An Economic Framework for Dynamic Spectrum Access and Service Pricing

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***Abstract*—The concept of dynamic spectrum access will allow the radio spectrum to be traded in a market like scenario allowing wireless service providers (WSPs) to lease chunks of spectrum on a short-term basis. Such market mechanisms will lead to competition among WSPs where they not only compete to acquire spectrum but also attract and retain users. Currently, there is little understanding on how such a dynamic trading system will operate so as to make the system feasible under economic terms.**

In this paper, we propose an economic framework that can be used to guide i) the dynamic spectrum allocation process and ii) the service pricing mechanisms that the providers can use. We propose a knapsack based auction model that dynamically allocates spectrum to the WSPs such that revenue and spectrum usage are maximized. We borrow techniques from game theory to capture the conflict of interest between WSPs and end users. A dynamic pricing strategy for the providers is also proposed. We show that even in a greedy and non-cooperative behavioral game model, it is in the best interest of the WSPs to adhere to a price and channel threshold which is a direct consequence of price equilibrium. Through simulation results, we show that the proposed auction model entices WSPs to participate in the auction, makes optimal use of the spectrum, and avoids collusion among WSPs. We demonstrate how pricing can be used as an effective tool for providing incentives to the WSPs to upgrade their network resources and offer better services.

***Index Terms*—Dynamic spectrum access, pricing, game theory, auction theory.**

I. Introduction

THE early success in wireless services can be attributed to the operational licensing of *multiple* wireless service providers (WSPs) in one geographic location. This has resulted in a competitive environment that allowed fast deployment of wireless infrastructures, better services, and competitive pricing. Currently, WSPs buy spectrum from the spectrum owner (for example, Federal Communications Commission (FCC) in the United States of America) and use it for providing services to the end users. Such static spectrum allocations are common in most countries [11]. These spectrum allocations are usually long-term and any changes are made under the strict guidance of a governmental agency, for example the FCC. However, studies have shown that the usage of spectrum is both space and time dependent.[1] Hence, the current practice of static spectrum allocation often leads to low spectrum utilization and results in fragmentation of the spectrum creating "white space" (unused thin bands) that cannot be used for either licensed or unlicensed services. With the disproportionate and time-varying usage of the spectrum, it is intuitive that the notion of fixed spectrum assignment to providers is questionable. Though it might be argued that the implementation and administration of static spectrum allocation is very easy, the fact remains that the current system is ineffective and deprives not only the service providers but also the end users.

In order to break away from the inflexibilities and inefficiencies of static allocation, the concept of *Dynamic Spectrum Allocation* (DSA) is being investigated by network and radio engineers, policy makers, and economists. In DSA, spectrum will be allocated dynamically depending on need of the service providers which in turn depends on end users' demands. The spectrum owner will create a common pool of *open* spectrum using parts of the spectrum band that are not allocated or are no longer used. These parts of the band that are open to all are known as the coordinated access band (CAB) [4]. Examples of such bands include the public safety bands (764–776, 794–806 MHz) and unused broadcast UHF TV channels (450–470 MHz, 470–512 MHz, 512–698 MHz, 698–806 MHz).

A. Dynamic WSP Switching

The presence of multiple WSPs in any geographic region together with the freedom of users in switching WSPs is forcing a competitive environment where each WSP is trying to maximize its profit. As far as the end users are concerned, there is still a strong association with a single WSP, i.e., a user usually connects to one provider for a period of time (e.g., 1–2 years) and gets the services as per the contractual agreement. However, it is anticipated that in the near future, the concept of service brokers, technically known as Mobile Virtual Network Operators (MVNO),[2] will evolve that will act as an interface between the providers and the users [29]. These service brokers will allow end users more freedom to move from long-term service provider agreements to more opportunistic service models.

With such a loose association between the users and the WSPs, the first question that arises is "how or which wireless service provider should be selected by a user?" Second, "what price must the WSPs charge such that they are able to attract the users and maximize their profit?" By introducing the providers

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[1] http://www.sharedspectrum.com/inc/content/measurements/nsf/NYC_report.pdf

[2] http://en.wikipedia.org/wiki/MVNO

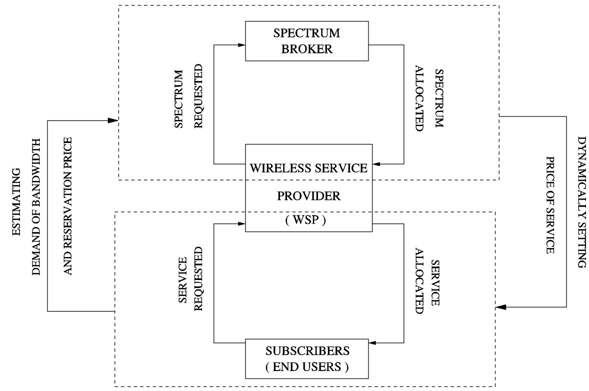


Fig. 1. Cyclic dependency.

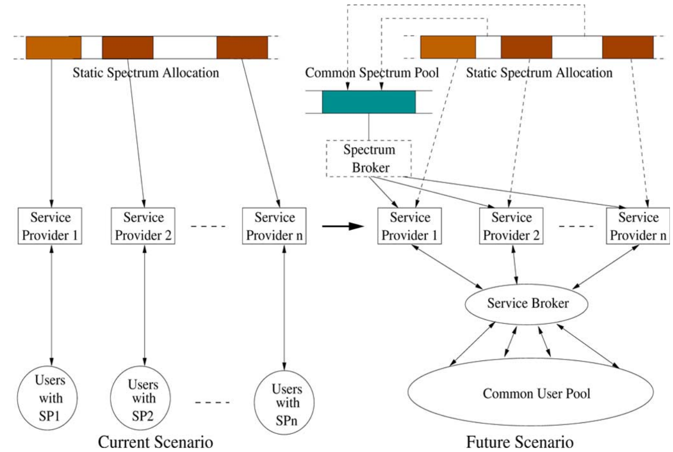


Fig. 2. The paradigm shift.

and users in a market like environment, it becomes convenient to leverage the concept of prices to regulate the demands of users who consume resources (bandwidth). In such a scenario, the goal of each service provider is twofold: acquire the necessary spectrum and serve as many users as possible. As both the number of end users and capacity of spectrum band are finite, this gives birth to an interrelated two-tier competitive behavior, where wireless service providers compete among themselves to acquire a large portion of the spectrum and also attract as many users as possible.

B. Cyclic Dependency

The trading system under consideration arises due to the presence of multiple competing WSPs in an *oligopoly*[3] market. The dynamic spectrum allocation is controlled by a spectrum broker [3] as shown in the upper half of Fig. 1. On the other hand, wireless service providers use the spectrum to offer services to the end users as shown in the lower half of Fig. 1. Though these two problems have been discussed separately, there is a strong correlation between them.

The most important factors that the WSPs need to consider are the *amount* of spectrum they need and the *price* they are willing to pay—both of which are determined by the demands of the users and the revenue generated from these users. In effect, estimation of the *demand for bandwidth* and the *expected revenue* will drive the provider's strategies. Service pricing by the providers, in turn, will affect the demand for the services by the users, thus resulting in a cyclic dependency in a typical supply–demand scenario shown by the arrows connecting the upper and lower halves of Fig. 1. As a result, these two tightly coupled problems must be analyzed together.

C. Economic Paradigm Shift

Currently, each provider gets a chunk of the spectrum and has a unique user pool that they cater to. In the future, a paradigm shift as depicted in Fig. 2, is very likely to occur where each provider will get a part of the spectrum from the common spectrum pool as and when they need through a *spectrum broker*. The users will be able to select their service provider as per their requirements through a *service broker*. In light of these new developments, it is important to investigate the economic issues that has a profound impact on the service quality and the prices paid by the end users.

D. Contributions of This Work

In this paper, we analyze the spectrum allocation scenario using the auction-theoretic model. We assume that the WSPs bid to acquire *extra* chunks of spectrum in addition to the statically allocated band and the spectrum broker tries to allocate the spectrum in such a manner so as to maximize their revenue. We map the winner determination problem to the knapsack problem and use a sealed-bid mechanism to find the optimal allocation. Through a game-theoretic model, the conflicting objectives that are inherent in the WSP-user interactions are captured. We deviate from the notion of per-service static prices [20], [24] and allow the providers to set the prices dynamically. Such a market mechanism is more flexible and realistic, as there does not exist any centralized authority to determine the price of a service [32]. One fundamental question that arises in such an environment is the existence of a price (Nash) equilibrium, where no provider finds it beneficial to change its price unilaterally [23]. Each service provider decides the price based on their current load and the service requested by the users. Obviously, the decision cannot be made unilaterally and the user must be involved. Through QoS and price preferences, a user selects a provider that best characterizes his preferences, usually given by the utility function.

