in \Re^{(R+1)\times(R+1)}$ *and* $\overline{P}'(k) \in \Re^{(R+1)\times(R+1)}$ *be defined as*

$$\overline{P}(k) = \begin{bmatrix} 1 & 0 \\ \overline{P}_{21}(k) & P(k) \end{bmatrix},$$

$$\overline{P}'(k) = \begin{bmatrix} 1 & 0 \\ \overline{P}'_{21}(k) & P'(k) \end{bmatrix}.$$

Suppose that

$$\lim_{k\to\infty} \prod_{l=0}^{k-1} \overline{P}(k-l) = \mathbf{1}_R c,$$

Then,

$$\lim_{k\to\infty} \prod_{l=0}^{k-1} \overline{P}(k-l)\overline{P}'(k-l) = \mathbf{1}_R c,$$

where $c = [1, 0, \cdots, 0] \in \Re^{1\times(R+1)}$.

Proof: The proof can be done by invoking lemma 3 and noting the lower triangular structure of $\overline{P}(k)$ and $\overline{P}'(k)$.

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