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A NOVEL ALTERNATIVE ALGORITHM FOR LIMITED ANGLE TOMOGRAPHY

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ABSTRACT

This paper studies incomplete data problems of circular cone-beam computed tomography, which occur frequently in medical imaging and industrial imaging. The incomplete data problems in which projection data are only available in an angular range can be attributed to the limited angle tomography. Limited angle tomography is a severely ill-posed inverse problem. In recent years, image reconstruction based on total variation (TV) was employed to reduce the problem and gave better performance on edge-preserving reconstruction. However, the artificial parameter can only be determined through considerable experimentation. In this paper, an alternating minimization method based on TV is proposed to reduce the data insufficiency in tomographic imaging. This novel alternating minimization method provides a robust and effective reconstruction without any artificial parameter in the iterative processes, by using the TV as a multiplicative constraint. The results demonstrate that this new reconstruction method brings satisfactory performance.

Index Terms—*Limited angle tomography, Alternating minimization, Total variation*

1. INTRODUCTION

Circular cone-beam CT (Computed Tomography) has been widely used in industrial non-destructive testing and medical imaging because it involves minimum complexity of hardware implementation. In circular cone-beam CT, the conventional and most commonly used method for reconstruction of complete projections is the standard filtered back projection (FBP) reconstruction technique [1]. However, complete projection data are not always available in many applications [1]. Some examples would be X-ray dose decrease, or time constraints when imaging a moving object, or X-rays being obstructed when passing through high-density region of objects. In these cases, the projection data can be acquired only from a view angle significantly less than 180^0, leading to the limited angle tomography.

An interesting approach to overcome the effects of limited angular range is to develop iterative image reconstruction algorithm. Much work has been done on developing iterative algorithms to overcome data insufficiency for CT image reconstruction over the past two decades [1]. Many researchers attempted several approaches including the Expectation-Maximization (EM) algorithm [1], statistical method [2], wavelet technique [3], algebraic reconstruct- tion methods [4, 5], level set [6] and variation methods [7]. For the limited angle problem, methods like the total variation in [8, 9] have been demonstrated to be very promising. Adaptive wavelet-galerkin method was introduced by the authors in [10] to solve the limited angle problem, and the reconstruction strategy has a comparable performance with a significant reduction in computational time. However, it is not enough to employ the prior information of reconstructed image. The authors in [11] reported a modified TV minimization method to reduce the data insufficiency and provide a robust and effective reconstruction in tomographic imaging by employing the prior of piecewise smoothness. However, the regularization parameter in [11] has a large effect on the quality of reconstruction. When the regularization parameter is too large, the image becomes uniform and inconsistent with the projection data. On the other hand, if the regularization parameter is too small, the reconstructed images suffer from the large artifacts.

This paper intends to make some further contribution to the subject in reconstructing an image from limited angle based on TV minimization method. The proposed minimization method combines the efficient computation of the gradient and robust reconstruction without any artificial parameter. Firstly, a new objective function is defined by adding the gradient to the new minimization problem to reduce the computational cost. Secondly, an alternating minimization algorithm is employed to implement the minimization problem. Compare to [11], the main advantage of the proposed method is that the proposed alternating minimization algorithm can avoid the initial artificial parameter setting in TV algorithm and reduce the computational cost. In the numerical examples, we find that the quality of reconstructed image by the proposed method is improved compared to the method in [11] and the algebraic reconstruction technique (ART).

The rest of this paper is organized as follows. In section II, we demonstrate discrete data model for circular cone-beam CT. In section III, we describe the new alternating algorithm for the limited angle tomography. Numerical results are presented in section IV, and conclusions are presented in section V.

