Lecture 22: Basic Image Formation
CAP 5415
Today

- We've talked about the geometry of scenes and how that affects the image
- We haven't talked about light yet
- Today, we will talk about image formation and how light interacts with surfaces
Term: Foreshortening

- From the light source's point of view, this surface
Term: Foreshortening

- looks the same as this surface
- Both surfaces receive the same amount of radiation
Term: Solid Angle

- Just as the angle subtended by a line can be found by projecting it onto a unit circle
- Unit: Radians
Term: Solid Angle

- Area subtended by a region on a unit sphere
- Unit: Steradians
Term: Radiance

- We need a term to describe the distribution of light
- Power of light in a region depends on
  - Foreshortened area of $r$
  - Solid angle subtended by the source
- Definition: Quantity of energy traveling at a point in a direction, per unit time, per unit area perpendicular to direction of travel, per unit solid angle
Term: Irradiance

- If we just care about how much light is arriving at a surface
- This is the incident power per unit area
How does light interact with surfaces? - The BRDF

- Now that we can talk about light, we want to know how it interacts with surfaces

- Local interaction model:
  - No Fluorescence – Light leaving at a wavelength is due to light arriving at a wavelength
  - Surfaces do not generate light internally
  - Radiance leaving point is only due to radiance arriving (light does not skip around)
The BRDF

- Can model a surface with these properties with a Bi-directional Reflectance Distribution Function

\[ \rho_{bd}(\theta_o, \phi_o, \theta_i, \phi_i) = \frac{L_o(x, \theta_o, \phi_o)}{L_i(x, \theta_i, \phi_i \cos \theta_i \, d\omega} \]

- Four dimensional function
Think of a hemisphere at a point
Think of a hemisphere at a point

- The BRDF is the ratio of light coming in to light coming out

\[
\rho_{bd}(\theta_o, \phi_o, \theta_i, \phi_i) = \frac{L_o(x, \theta_o, \phi_o)}{L_i(x, \theta_i, \phi_i) \cos \theta_i d\omega}
\]
The BRDF
• Symmetric with respect incoming and outgoing angle
  – Known as The Helmholtz Reciprocity Principle

\[
\rho_{bd}(\theta, \phi, \theta_i, \phi_i) = \frac{L_o(x, \theta, \phi, \phi_o)}{L_i(x, \theta_i, \phi_i) \cos \theta_i d\omega}
\]
How do I get the BRDF?

- Can be measured using a gonioreflectometer
Research in Shortcuts

MITSUBISHI ELECTRIC RESEARCH LABORATORIES
http://www.merl.com

Efficient Isotropic BRDF Measurement

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Figure 1: A photograph of our isotropic BRDF measurement device.

Figure 2: Pictures of some acquired materials.
Important BRDF's

- Ideal Diffuse Surface or Lambertian Surface
- Radiance leaving at an angle is independent of the angle
- Also called a matte surface
- The reflectance (or how it reflects light) is determined by the albedo
Important BRDF's - Diffuse

- Diffuse or Lambertian
- Surface reflects light uniformly in all directions
Important BRDF's - Specular

- Think of a mirrored ball
Specular surfaces

- Another important class of surfaces is specular, or mirror-like.
  - radiation arriving along a direction leaves along the specular direction
  - reflect about normal
  - some fraction is absorbed, some reflected
  - on real surfaces, energy usually goes into a lobe of directions
  - can write a BRDF, but requires the use of funny functions

(Slide from Forsyth)
All surfaces are not diffuse or specular

• But many can be modeled as the combination of a diffuse component and a specular component

(Image from Fleming et al)
Specular reflection

- Popular models:
  - Phong Model
  - Ward Model
    - More physically realistic

- Include a specular mirror-like component and a diffuse component

(Image from Ron Dror)
The Ward Model

\[ f(\theta_i, \phi_i; \theta_r, \phi_r) = \frac{\rho_d}{\pi} + \rho_s \frac{1}{\sqrt{\cos \theta_i \cos \theta_r}} \frac{\exp(-\tan^2\delta/\alpha^2)}{4\pi\alpha^2}. \]

**Figure 2.3.** Geometry used to define the specular component of the Ward model. As in Figure 2.1, \(N\) is the surface normal, \(S\) is the light source direction, and \(V\) is the view direction. The half-angle vector \(H\) bisects \(S\) and \(V\). The direction of ideal specular reflection \(R\) is such that \(N\) bisects \(R\) and \(S\).
What can we use this for?