In particular, we answer the following questions: i) how the spectrum will be allocated from the coordinated access band (CAB) to the service providers, ii) how service providers will determine the price of their services, and iii) how are the above two inter-related. More specifically, the contributions are:

- We devise a "winner determining sealed bid knapsack auction" mechanism that is used to dynamically allocate spectrum to multiple competing WSPs.
- The bidding strategies of the WSPs are investigated under first price and second price schemes in this knapsack auction.
- For WSP-user interaction, we provide the conflict and decision models under an incomplete information game situation, and propose utility functions for both WSPs and users.

[3]An oligopoly is a market form in which a market is dominated by a small number of sellers (oligopolists).

- We investigate threshold conditions that determine the Nash equilibrium and propose a dynamic pricing policy that helps both providers and users to maximize their utilities.
- We estimate the demand for bandwidth and the expected revenue generated from the users under the equilibrium condition; we argue that the revenue generated from users is the prime factor that governs the reservation price for the WSPs.
- Through a simulation study, we show that the proposed auction mechanism entices the WSPs to participate in the auction, makes optimal use of the common spectrum pool, and avoids collusion among WSPs. Also, we demonstrate that there are incentives for the providers to upgrade their network and radio resources.

The rest of the paper is organized as follows. In Section II, we discuss the body of work that relates to this research. We describe the proposed auction method in Section III. The game models and equilibria conditions are presented in Section IV. The demand for bandwidth is estimated in Section V. The simulation model and results are presented in Section VII. Conclusions are drawn in the last section.

II. Related Work

Economic theories have been used to analyze networking and communications problems where interacting decision-makers have conflicting objectives. In particular, auction and game theories have been proved to be very powerful tools to deal with distributed problems from an economic point of view. This is because the service quality that each user receives in a competitive environment is often affected by the action of other users who also try to contend for the same pool of resources.

Auction theory has been used to understand markets, especially to model auction participants who bid to win and maximize profit [25]. Currently, most auction sites (e.g., eBay[4]) support a basic bidding strategy through a proxy service for a single-unit auction where bidding continues till a winner evolves. In a single unit auction, Vickrey proved that "English" and "Dutch" type auctions yield the same expected revenue under the assumptions of risk neutral participants and privately known value drawn from a common distribution [33]. Vickrey's result is embodied in the "Revenue Equivalence Theorem" (RET) [14]. However, with emerging markets like electricity and spectrum band, single unit auctions are falling short to address the issues where bidders bid for multi-units and multiple winners emerge [1]. As bidders compete for a part of the available resource and are willing to pay a price for *that* part only, the auction model needed must be more generalized and is being currently investigated [2]. The implications of flat pricing and congestion pricing for capacity expansion are studied in [18]. Maille *et al.* designed a bandwidth pricing mechanism based on second-price auctions that solves congestion problems in communication networks [19]. A decentralized auction-based approach to pricing of edge-allocated bandwidth in a differentiated services Internet is presented in [27]. In [9], microeconomics based mechanisms are followed to manage dynamic spectrum allocation among the collaborating "agents", where the agents can be either the service providers or base stations or user terminals. The optimized allocation strategy for spectrum investigated in this paper follows a discriminatory pricing for both bargaining and auctions. Secondary pricing and allocation of spectrum from a primary license holder to other service providers is considered in [7]. Their proposed charging model is based on per admitted call in proportion to the interference the buyer networks generate. Though this is a simplistic pricing approach, the objective is still to maximize the profit by the primary lease holder. In contrast to the simplified approach, a real-time spectrum auction framework is presented in [8] under interference constraints. A piecewise linear demand curve for spectrum is assumed from the providers where maximization of revenue and spectrum utilization are attempted. However, the issue of collusion is not addressed. Collusion-resistant dynamic spectrum allocation is investigated in [12] with the help of pricing, where the aim is to optimize overall spectrum efficiency in the presence of user collusion. Though this approach maximizes the wireless networks' utilities, it does not consider the revenue maximization objective from spectrum owner's perspective. On the other hand, in a totally separate approach, the problem of dynamic spectrum access in cellular networks is investigated through interference constrained graphs and max-K-cut formulations in [31]. Secondary spectrum sharing with the similar QoS constraints is studied in [21], [36]. However, pricing issues are not considered.

As far as game theory is concerned, there is an emerging body of work that deals with decision making in a multi-provider setting. A broad overview of game theory and its application to different problems in networking and communications can be found in [35] and the references therein. Network services, including pricing issues, have been studied with the help of stability and fairness in [13]. In [5], a market in the form of a "bazaar" was introduced where infrastructure-based wide area wireless services are traded in a flexible manner and at any time scale. The mobile bazaar architecture allows fine-grained service through cooperative interactions based on user needs. The problem of dynamically selecting ISPs for forwarding and receiving packets has been studied in [34]. A multi-homed user, i.e., user with access to multiple ISPs, has the freedom to choose a subset of ISPs from the available ones. Shakkottai *et al.* examine how transit and customer prices are set in a network consisting of multiple ISPs and propose the equilibrium based on threat strategies [30]. Zemlianov *et al.* assumed the existence of two orthogonal technologies that were overlaid [37]. In particular, cellular and WLANs were considered where users were vertically transferred from one network to another based on the load of each network. In [17], service admission control was done based on the outcome of a game and Nash equilibrium was reached using pure strategy. Users were offered differentiated services based on the price they paid and the service degradation they could tolerate. However, dynamic pricing was not explored in [17]. In [22], Musacchio *et al.* study the economic interests of a wireless access point owner and his paying client, and model their interaction as a dynamic game. Resource allocation and base-station assignment problems for the downlink

[4]http://www.ebay.com/

in CDMA networks is studied based on dynamic pricing in [16]. He *et al.* presented a non-cooperative game for pricing Internet services but concluded with an unfair Nash equilibrium where future upgradation of the networks were discouraged [10].

In spite of all the above-mentioned works, there is still no unified framework that studies the interdependencies between spectrum owner–service provider interaction and service provider–end-user interaction [28]. In this work, we propose an economic framework that deals with dynamic spectrum access and service pricing. To the best of our knowledge, this is the first attempt to present the cyclic dependency among the two interactions and solve both the competition problems simultaneously.

III. Spectrum Allocation Through Auctions

In this section, we analyze a part of the logical model presented in Fig. 2, i.e., the interaction between the spectrum broker and the service providers. Auction is invoked only when the total demand of spectrum *exceeds* the total spectrum available in the CAB. The spectrum auction should be conducted on a periodic basis and on a small time granularity (e.g., every 1, 12, or 24 hours). Such synchronous auctions, i.e., allocations and de-allocations done periodically, will allow the spectrum broker to maximize revenue since the spectrum broker will be in a position to compare all the bids. If the auctions were to be conducted in an asynchronous manner, i.e., the providers can make requests at *any point of time*, the spectrum broker cannot choose the optimal winner set among all the bids. For example, if service provider B's higher bid comes after service provider A's bid (who has already been allocated), then the spectrum broker looses revenue if B's request cannot be granted due to unavailability of spectrum at that instant.

A. Auction Issues

A good auction design is important for any type of successful auction and often varies depending on the item on which the auction is held. Unlike classical single-unit auctions, spectrum auctions are multi-unit where bidders bid for a part of the spectrum band, i.e., the bids are for different amounts of bandwidth. (Note that we do not differentiate between bands with regard to the frequency. We assume that total spectrum is homogeneous and thus no band is superior or inferior than any other band. Considering frequency constraint to the spectrum allocation problem is beyond the scope of this research.) Also, multiple winners evolve constituting a winner set. Thus, determination of winner set depends heavily on the auction strategy adopted. In our auction model, the spectrum broker is the seller who owns the coordinated access band and service providers are the buyers/bidders. For designing the auction, we consider following important issues:

- How to maximize the revenue generated from bidders.
- How to maximize the spectrum usage.
- How to entice bidders by increasing their probability of winning.
- How to prevent collusion among providers.

B. Formulation of Auction Rules

Recall, the service providers already have some spectrum that was statically allocated. It is the additional spectrum that is sought from the CAB. In such model, the amount of extra spectrum that the WSPs are requesting from the CAB depends on the bandwidth demand from the end-users. We assume that the spectrum requested by the WSPs is a hard requirement to satisfy their end-users, i.e., the WSPs need at least the spectrum amount requested. Obtaining anything less than the minimum requirement will not provide positive payoff with the corresponding bid submitted. So, it is better from a rational WSP's perspective to obtain no spectrum instead of getting less than the minimum requirement that yields negative utility.