2. DATA MODEL IN CIRCULAR CONE-BEAM CT

In circular cone-beam CT, the task in image reconstruction is to recover the density of an object $f(x)$ under examination provided by a set of line integrals. When the rotation angle of the X-ray source is defined as a, the cone-beam projection of the object function $f(x)$ at a point (u,v) on the detector can be expressed as

$$p(u,v,a)=\int_0^\infty f(s+\lambda(a,u,v))d\lambda, \qquad (1)$$

where source s is defined as

$$s=s(a)=(R\cos a, R\sin a, 0), \qquad (2)$$

where R denotes the distance from the source point to the rotation axis. And the ray direction vector $\theta(a,u,v)$ indicates the direction of the ray starting from source point $s(a)$ and passing through the point (u,v) on the detector. By discretizing the projection acquisition model, equation (1) can be approximated by following discrete linear system:

$$P=kf. \qquad (3)$$

The projection vector P consists of M-length measured projection data. f is a vector of length N with individual elements f_j, $j=1,2,\cdots N$. The system matrix k is a ray-driven projection operator which depends on the model for the integration in equation (1).

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When considering the error of measurement or noise, the projection acquisition model in equation (3) is described by the following formula:

$$P=kf+n, \qquad (4)$$

where n is the additive noise associated with the measurement. Because limited angle tomography is actually an incomplete reconstruction, formula (4) is underdetermined system of linear equations.

In general, the effectiveness of the TV-norm formulation relies on the fact that the imaged object has a relatively sparse gradient image. The addition of the TV-norm formulation has a very positive effect on the quality of reconstruction for piecewise smoothness of an unknown image. Despite of the success of reconstructions in limited angle tomography based on total variation method, a drawback is the presence of an initially artificial parameter in the minimization of the TV norm, which can only be determined by considerable numerical experimentations and a priori information of the desired reconstruction. This is often very time-consuming. One may save lots of time and computational cost if there is an iterative method that can give us a reasonable parameter within a practically acceptable number of iterations. Hence, we propose a new alternating optimization program for limited angle tomography by combining the gradient with the TV-norm in this section. The proposed alternating optimization program is given by:

$$\arg\min c_1(f)=\arg\min(z-f)^T\nabla H(f)+\frac{1}{2}\|z-f\|_2^2 \qquad (5)$$

$$\arg\min c_2(f)=\arg\min\frac{\|kz-P\|_2^2}{\|P\|_2^2}\|f\|_{TV}, \qquad (6)$$

where z is a set of images satisfying the data fidelity, $H(f)=\frac{1}{2}\|kf-P\|_2^2$, T denotes the transpose $\|z\|_{TV}=\sum_{s,t}\sqrt{(z_{s,t}-z_{s-1,t})^2+(z_{s,t}-z_{s,t-1})^2}$, s and t are the horizontal and vertical position of image z. The proposed optimization program is based on the following things: since the matrix k of data model is too large to be handled explicitly, computation of ∇H may be carried out efficiently. It is much less expensive to compute the gradient ∇H and to solve the optimization program (5) than it is to solve the original problem (4) by other means [12]. The algorithm based on TV is more suitable for limited angle tomography because it can reconstruct sharp discontinuities or edges with sparse or insufficient data that may occur due to practical issues of CT scanning. When the projection data P are incomplete in limited angle tomography, a large set of images may be consistent with the available data. That is, there exists a set of images which satisfy the equation (5). The minimization of the TV norm in equation (6) is used to select a unique image from the set of images. At the early stages of the iterations, the presence of the large data mismatch $c_2(f)$ in equation (6) takes an image outside the feasible region. In this case, the penalty of TV norm must be strengthened so that the images can return to the feasible region in the direction of lowered image TV. When the error of the estimated data with respect to the available data approaches ε in the iteration process, it shows that an estimated image belongs to the feasible region. In this case, the penalty of TV norm must be decreased to prevent an iterative image from the feasible region. Thus, it can accelerate the convergence of solution. Hence, we attempt to find a new optimization scheme to balance TV norm with data constraints in order to satisfy adaptive penalty of TV norm by introducing the equation (6). The balance is achieved by achieving a new TV norm as a product of TV norm and data constraints in this paper.

