Lecture 15: Basic Multi-View Geometry

A bit more on perspective

One More

Stereo

- If I needed to find out how far point is away from me, I could use triangulation and two views
To see this

- I have three unknowns – (x, y, z)
- I have 4 equations
  - P projects to (u and v)
  - P' projects to (u and v)

Today

- For the rest of the lecture we will talk about the geometry of multiple views
- To begin we will talk about epipolar geometry

Epipoles

- The projection of the optical centers of each camera

Epipoles and epipolar lines

- For a point P, we may not know where it projects to (P')
- We do know that it lies on an epipolar line

When the cameras are calibrated

- The vectors \( \overrightarrow{OP}, \overrightarrow{OO'}, \) and \( \overrightarrow{O'P'} \) are coplanar
- Can be expressed as \( \overrightarrow{OP} \cdot [\overrightarrow{OO'} \times \overrightarrow{O'P'}] = 0 \)

How you can see this

- The vector returned by the cross product is perpendicular to the two vectors
- Can be thought of as a normal to a plane
- If \( \overrightarrow{OP} \) lines in the plane, it should also be perpendicular to that normal

\[ \overrightarrow{OP} \cdot [\overrightarrow{OO'} \times \overrightarrow{O'P'}] = 0 \]
Relating the two views

- First, remember that we have have points in two coordinate systems
- Can express a rigid transformation (translation and rotation) between the two systems
  \[ \overrightarrow{O}p' = \overrightarrow{O}' \mathcal{R}p' + t \]
  \( R \) is a Rotation Matrix

(now, rewrite constraint in terms of the coordinates of the left camera)

\[ \overrightarrow{O}p \cdot \left[ \overrightarrow{O}\overrightarrow{O}' \times \overrightarrow{O}'p' \right] = 0 \]
\[ p \cdot \left[ t \times \overrightarrow{O}' \mathcal{R}p' \right] \]
For simplicity:
\[ p \cdot \left[ t \times \mathcal{R}p' \right] \]

The essential matrix

- Starting with:
  \[ p \cdot \left[ t \times \mathcal{R}p' \right] \]
- A cross product can be rewritten as a matrix multiplication, leading to the constraint
  \[ p^T \mathcal{E}p' = 0 \]
  \( \mathcal{E} \) is called the essential matrix

Properties of the essential matrix

\[ p^T \mathcal{E}p' = 0 \]

- 3x3 matrix
- Rank 2
- Only relates the extrinsic parameters of the two views

What if the cameras aren’t calibrated

- The relationship still holds, but we have to calibrate the cameras first.
- Those calibration matrices, combined with the essential matrix, are known as the fundamental matrix
  \[ p^T \mathcal{F}p' = 0 \]
- Encodes information from the intrinsic and extrinsic parameters
- Also Rank 2

Finding the fundamental matrix

- Basic algorithm: 8-point algorithm
- Find 8 corresponding points in the images
- Once you have the corresponding p and p’ points,
  \[ p^T \mathcal{F}p' = 0 \]
- Is linear in \( \mathcal{F} \)
Finding the fundamental matrix

- Minimize over the coefficients of F

\[ \sum_{i=1}^{n} (p_i^T \mathcal{F} p'_i)^2 \]

- Improvements proposed by Hartley (1995)
  - Normalize corresponding points
  - After you compute the first estimate of \( \mathcal{F} \), enforce Rank-2 constraint using SVD (see text)
- Significantly improves results

Adding more views

- Text also describes the geometry of 3 or more

(From Forsyth and Ponce)

How do we use the fundamental matrix?

- It can tell us where to look for points in the other image
- The quantity \( p^T \mathcal{F} \) is a vector
- So, \( p^T \mathcal{F} p' \) defines a line
- This tells us where to look for the point that corresponds to \( p \)
- (Demo)

Another Application: View Morphing

**View Morphing**

**Abstract**

**Figure from Dyer and Seltz**

The Goal

- Interpolate between views to get new view

**Figure from Dyer and Seltz**

Can't use normal interpolation
Solution: Pre-warp Images

- If the views are parallel, interpolation won't distort objects

Solution: Pre-warp Images

- Can warp images so that the appear to be parallel

Can now do a morph

- Morphing is warping and cross-dissolving

Results

- Use Fundamental matrix to figure out the pre-warping transform
- Identify eight corresponding points
- Estimate fundamental matrix
- Figure out transformations such that the fundamental matrix becomes

\[
\mathbf{F} = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & -1 \\
0 & 1 & 0 \\
\end{bmatrix}
\]
The pre-warping transformation

- Can express as rotation translation and scale
- Rotate around direction orthogonal to epipole

Next Time:

- Using the views to get the depth of points in the scene