Though the objective of the spectrum broker is to sell the CAB and earn revenue, it is not at all intended that only big companies with higher spectrum demand are given additional spectrum. The goal here is to increase competition and bring new ideas and services at the same time. As a result it is necessary to make the small companies, who also have a demand of spectrum, interested in taking part in the auction.

The problem described here has a very close connection to the classical knapsack problem, where the goal is to fill a sack of finite capacity with several items such that the total valuation of the items in the sack is maximized. Here, the sack represents the finite capacity of spectrum in the CAB that is to be alloted to the WSPs in such a manner that the revenue generated from these WSPs is maximized. In this regard, we propose the "Winner Determining Sealed Bid Knapsack Auction".

To formalize mathematically, we consider L WSPs (bidders) who compete for a total spectrum W in a particular geographic region. All the service providers submit their demands at the same time in a sealed bid manner. We follow sealed bid auction strategy, because sealed bid auction has shown to perform well in all-at-a-time bidding and has a tendency to prevent collusion [26]. Each service provider has knowledge about its own bidding quantity and bidding price but do not have knowledge about other's quantity and price.

We formulate the auction as follows. We denote the strategy adopted by service provider i by a tuple $q_i = \{w_i, x_i\}$ where w_i denotes the amount of spectrum requested and x_i denotes the corresponding price that the service provider is willing to pay. If the sum of the bidding quantities do not exceed the spectrum available, W, then the requested quantities are allocated. Otherwise, auction is initiated when

$$\sum_{i=1}^{L} w_i > W. \tag{1}$$

Our goal is to solve the winner-determination problem in such a way so that the spectrum broker maximizes revenue by choosing a bundle of bidders (q_i), subject to condition that the total spectrum allocated does not exceed W, i.e.,

$$\text{maximize} \sum_i x_i \quad \text{such that} \sum_i w_i \leq W. \tag{2}$$

Note that a more realistic approach would have been a multiple-choice knapsack formulation with each provider (bidder) submitting a continuous demand curve. However, optimizations with continuous demand curve is hard. Approximation of continuous demand functions such as piecewise linear demand curve has been considered by Gandhi *et al.* [8]. Our approach of a single point demand can be thought of as a special case of Gandhi's approach. Though this is a simplified approach, our emphasis is on the determination of that point, i.e., what should be ideal bandwidth request and what would be the corresponding price.

C. Bidders' Strategies

We investigate bidders' strategies for both first and second price bidding schemes under knapsack model. In first price auction, bidder(s) with the winning bid(s) pay their winning bid(s). In contrast, in second price auction, bidder(s) with the winning bid(s) do not pay their winning bid but pay the second highest bid.

Let each bidder i submit its demand tuple q_i. Then the optimal allocation of spectrum is done by considering all the demand tuples. We denote this optimal allocation as M, where M incorporates all the winning demand tuples q_i and is subject to condition given in (2). Without loss of generality, we assume bids can take only integer values (as bids in dollar values are always expressed as integer) and number of bidders (providers) is typically of the order of 10.[5] Thus, we are able to solve the winner determination problem through dynamic programming with reasonably low computation. The aggregate bid can be obtained by summing all the bids from winning bidders, i.e.,

$$\sum_{i \in M} x_i. \tag{3}$$

Let us consider a particular bidder j who was allocated spectrum and thus belongs to M. Then the aggregate bid generated from the optimal allocation M minus the bid of bidder j is

$$\sum_{i \neq j, i \in M, j \in M} x_i. \tag{4}$$

Now consider that bidder j does not exist and the auction is among the remaining $L-1$ bidders. Let the optimal allocation be denoted by M^*. The aggregate bid generated in this case is

$$\sum_{i \neq j, i \in M^*, j \notin M^*} x_i. \tag{5}$$

Therefore, minimum winning bid of bidder j must be at least greater than

$$X_j = \sum_{i \neq j, i \in M^* j \notin M^*} x_i - \sum_{i \neq j, i \in M j \in M} x_i. \tag{6}$$

Through (6), we find that there exists a minimum winning bid X_j of bidder j, given the strategies of other bidders and auctioneer are fixed. We also find that the existence of this minimum winning bid signifies that bidder j will be granted the request if the corresponding bid $x_j > X_j$, and not granted if $x_j < X_j$. If $x_j = X_j$, bidder j is indifferent between winning and losing. This implies that in an ideal hypothetical world, if bidder j had any way of knowing the bids of other bidders, the computation of X_j would have been helpful in presenting his own bid and thus would govern the strategy of bidder j.

[5]http://en.wikipedia.org/wiki/Wireless_service_provider

However, in the real world scenario, the knapsack auction is conducted in a sealed bid manner and thus bidder j has no way of knowing what other bidders are bidding. Clearly, this is an example of incomplete information game among the bidders where bidder j has no knowledge of X_j. The only information with which bidder j is playing is his own reservation price. After the game is played, if bidder j wins and is granted allocation, then the concept of X_j is necessary to evaluate the payoff obtained from this game. In such a game scenario, it is important to find if there exists any Nash equilibrium strategy of the bidders so that bidders cannot dominate the game. In other words, we investigate for the dominant best response from the bidders' perspective which is considered as the bidders' strategies in this game.

In this regard, we present two lemmas (lemma 1 and 2) to study the dominant best response from the bidders under both second price and first price knapsack auctions. The auctioneer may choose any of the first or second price bidding strategy; however, there are certain differences between the the two when it comes to the perspective of the bidders. In first price bidding, bidders continuously need to guess how their opponents will bid instead of bidding the exact value of the resource under auction. In second price bidding, bidders do not need to guess other bidder's bids but can offer a bid request which reflects their own valuation of the resource. As a result, first price auctions make bidding more complex than the second price bidding for the bidders. Thus, it is found that first price bidding does not present any dominant strategy for the bidders while second price bidding strategy does provide dominant strategies for bidders. This difference in the bidding strategy leads to lemmas 1 and 2. Before we proceed with the proofs, we first define bidder's reservation price.

Bidder's Reservation Price: Bidder's reservation price is defined as the maximum price a bidder would be willing to pay. When a service provider buys spectrum from the spectrum broker, the service provider needs to sell that spectrum in form of services to the end users who pay for these services. The revenue thus generated helps the provider to pay for the fixed (static) cost for the statically assigned spectrum and the extra spectrum that the provider might need from the CAB. If the total revenue generated from the users is R and R_{static} goes towards the fixed cost, then the difference, R_{dynamic}, is the maximum amount that the provider can afford for the extra spectrum from CAB, i.e.,

$$R_{\text{dynamic}} = R - R_{\text{static}}. \tag{7}$$

Note that R_{dynamic} is *not* the bidder's reservation price but is a prime factor that governs this reservation price.

Lemma 1: In the second price knapsack auction, the dominant strategy of the bidder is to bid bidder's reservation price.

Proof: Let us assume the jth bidder has the demand tuple $q_j = \{w_j, x_j\}$ and its reservation price for that amount of spectrum requested is r_j. Now, as shown above in (6), the jth bidder's request will be granted and consequently belong to optimal allocation M, only if the bid generated by the jth bidder is at least X_j. Then according to the second price bidding policy, the jth bidder will pay the second price which is X_j in this case. Then the payoff obtained by the jth bidder is

$$E_j = r_j - X_j. \tag{8}$$

Through proof by contradiction, we show that the jth bidder's true bid is its reservation price r_j.

We assume that the jth bidder does not bid its true evaluation of the spectrum requested, i.e., $x_j \neq r_j$. Accordingly, bidder j has two options of choosing x_j.

Option 1: Bid is less than the reservation price, i.e., $x_j < r_j$. The values of x_j, r_j, and X_j are such that:

- $r_j > x_j > X_j$, then bidder j falls inside the optimal allocation M and its request is granted. The expected payoff obtained by jth bidder is still given by $(r_j - X_j)$.
- $r_j > X_j > x_j$, then bidder j loses and its request is not granted. Accordingly, the expected payoff becomes 0.
- $X_j > r_j > x_j$, bidder j still loses and the expected payoff is again 0.

Option 2: Bid is more than the reservation price, i.e., $x_j > r_j$. The values of x_j, r_j, and X_j are such that:

- $x_j > r_j > X_j$, then bidder j falls inside the optimal allocation M and its request is granted. The expected payoff obtained by jth bidder is still given by $(r_j - X_j)$.
- $x_j > X_j > r_j$, though bidder j wins but the expected payoff becomes negative in this case. The expected payoff obtained by jth bidder is given by: $(r_j - X_j) < 0$. Bidder j will not be interested in this scenario.
- $X_j > x_j > r_j$, bidder j loses and the expected payoff is again 0.