Proposition 1. When the matrix $A=-\frac{1}{2}kk^T$ is defined, we have the following identity:

$$(z-f)^T\nabla H(f)+\frac{1}{2}\|z-f\|_2^2=(kf-P)^T A(kf-P). \quad (7)$$

Proof: we can derive the following identities

$$\nabla H(f)=k^T(kf-P). \quad (8)$$

Equation (9) can be obtained by considering the minimization of the equation (5) with respect to z

$$z=f-\nabla H(f)=f-k^T(kf-P). \quad (9)$$

And equation (10) can be obtained by considering the equation (9) and the second term of the equation (5)

$$\frac{1}{2}\|z-f\|_2^2=(kf-P)^T A(kf-P). \quad (10)$$

Hence, combining the equation (9) and equation (10), the equation (11) is replace by

$$\begin{aligned}&(z-f)^T\nabla H(f)+\frac{1}{2}\|z-f\|_2^2\\&=(kf-P)^T A(kf-P)\\&=\|kf-P\|_A\end{aligned}, \quad (11)$$

where $A=-\frac{1}{2}kk^T$. The results follow.

By comparing the equation (5) and equation (7), we use the weighted matrix A in the data fitting term of equation (9) instead of the Euclidean norm in that of equation (5). Namely, we introduce the least-squares combined with the weight norm to the data fitting term in the equation (5). From the equation (9), we can find that when A is equal to the identity matrix I , the weight data fitting term is the same as $\|kf-P\|_2^2$.

In this paper, the alternating iterative algorithm is used to solve the proposed program. In the i th iteration, the update z_i and f_i are obtained by minimizing the equation (5) and equation (6) in the alternating manner respectively, shown as equation (12) and (13).

$$z^i=\arg\min(z-f^{i-1})^T\nabla H(f^{i-1})+\frac{1}{2}\|z-f^{i-1}\|_2^2 \quad (12)$$

$$f^i=\arg\min\frac{\|kz^{i-1}-P\|_2^2}{\|P\|_2^2}\|f\|_{TV}. \quad (13)$$

Equation (12) can be transformed as follows:

$$z^i=f^{i-1}-\nabla H(f^{i-1})=f^{i-1}-k^T(kf^{i-1}-P). \quad (14)$$

To establish the global convergence, the modified conjugate gradient formulas [13] is introduced to solve the equation (13). Because the image function is non-negative, we use f^i to denote the estimated image after projection onto the non-negative half-plane in the i th iteration. The image estimate f^i is a vector of length N with individual elements f_j^i , $j=1,2,\cdots,N$. N is the number of pixels in the reconstructed image:

$$f_j^i=\begin{cases}f_j^i & f_j^i\ge 0\\ 0 & f_j^i<0\end{cases} \quad j=1,2,\cdots,N. \quad (15)$$

The alternating iterative algorithm of the equation (12)-(13) computes a sequence of iterates by staring from an initial value $f^0=0$

$$f^0,z^1,f^1,z^2,f^2,z^3,f^3,\cdots z^i,\cdots.$$

4. EXPERIMENTAL RESULTS

To verify the performance of the method introduced above, we compare the proposed method with the method in [11] and the ART algorithm, which have been widely applied to solve the under-determined or unstable linear systems in tomographic imaging. In this section, we demonstrate and validate our algorithm under "ideal" and "noisy" conditions in order to verify the effectiveness of the proposed algorithm. The proposed algorithm is implemented in a MATLAB 6.5[TM]environment to test the performance using the simulated Shepp-Logan phantom and a real bee phantom as the test images. And its imaging parameters of the typical scanner geometry are defined as:

(1) The distance from the X-ray source to the centre of reconstructed image is 400 mm.
(2) The distance from the X-ray source to the detector is 800 mm.
(3) The projection interval is 1°.
(4) The reconstruction matrix is 128 × 128× 128.
(5) The pixel matrix of the detector is 99 × 300.
(6) The pixel pitch of the detector is 0.50647 mm.
(7) The pixel pitch of the object is 0.3125 mm.