- Once we can express how the illumination interacts with the surface, we can try and invert the equations to recover the surface properties.

- One application:
  - Photometric stereo
Basic Setup

- Multiple images of a scene
- We know where the illumination is
Assumptions

- Illumination is an infinitesimally small point
- Illumination is far from camera
- Surface is Lambertian

Result:

\[ B(x) = \rho(x) \mathbf{N}(x) \cdot S_1 \]

- Brightness of a point only depends on the angle between its normal and the illumination

\[ B(x) = \rho(x) \mathbf{N}(x) \cdot S_1 \]

The albedo
Solving Photometric Stereo

• Input:
  – Intensity measurements

• Model:
  \[ I(x, y) = k\rho(x, y)N(x, y) \cdot S_1 \]
  \[ = g(x, y) \cdot V_1 \]

• \( g(x, y) \) describes the surface

• How can we recover these \( g(x, y) \) vectors?
Photometric Stereo

- We have $n$ views that we can stack into a matrix

$$\mathbf{v} = \begin{pmatrix} \mathbf{v}_1^T \\ \mathbf{v}_2^T \\ \vdots \\ \mathbf{v}_n^T \end{pmatrix}$$

- The image measurements can also be stacked

$$\mathbf{i}(x, y) = \{I_1(x, y), I_2(x, y), \ldots, I_n(x, y)\}^T$$

- Leaving us with

$$\mathbf{i}(x, y) = \mathbf{V} \mathbf{g}(x, y)$$
Extracting Albedo and Shape

- To get albedo, use fact that normal has unit length

\[ \| g(x, y) \| = \rho(x, y). \]

- That leaves us with the normal at each point
How do we go from

• to
My favorite method

- Use this relationship to estimate derivatives of the surface

\[ N(x, y) = \frac{1}{\sqrt{1 + \frac{\partial f}{\partial x}^2 + \frac{\partial f}{\partial y}^2}} \left\{ \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, 1 \right\}^T \]

- Set up linear approximation
- Solve
What if we only have one image?

- How do we begin?
- Look at \( p-q \) map
Using intensity

- The intensity induces a cost function that constrains the derivative at a point
Check by estimating shape and re-rendering

(a) Our result. RMS=0.049.
(b) [22]'s result. RMS=0.008.
(c) Ground-truth.
What if we don't know the $p$-$q$ map?
Let's look at an artificial illumination
Surface curvature and images

- In a mirror, the intensity of a point is determined by its surface orientation
- Why do you see lines?
See something similar in images in real illuminations
Tied to Curvature
Results (fresh!)
Results

Figure 6. Result panel
Working with Intensity Values

- Simple Non-Linearities can have a great effect on image appearance
Working with Intensity Values

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- Simple Non-Linearities can have a great effect on image appearance
Working with Intensity Values

- This is not just linear scaling
Working with Intensity Values

- Can go the other way
Gamma correction

- Model this non-linearity using this type of curve:

\[ V_{in} = V_{out}^{\gamma} \]
Cameras and Gamma

- Consumer cameras put a similar non-linearity into photos
- Makes images look better
- You (as a vision person) don't want nice-looking images
- You want ACCURATE images
- Need to calibrate camera to remove gamma
Method 1: Use a target
Method 2: Use multiple images of same scene
Which leads to a second problem

- Most camera only give you 8 bits of precision
- Hard to capture both bright and dim regions

Solution:
  - Use multiple images to create a High Dynamic Range Image

Can be solved as a least-squares problem if you account for saturation and dimness