It is evident that if bidder j wins, then the maximum expected payoff is given by $E_j = r_j - X_j$ and bidding any other price (higher or lower) than its reservation price r_j will not increase payoff. Thus, the dominant strategy of a bidder in second price bidding under knapsack model is to bid its reservation price. ■

Comments: Our result corroborates with the result shown in other contexts in the economics literature, e.g., in Clarke's tax [6]. Thus, it is clear that bidders have no option of manipulating this auction.

Lemma 2: In first price bidding, reservation price is the upper bidding threshold.

Proof: Contrary to the Lemma 1, in first price bidding, the expected payoff obtained by jth bidder can be given by, $E_j = r_j - x_j$, as the actual price paid by the bidder is the same as the bid. Then, to increase the expected payoff, i.e., to keep $E_j > 0, x_j$ must be less than r_j.

Again at the same time, to win, bid x_j must be greater than X_j [(6)]. Thus, the weak dominant strategy for the bidder in first price auction is to bid less than the reservation price. ■

IV. Service Provisioning Using Games

In this section, we consider the most generic abstraction of "always greedy" and "profit seeking" model that exists between WSPs and end-users. The WSPs compete among themselves to provide service to a common pool of users. The resource for the WSPs are the spectrum bands that have been statically allocated and the additional spectrum that they buy on a dynamic basis. Users on the other hand select service providers depending on the benefit they obtain for the prices they pay. Let us discuss the conflict that arises between the WSPs and the users.

A. Conflict Model

We consider the model as shown in the lower half of Fig. 2, where any user can access any WSP. The users are the potential buyers who buy services from the WSPs. The selection of a WSP is done on a dynamic basis, i.e., a user compares the offerings both in terms of QoS and price for a particular service. Once a service is completed, the user relinquishes the radio resources. As the prices offered are not static, the users do not have any information about other users' strategies, i.e., demand for resources or price willingness to pay. In such an incomplete information scenario, the benefit of a user depends not only on its own strategy but also on what others do. Since we assume that every user is selfish, the problem is modeled as a non-cooperative game.

Service providers, very much like the users, also act in their self-interest. As a seller of the services, they determine the price for its services depending on the amount of spectrum acquired and the price paid. Similar to the non-cooperative incomplete information game among the users, the service providers also do not have any information about other providers' strategies, such as, price assigned for services, alloted resource, remaining resource, existing load, etc. Based on this conflict model, we need to define the decisions that we need to make. First, let us state the assumptions.

Assumptions: The devices carried by the users have the capability of measuring the received signal strengths from the base stations belonging to different service providers. The wireless service providers are selected on a session by session basis. For every session, a user chooses one of multiple service providers that has the capability of providing the resource (bandwidth) demanded by the user application.

B. Decision Model

As a user, the decision problem is to select the best service provider for the session requested. Now the question arises, how to select the best service provider or rather what criteria determines the best. The quality of service perceived by a user in a network must be considered in this regard. As quality of service depends on the traffic load, pricing strategies, and channel conditions, we must therefore perform a cost benefit analysis to find the best service provider.

As a service provider, the decision problem is to advertise a price for a service without knowing what prices are being advertised by its competitors. The optimization is to find a price such that the provider is able to sustain profit in spite of offering a low price, i.e., is there any price threshold to reach equilibrium?

C. Utility Function

An utility function is a mathematical characterization that represents the benefits and cost incurred. Here, we define the utility functions for both WSP and users.

We consider L service providers that cater to a common pool of $\mathcal{N}$ users. Let the price per unit of resource advertised by the service provider j, $1 \leq j \leq L$, at time t be $p_j(t)$. Let $b_{ij}(t)$

be the resource consumed by user i, $1 \leq i \leq \mathcal{N}$, served by provider j. We further assume that the total resource (capacity) of provider j is C_j.

The utility obtained by user i under the provider j can be given by [35]

$$u_{ij}(t) = a_{ij} \log(1 + b_{ij}(t)) \tag{9}$$

where the coefficient a_{ij} is a positive parameter that indicates the relative importance of benefit and acts as a weightage factor.

Note that we could have chosen any other form for the utility that increases with $b_{ij}(t)$. But we chose the log function because the benefit increases quickly from zero as the total throughput increases from zero and then increases slowly. This reflects the intuition that the initial increase in the perceived throughput is more important to a user. Moreover, log function is analytically convenient, increasing, strictly concave and continuously differentiable.

Next, we consider the cost components incurred by the user. The first cost component is the direct cost paid to the provider for obtaining $b_{ij}(t)$ amount of resource. If $p_j(t)$ is the price per unit of resource, then the direct cost paid to the jth provider is given by

$$p_j(t) b_{ij}(t). \tag{10}$$

This direct cost component decreases user i's utility. Note that in expression (10), both price per unit resource and the resource amount requested are variables.

The second cost component incurred by the user is the perceived quality of service, one of the manifestations of which is the queuing delay. We assume the queuing process to be $M/M/1$ at the links. Thus, the delay cost component can be written as

$$\begin{cases} \psi\left(\frac{1}{C_j - \sum_i^{N_j} b_{ij}(t)}\right), & \text{if } \sum_i^{N_j} b_{ij}(t) < C_j \\ \infty, & \text{if } \sum_i^{N_j} b_{ij}(t) \geq C_j \end{cases} \tag{11}$$

where N_j is the number of users currently served by provider j and $\psi(\cdot)$ is a mapping cost function of delay.

The last cost component is the cost due to the inherent characteristics of the wireless medium, *viz.* channel condition. Due to the relative distances of the user from the base stations of different networks (providers), and due to the various radio propagation effects, the signal received from different base stations will be different. If the channel quality is good, then the loss due to the channel will be less leading to higher empirical benefit to the users. On the other hand, if the quality of the channel is poor, then loss probability will be more leading to lower empirical benefit. We assume that Q_j denotes the wireless channel quality received from the base station of the jth provider. We model this cost component as an inverse function of Q_j and write as

$$\phi\left(\frac{1}{Q_j}\right). \tag{12}$$

Combining all the components obtained in (9), (10), (11), and (12), we get the net utility as

$$\begin{aligned} U_{ij}(t) = a_{ij} &\log(1 + b_{ij}(t)) \\ &- p_j(t) b_{ij}(t) - \psi\left(\frac{1}{C_j - \sum_i^{N_j} b_{ij}(t)}\right) \\ &- \phi\left(\frac{1}{Q_j}\right). \end{aligned} \tag{13}$$

We also obtain the utility as obtained by the service providers. The utility of service provider j at time t is

$$V_j(t) = p_j(t) \sum_i^{N_j} b_{ij}(t) - K_j \tag{14}$$

where K_j is the cost incurred to provider j for maintaining network resources. For the sake of simplicity, we assume this cost to be constant.

With the utility functions for the providers and users known, we investigate whether there exists any strategy that will help the users and providers to reach an equilibrium in the incomplete information game. The equilibrium we seek in this kind of a game is the Nash equilibrium [23]. Nash equilibrium is an equilibrium point where none of the players (i.e., the users and service providers) will find it beneficial to change the strategy unilaterally. Thus, we need to examine conditions that will yield the Nash equilibrium.

D. Price Threshold

Consider user i has a certain resource demand and wants to connect to a provider at time t. All the providers advertise their price per unit of resource amount and the existing load. Being rational, user i wants to maximize his net utility (potential benefit minus cost incurred).

To simplify our analysis, we assume all the users maintain a channel quality threshold. Later, in Section IV, we no longer hold this assumption and investigate the channel quality threshold strategy from users' perspective. Then without loss of generality, we combine the cost components $\psi(\frac{1}{C_j - \sum_i^{N_j} b_{ij}(t)})$ and $\phi(\frac{1}{Q_j})$ and modify (13) as

$$U_{ij}(t) = u_{ij}(t) - p_j(t) b_{ij}(t) - \xi\left(\frac{1}{C_j - \sum_i^{N_j} b_{ij}(t)}\right) \tag{15}$$

where

$$\xi\left(\frac{1}{C_j - \sum_i^{N_j} b_{ij}(t)}\right) = \psi\left(\frac{1}{C_j - \sum_i^{N_j} b_{ij}(t)}\right) + \phi\left(\frac{1}{Q_j}\right). \tag{16}$$

Basically, the function $\xi(\cdot)$ absorbs $\psi(\cdot)$ and $\phi(\cdot)$, and we assume that $b_{ij}(t)$ in $\xi(\cdot)$ captures the behavior of channel quality. If the channel quality is good, less amount of $b_{ij}(t)$ (resource) would be required; and a possibility of serving more number of users by that provider. On the other hand, if the channel quality is poor, then more resources (e.g., more time slots) will be required to maintain the QoS of user i.