The original mid-plane slice of the Shepp-Logan phantom is shown in Fig.1 (a). The display gray scale is $[0,255]$. In this study, the number of iterations for the proposed algorithm and the method in [11] is 30, and ART algorithm is 50. We define the scanning angular range of 20^0 in which the projection data at 20 uniformly distributed views are available. Fig.1 (b) shows that the mid-plane image is reconstructed by using our method from 20 noiseless projection data. Fig.2(c)(d) are the reconstructed images by using the method in [11] and ART algorithm in the same situation, respectively. In the second simulation, additive Gaussian noise with standard deviation 3% of the maximum value of the generated projection is added to the projection data, leading to a peak-signal-to-noise ratio (PSNR) of 21.452dB. Fig.2 (b) shows that the mid-plane images reconstructed by using our method with noisy projection data.

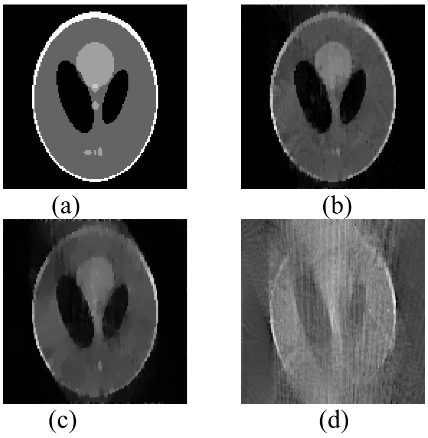

Fig.1 The 128×128 Shepp-Logan image reconstruction with noiseless projections which are available in angular range 20^0. (a)The original mid-plane image. (b) Reconstruction using our method. (c) Reconstruction using the method in [11]. (d) Reconstruction using ART method.

Fig.2(c)(d) are the reconstructed images by using the method in [11] and ART algorithm in the same situation, respective- ly. It can be seen that Fig.1 (b) and Fig.2 (b) contain a tiny deviation from the original phantom. Fig.1(c)(d) and Fig.2(c)(d) are much distorted. It is shown in Fig.1(c)(d) that the reconstructed images suffer from the different artifacts in image reconstruction compared to our method in the "ideal" condition. Moreover, the reconstructed images by using our method have a high contrast and are clearer than those by using the other algorithms in the same situation. Compared to other algorithms, the reconstructed images by our method display better results in "noisy" condition as shown in Fig.2.

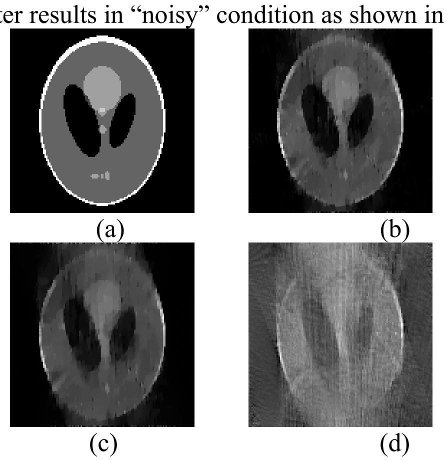

Fig.2 The 128×128 Shepp-Logan image reconstruction with noisy projections which are available in angular range 20^0.(a)The original mid-plane image. (b) Reconstruction using our method. (c) Reconstruction using the method in [11]. (d) Reconstruction using ART method.

5. CONCLUSION

In this paper, we have proposed a new alternating optimization program for the reconstruction of object in limited angle tomography. The performances in terms of image quality have been shown in the previous section. In general, the comparable image quality performance between the proposed algorithm and other algorithms is that the proposed algorithm has a very positive effect on the quality of reconstruction. The effectiveness of the proposed algorithm relies on the fact that the new TV norm can be implemented in an adaptive manner.

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