Let us investigate if there exists any optimal resource amount for the users and any pricing bound from the providers that will maximize the users net utility. To do so we need to find whether the net utility given in (15) can be maximized with respect to the resource amount. If so, then a unique maximization point exists for $U_{ij}(t)$ with respect to $b_{ij}(t)$.

Differentiating (15) with respect to $b_{ij}(t)$, and finding the first and second derivatives, we find that $U_{ij}(t)$ contains a unique maximization point. Detailed derivations are presented in the Appendix. Thus, equating first derivative to 0, and solving for $b_{ij}(t)$ gives the optimal amount of resources needed by the users for a certain price $p_j(t)$ and this resource amount will maximize the utility of the user. From the reverse point of view, it is also clear that there exists a maximum threshold for the price $p_j(t)$, which is given by

$$\frac{a_{Ij}}{m_{Ij}(t)} - \xi'\left(\frac{1}{C_j + N_j - m_{Ij}(t)}\right). \tag{17}$$

For notational simplicity, we represent $a_{Ij} = \sum_i^{N_j} a_{ij}$ and $m_{Ij}(t) = \sum_i^{N_j} m_{ij}(t)$. Again, the derivations for finding the price upper bound is provided in the Appendix.

With the users' maximization strategy in effect, it is clear that the best response from the service providers' perspective would be to maintain the non-zero, non-negative price threshold so that the users's net utility can be maximized. We need to investigate if this price upper bound also helps the providers in maximizing their utilities. In other words, we are interested in finding mutual best responses from both users and service providers so that they do not find better utility by deviating from the best responses unilaterally, i.e., we seek the Nash equilibrium. With users' maximization strategy known, we find if providers' net utility equation has any maximization point. Replacing $\sum_i^{N_j} b_{ij}(t)$ by $m_{Ij}(t) - N_j$, we get

$$V_j(t) = \left(\frac{a_{Ij}}{m_{Ij}(t)} - \xi'\left(\frac{1}{C_j + N_j - m_{Ij}(t)}\right)\right) \times (m_{Ij}(t) - N_j) - K_j. \tag{18}$$

Differentiating (18) with respect to $m_{Ij}(t)$, we get

$$V_j'(t) = \left(\frac{a_{Ij}}{m_{Ij}(t)} - \xi'\left(\frac{1}{C_j + N_j - m_{Ij}(t)}\right)\right) + \left(-\frac{a_{Ij}}{(m_{Ij}(t))^2} - \xi''\left(\frac{1}{C_j + N_j - m_{Ij}(t)}\right)\right) \times (m_{Ij}(t) - N_j). \tag{19}$$

Differentiating again, and studying the expression for $V_j''(t)$, we get, $V_j''(t) < 0$; which implies that utility for the providers has a maximization point obeying the price bound.

Thus, it is clear that this pricing upper bound from the providers helps both the users and the providers to maximize their respective utilities and reach the Nash equilibrium point. If all of the other providers and users keep their strategies unchanged, and a provider decides not to maintain its pricing upper bound unilaterally, then that provider will not be able to maximize its users' utility. Thus, the users will not connect to this provider decreasing provider's revenue and the provider will not be able to reach the Nash equilibrium point.

With the existence of the Nash equilibrium known, we conduct a detailed analysis for finding the expected optimal resource amount requested by the users and the optimal price advertised by the providers in the next section.

V. Estimating the Demand for Bandwidth

The amount of extra (dynamic) spectrum that a provider needs depends on the demand for services by the users it supports. Therefore it is essential to estimate the resources consumed by the users and the price that is recovered from them. These estimates will help a provider determine the tuple $q_i = \{w_i, x_i\}$.

For finding the optimal resources, we proceed the following way. We equate (19) to 0, which gives the optimal value of $m_{Ij}(t)$. Equation (19) is not in closed form because the exact nature of $\xi(\cdot)$ is not known. We assume the solution of the above equation to be $m_{Ij(\mathrm{opt})}(t)$. Of course, for a given $\xi(\cdot)$, the value of $m_{Ij(\mathrm{opt})}(t)$ can always be obtained.

The optimal price that will maximize provider j's utility can be obtained by substituting $m_{Ij(\mathrm{opt})}(t)$ in (17). Thus, we get the optimal price as

$$p_{j(\mathrm{opt})}(t) = \frac{a_{Ij}}{m_{Ij(\mathrm{opt})}(t)} - \xi'(\frac{1}{C_j + N_j - m_{Ij(\mathrm{opt})}(t)}). \tag{20}$$

Note that $p_{j(\mathrm{opt})}(t)$ is clearly dependent on N_j.

To have a better insight into the analysis, we assume a simple closed form of $\xi(\frac{1}{C_j+N_j-m_{Ij}(t)})$ as $\frac{1}{(C_j+N_j-m_{Ij}(t))^\alpha}$, where α is a power coefficient in the delay and congestion component. While taking an exact form of $\xi(\cdot)$, we made sure that it satisfies the constraint of its first, second, and third derivatives to be positive. Any other form of $\xi(\cdot)$ would also suffice if the derivatives are positive. Rewriting (19), we get

$$V_j'(t) = \left(\frac{a_{Ij}}{m_{Ij}(t)} - \frac{\alpha}{(C_j + N_j - m_{Ij}(t))^{\alpha+1}}\right) + \left(-\frac{a_{Ij}}{(m_{Ij}(t))^2} - \frac{\alpha(\alpha+1)}{(C_j + N_j - m_{Ij}(t))^{\alpha+2}}\right) \times (m_{Ij}(t) - N_j). \tag{21}$$

Equating (21) to 0, we can find the solution of $m_{Ij}(t)$ for finding the maxima. It can be seen that the equation is not in its closed form. Thus, to solve the equation, we consider a special case.

Special case: We assume $\alpha = 1$ and $N_j = C_j$ and equate (21) to 0 to obtain

$$(2C_j - m_{Ij}(t))\sqrt{[3]a_{Ij}C_j} = m_{Ij}(t). \tag{22}$$

Solving the above equation for optimal $m_{Ij}(t)$, we get $m_{Ij(\mathrm{opt})}(t) = \frac{2C_j\theta}{1+\theta}$, where $\theta = \sqrt{[3]a_{Ij}C_j}$.

Using the optimal value of $m_{Ij}(t)$, we get the optimal value of $p_j(t)$ as

$$p_{j(\mathrm{opt})}(t) = \frac{a_{Ij}}{2C_j}\left(1 + \frac{1}{\theta}\right) - \left(\frac{1+\theta}{2C_j}\right)^2. \tag{23}$$

Thus, we see that the providers can achieve Nash equilibrium under the given pricing constraint and at the same time they can maximize their utility if the price is set as given by (23). Next, we use this pricing strategy as an incentive for the providers to upgrade their resources and users to improve their utility.

A. Pricing as an Incentive

With the strategies to determine the prices and the expected profit known, let us investigate if there is any incentive for the providers to upgrade their radio/network resources, and if this additional resource provides any incentive to the users too.

Substituting $m_{Ij(\text{opt})}(t)$ in (46), we get

$$m_{ij(\text{opt})}(t) = \frac{a_{ij}}{a_{Ij}}\left(\frac{2C_j\theta}{1+\theta}\right). \quad (24)$$

We know $m_{ij}(t) = 1 + b_{ij}(t)$; the optimal resource consumed by user i under provider j is given by

$$b_{ij(\text{opt})}(t) = \frac{a_{ij}}{a_{Ij}}\left(\frac{2C_j}{1+\frac{1}{\theta}}\right) - 1. \quad (25)$$

Thus, the optimal amount of resources for a provider to be demanded from a spectrum broker in the equilibrium can be given by $\sum_i b_{ij(\text{opt})}(t)$. Moreover, the utility of provider j can be written as $p_{j(\text{opt})}(t)\sum_i b_{ij(\text{opt})}(t)$.

Note that by using a proper transformation function (which is beyond the scope of this research), the total utility of provider j can be converted to a dollar value denoted by R (refer (7))—the total revenue obtained by provider j.

$$R = T\left(p_{j(\text{opt})}(t)\sum_i b_{ij(\text{opt})}(t) - K_j\right) \quad (26)$$

where $T(\cdot)$ is some transformation function.

Thus, provider's reservation price is governed by $T(p_{j(\text{opt})}(t)\sum_i b_{ij(\text{opt})}(t) - K_j) - R_{\text{static}}$.

VI. Channel Threshold Based Provider Selection

In this section, we analyze whether there exists any strategy for the users in choosing wireless service providers with respect to channel condition. We investigate if there exists any channel quality threshold, i.e., any minimum acceptable channel quality below which it will not be beneficial to select a network. Also, we show that any unilateral decision to deviate from the minimum threshold will not help a user, i.e., the threshold is the Nash equilibrium.

Theorem 1: Under varying channel conditions, a rational user should be active (transmit/receive) only when the channel condition is better than the minimum channel quality threshold set by the system to achieve Nash equilibrium.

Proof: We modify the net utility equation given by (13) to emphasize the channel quality cost component. We present the modified equation as

$$U_{ij}(t) = \begin{cases} u_i(b_i, \mathbf{b_{-i}}) - \phi\left(\frac{1}{Q_j}\right), & \text{if active} \\ 0, & \text{if not active} \end{cases} \quad (27)$$

where

$$u_i(b_i, \mathbf{b_{-i}}) = a_{ij}\log(1 + b_{ij}(t)) - p_j(t)b_{ij}(t)$$
$$-\psi\left(\frac{1}{C_j - \sum_i^{N_j} b_{ij}(t)}\right). \quad (28)$$

The notation $u_i(b_i, \mathbf{b_{-i}})$ emphasizes that the ith user's utility not only depends upon his own strategy but also on the strategies taken by the rest of the users denoted by $\mathbf{b_{-i}}$. For notational simplicity, we use Q instead of Q_j throughout the proof, where Q defines the channel condition perceived by the user when the intended WSP is the jth one.

1) Attaining Nash Equilibrium: From (27), it is clear that due to the inverse nature of cost due to channel condition, utility decreases monotonically with decrease in channel condition. We hypothesize that a user should be active with jth service provider, only if its channel quality is better than a given threshold. Let this threshold be Q_T. Note that this threshold is effective with regard to jth wireless service provider only. The threshold might be different for different WSPs. Therefore, the probability that a user is active with provider j is

$$\int_{Q_T}^{\infty} f_Q(x)dx = p'(Q_T) \quad (29)$$

where $f_Q(x)$ is the probability density function of Q. Now, if we assume that all the other users in the network act rationally and maintain the minimum channel quality threshold Q_T, then the probability that l users out of N other users in the jth network will be active is given by

$$p_l = \binom{N}{l}(p'(Q_T))^l(1 - p'(Q_T))^{N-l}. \quad (30)$$

Then, the expected net utility of the ith user (if the user is active) is given by

$$E[U_{ij}(t)] = \sum_{l=0}^{N}\left(u_i(b_i, \mathbf{b_{-i}}) - \phi(\frac{1}{Q})\right)p_l. \quad (31)$$

As $\sum_{l=0}^{N}\binom{N}{l}(p'(Q_T))^l(1-p'(Q_T))^{N-l} = 1$, the above equation can be written as

$$E[U_{ij}(t)] = \sum_{l=0}^{N} u_i(b_i, \mathbf{b_{-i}})p_l - \phi\left(\frac{1}{Q}\right). \quad (32)$$

If we define $u'_i(Q_T)$ as

$$u'_i(Q_T) = \sum_{l=0}^{N} u_i(b_i, \mathbf{b_{-i}})p_l \quad (33)$$

then the expected net utility of user i, if active, is given by

$$E[U_{ij}(t)] = u'_i(Q_T) - \phi(\frac{1}{Q}). \quad (34)$$

If the user is not active then by definition the expected net utility is 0. Thus, the *achievable gain* net utility considering both modes (active and not active) obtained by user i is

$$G'_i(Q_T) = \int_{Q_T}^{\infty}[u'_i(Q_T) - \phi\left(\frac{1}{x}\right)]f_Q(x)dx$$
$$= u'_i(Q_T)p'(Q_T) - B'(Q_T) \quad (35)$$

where $B'(Q_T) = \int_{Q_T}^{\infty} \phi(\frac{1}{x}) f_Q(x)dx$.

Now we will show that if the users act rationally and are active only when the channel condition is better than Q_T, then Nash equilibrium can be reached, i.e., they will reach a stable state where the *gain* of a user cannot be increased further by unilaterally changing the strategy of that user. For a user, the expected net utility for being active and not being active should be equal at the threshold. Therefore, the solution to

$$u_i'(Q_T) - \phi\left(\frac{1}{Q_T}\right) = 0 \tag{36}$$

gives the value of the threshold. We will now show why maintaining this threshold will help reach the Nash equilibrium.

Let Q_1 be the equilibrium solution to (36). Suppose, a user now unilaterally changes his strategy and decides the threshold to be Q_2. All the other users keep their threshold at Q_1. Then plugging in appropriate values in the integration in (35), the difference, $(G_i'(Q_1) - G_i'(Q_2))$, in the gain is given by

$$\begin{aligned}&[u_i'(Q_1)p'(Q_1) - B'(Q_1)]\\ &\quad - [u_i'(Q_1)p'(Q_2) - B'(Q_2)]\\ &\quad = u_i'(Q_1)[p'(Q_1) - p'(Q_2)] - [B'(Q_1) - B'(Q_2)]\\ &\quad = \phi\left(\frac{1}{Q_1}\right)[p'(Q_1) - p'(Q_2)] - [B'(Q_1) - B'(Q_2)].\end{aligned} \tag{37}$$

Two cases might arise depending on the relative values of Q_1 and Q_2.

Case 1: $Q_1 > Q_2$

$$\begin{aligned}&G_i'(Q_1) - G_i'(Q_2)\\ &\quad = -\left[\int_{Q_2}^{Q_1}\left[\phi\left(\frac{1}{Q_1}\right) - \phi\left(\frac{1}{x}\right)\right] f_Q(x)dx\right]\\ &\quad > 0.\end{aligned} \tag{38}$$

Case 2: $Q_1 < Q_2$

$$\begin{aligned}G_i'(Q_1) - G_i'(Q_2) &= \left[\int_{Q_1}^{Q_2}[\phi(\frac{1}{Q_1}) - \phi(\frac{1}{x})] f_Q(x)dx\right]\\ &> 0.\end{aligned} \tag{39}$$

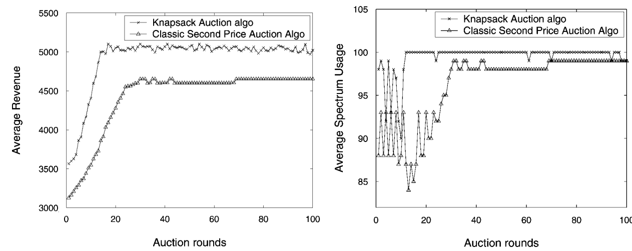
Thus, we find that a user cannot increase his gain by unilaterally changing his strategy. As a result, it becomes evident that a channel quality constraint exists for the users and maintaining this threshold will help the users to reach Nash equilibrium. ■

VII. Numerical Results and Interpretation

We present our results in this section. In Section VII.A, we simulate our auction model and show how the knapsack synchronous auction outperforms the classical highest bid auction models. In Section VII.B, we model the interaction between WSPs and users.

A. Spectrum Auctioning

The main factors that we consider for demonstrating the performance of the proposed knapsack auction are: revenue generated by spectrum broker, total spectrum usage, and probability of winning for bidders. For the simulation model we follow second price sealed-bid mechanism. In second price bidding, bidders do not need to guess other bidder's bids and thus can offer a bid request which reflects their own valuation of the resource. Thus, second price bidding does provide dominant strategies for bidders as shown in lemma 1 earlier. Moreover, second price bidding reduces the risk that the winning bidders might be subject to *winner's curse* [15]. We assume that all the bidders are present for all the auction rounds; bidders take feedback from previous rounds and generate the bid tuple for next round. The bid tuple q_i generated by bidder i consists of i) amount of spectrum requested, w_i and ii) the price the bidder is willing to pay, x_i.



(a) (b)

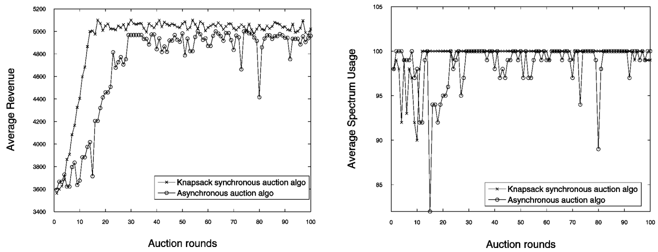
Fig. 3. (a) Revenue and (b) usage maximization with auction rounds for knapsack synchronous and classical highest bid.

For simulation purpose, the parameters considered are as follows. Total amount of spectrum in CAB is assumed as 100 units, whereas minimum and maximum amount of spectrum requested by each bidder is 11 and 50 units, respectively. Minimum bid per unit of spectrum is considered as 25 units.

Revenue and spectrum usage: Fig. 3(a) and (b) compares revenue and spectrum usage for knapsack synchronous and classical highest bid strategies for each auction round. The number of bidders considered is 10. Note that both revenue and usage are low in the beginning and subsequently increases with rounds. In the initial rounds, bidders are dubious and make low bids. With increase in rounds, potential bidders emerged as expected and raised the generated revenue. We observe that the proposed auction generates 10%–15% more revenue compared to the classical model and also reaches steady state faster. Similarly, in Fig. 4(a) and (b) we compare revenue and spectrum usage for both the synchronous and asynchronous strategies. It is observed that knapsack synchronous strategy provides better revenue and spectrum usage compared to asynchronous bid submission mechanism as spectrum broker can synchronize all the bids.

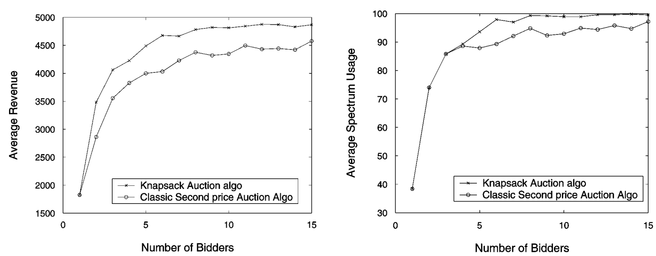
Figs. 5(a), (b) and 6(a), (b) show the average revenue and spectrum usage with varying number of bidders for both comparisons (knapsack synchronous with classical highest price and knapsack synchronous with knapsack asynchronous).

Bidder participation: In Fig. 7(a) and (b), we look at our auction model from the bidders' perspective. Higher revenue requires high participation in number of bidders. However, classical auctions always favor bidders with high spectrum request and/or high bid, thus discouraging low potential bidders and giving the higher potential bidders a chance to control the



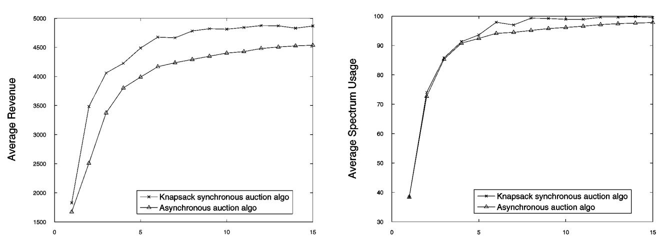
(a) (b)

Fig. 4. (a) Revenue and (b) usage maximization with auction rounds for synchronous and asynchronous bid requests.



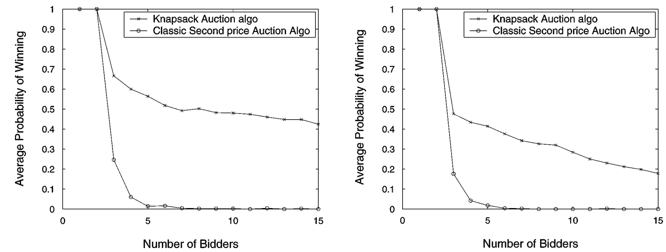
(a) (b)

Fig. 5. (a) Revenue and (b) usage for knapsack synchronous and classical highest bid.



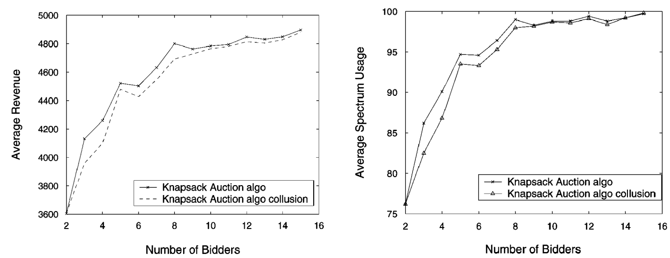
(a) (b)

Fig. 6. (a) Revenue and (b) usage for synchronous and asynchronous bid requests.



(a) (b)

Fig. 7. (a) Probability of winning with lowest spectrum request. (b) Probability of winning with lowest value bid.



(a) (b)

Fig. 8. (a) Average revenue with and without collusion. (b) Average usage with and without collusion.

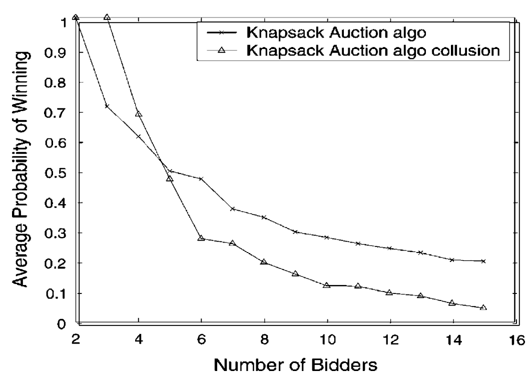


Fig. 9. Average probability of winning with and without collusion.

auction. In order to evaluate the bidder participation, we consider two cases: 1) bidder with the lowest spectrum request and 2) bidder with the lowest bid. For these two cases, we compare knapsack synchronous with classical highest bid strategies in terms of probabilities to win a bid. We observe that the proposed auction strategy has a significantly high probability of winning compared to classical strategy. Note that probability of winning in classical strategy almost reaches zero with increase in bidders.

Collusion prevention: The occurrence of collusion must be prevented in any good auction so that a subset of bidders can not control the auction that might decrease the spectrum broker's revenue. We consider two cases: 1) when bidders collude and 2) when bidders do not collude. In our simulation model, we assume bidders randomly collude in pair in all possible combinations with others.

In Fig. 8(a), we show the average revenue generated by spectrum broker with increase in number of bidders both in presence and absence of collusion. Though at the beginning with less number of bidders, presence of collusion reduces the average revenue slightly, but with increase in number of bidders the effect due to collusion decreases. Thus, with increase in number of bidders, i.e., with increase in (perfect) competition, revenue generated even in the presence of collusion reaches almost the same value as that of without collusion. Fig. 8(b) presents the usage of spectrum in the presence and absence of collusion. The most interesting result from bidders' perspective is shown in Fig. 9. When the number of bidders is low (less than or equal to 4 in our case) collusion provides better probability of winning but as the number of bidders increases, probability of winning with the help of collusion decreases, discouraging bidders to collude.

B. Pricing: Numerical Results

Here, we provide some insights on how the pricing strategies proposed for WSP and end users interaction work as incentives for both. We consider two cases—fixed and increasing number of users.

- **Fixed number of users:** We keep the number of users fixed with the total resource of the provider increasing. Recapitulate from (23) and (25) that increasing resource implies

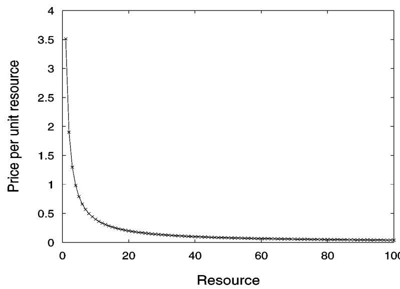


Fig. 10. Price per unit of resource versus resource (with number of users fixed).

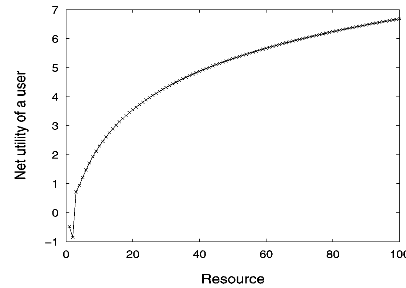


Fig. 12. Net utility of a user versus resource (with number of users fixed).

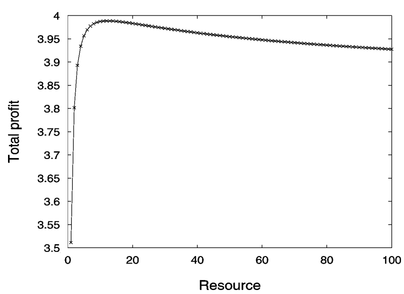


Fig. 11. Total profit of a provider versus resource (with number of users fixed).

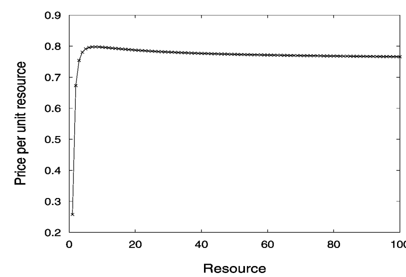


Fig. 13. Price per unit of resource versus resource (with increasing number of users such that the ratio of a_{Ij} and C_j is fixed).

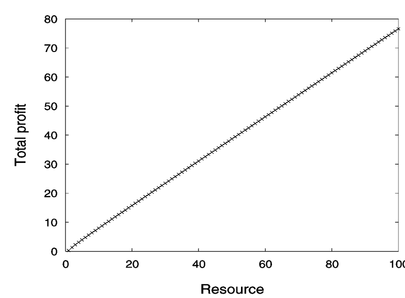


Fig. 14. Total profit of a provider versus resource (with increasing number of users such that the ratio of a_{Ij} and C_j is fixed).

increasing C_j and fixed number of users implies a fixed value of a_{Ij}. We consider all users have equal weightage factor $a_{ij} = 1.5$ and the value of C_j varies from 1 to 100 units. These values used for obtaining the numerical results are arbitrary and are merely for the sake of demonstration. Any other values of a_{ij} and C_j can be used as long as they satisfy the constraint that the price per unit resource is positive.

Fig. 10 shows how the provider must decrease the price per unit of resource if the total amount of resources increases with the same user base. This decrease in unit price is necessary if resource utilization is to be maximized which also serves as an incentive for the users.

The total profit of the provider is presented in Fig. 11. With the number of users fixed, we observe that the total profit of the provider increases till a certain resource and then decreases. For a fixed number of users, this result allows us to estimate the amount of resource that the provider must have such that its profit is maximized.

We show how the net utility of users increases with more resources in Fig. 12. It is clear that for fixed number of users, more resources is an incentive for the users. An important aspect to note here is that, for initial increase in resource the utility increased very quickly from 0 but the utility slowly saturates indicating that more resources have limited value beyond a certain point, i.e., the users will not find ways to utilize abundant resources.

- **Increasing number of users:** The number of users (population base) is increasing in this case which is typical of any market. We start with 1 user under a provider. For fair comparison with the previous case (i.e., with fixed number of users), we increase resources from 1 to 100 units. Note that a_{Ij} is no longer fixed and increases with increasing number of users. For this simulation, we assume that the increase in number of users is such that the ratio of a_{Ij} and C_j is fixed.

In Fig. 13, the price per unit of resource is presented where both users and resources increase. As the initial number of users is very low, increasing resource necessitates an initial increase in price per unit of resource. But as the number of users increase, it is imperative that price per unit resource decreases providing incentive for the users.

In Fig. 14, we present the total profit of the provider. Unlike the previous case (Fig. 11), we see that with users increasing proportionally with resources, the total profit is always increasing which presents a better incentive for the providers than the case with fixed number of users. It is evident that to increase total profit, the providers would prefer more number of users, each getting less resource, rather than having less number of users, each having more resource. Note that the linear increase in profit is just due to the assumption: the ratio between the users and resources is fixed. Fig. 15 justifies the hypothesis that with increasing number of users and resources, the increase in the amount of resource for users saturates.

In Fig. 16, we show the net utility of the users. We see that the net utility increases with increasing resources; thus providing

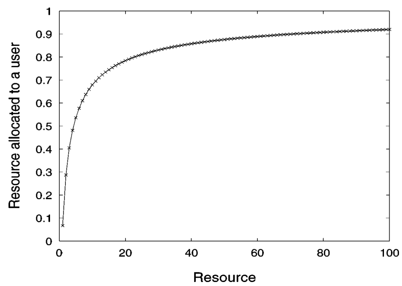


Fig. 15. Resource allocated to a user versus resource (with increasing number of users such that the ratio of a_{Ij} and C_j is fixed).

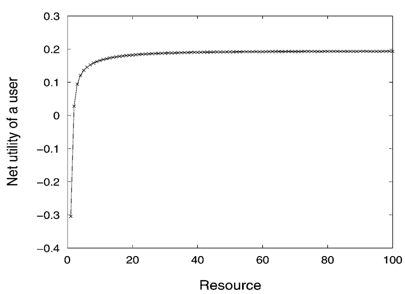


Fig. 16. Net utility of a user versus resource (with increasing number of users such that the ratio of a_{Ij} and C_j is fixed).

incentive for the users but unlike Fig. 12, the increase in net utility saturates early. It is evident that to increase net utility, users would like to be in a lightly loaded system (i.e., with less number of users), thereby getting a larger share of the resources, rather than having to share the resources with more number of other users. This presents the classic case of conflict between the providers and users.

VIII. Conclusion

Dynamic spectrum allocation coupled with fine granularity switching of services by end-users will engender a flexible and competitive environment for trading wireless services. In this research, we provide a framework based on auction and game theories that capture the interaction among spectrum broker, service providers, and end-users in a multi-provider setting. We propose a winner determining sealed bid knapsack auction that dynamically allocates spectrum from CAB and at the same time maximizes revenue generated, entices WSPs by increasing their probability of winning, and prevents collusion. We construct utility functions for users and service providers considering their conflicts with each other. We show that both of them can reach Nash equilibrium if they maintain certain threshold conditions. We also demonstrate how proper pricing can provide incentives to providers to upgrade their resources and users to opt for better services.

Appendix

We provide the detailed derivation steps for finding the price upper bound for WSPs as continuation from Section IV. The modified utility function of the user as presented in (15) is

$$U_{ij}(t) = u_{ij}(t) - p_j(t)b_{ij}(t) - \xi\left(\frac{1}{C_j - \sum_i^{N_j} b_{ij}(t)}\right). \tag{40}$$

Differentiating (40) with respect to $b_{ij}(t)$,

$$U'_{ij}(t) = \frac{a_{ij}}{1 + b_{ij}(t)} - p_j(t) - \xi'\left(\frac{1}{C_j - \sum_i^{N_j} b_{ij}(t)}\right). \tag{41}$$

Similarly, the second derivative is

$$U''_{ij}(t) = -\frac{a_{ij}}{(1 + b_{ij}(t))^2} - \xi''\left(\frac{1}{C_j - \sum_i^{N_j} b_{ij}(t)}\right). \tag{42}$$

If we assume delay and congestion component, such that $\xi''(\frac{1}{C_j - \sum_i^{N_j} b_{ij}(t)}) > 0$, then $U''_{ij}(t) < 0$ and it is clear that $U_{ij}(t)$ is strictly concave in the region bounded by $\sum_i^{N_j} b_{ij}(t) = C_j$; and $U_{ij}(t) \to -\infty$ as $\sum_i^{N_j} b_{ij}(t) \to C_j$. Moreover, it can be inferred from (42) that as $U''_{ij}(t) < 0, U_{ij}(t)$ contains a unique maximization point.

Thus, equating first derivative (41) to 0, and solving for $b_{ij}(t)$ gives the optimal amount of resources needed by the users for a certain price $p_j(t)$ and this resource amount will maximize the utility of the user.

As the users are homogeneous, to maximize users' utility, the first derivative of all the users can be equated to zero:

$$U'_{1j}(t) = U'_{2j}(t) = \cdots = U'_{N_j j}(t) = 0. \tag{43}$$

Recall that N_j is the number of users currently served by provider j. Thus, (43) reduces to

$$\frac{a_{1j}}{1 + b_{1j}(t)} = \frac{a_{2j}}{1 + b_{2j}(t)} = \cdots = \frac{a_{N_j j}}{1 + b_{N_j j}(t)}. \tag{44}$$

If $1 + b_{ij}(t) = m_{ij}(t)$ and with the help of identity, we get

$$\frac{a_{ij}}{m_{ij}(t)} = \frac{\sum_i^{N_j} a_{ij}}{\sum_i^{N_j} m_{ij}(t)}. \tag{45}$$

For notational simplicity, we represent $a_{Ij} = \sum_i^{N_j} a_{ij}$ and $m_{Ij}(t) = \sum_i^{N_j} m_{ij}(t)$. Thus, (45) can be written as

$$\frac{a_{ij}}{m_{ij}(t)} = \frac{a_{Ij}}{m_{Ij}(t)}. \tag{46}$$

Putting the above form into (41), we get

$$U'_{ij}(t) = \frac{a_{Ij}}{m_{Ij}(t)} - p_j(t) - \xi'\left(\frac{1}{C_j + N_j - m_{Ij}(t)}\right). \tag{47}$$

Note $U'_{ij}(t)$ is strictly decreasing with the values of $m_{Ij}(t)$ lying in the interval $(C_j, C_j + N_j)$. Then for achieving the Nash equilibrium by the providers, the pricing constraint $p_j(t)$ is upper bounded by

$$\frac{a_{Ij}}{m_{Ij}(t)} - \xi'(\frac{1}{C_j + N_j - m_{Ij}(t)}). \tag{48}$$

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