Forcing Behavioral Subtyping Through Specification Inheritance
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Forcing Behavioral Subtyping Through Specification Inheritance

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Abstract
A common change to object-oriented software is to add a new type of data that is a subtype of some existing type in the program. However, due to message passing unchanged parts of the program may now call operations of the new type. To avoid re-verification of unchanged code such operations should have specifications that are related to the specifications of the appropriate operations in their supertypes. This paper presents a specification technique that uses inheritance of specifications to force the appropriate behavior on the subtype objects. This technique is simple, requires little effort by the specifier, and avoids re-verification of unchanged code. We present two notions of such behavioral subtyping, one of which is new. We show how to use these techniques to specify examples in C++.

1 Introduction

Object-oriented (OO) software can be extended by adding new subtypes to existing types. Such extensions provide reuse of existing functions by allowing one to use subtype objects in place of supertype objects. However,

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due to message passing unchanged functions will then execute operations of
the newly added subtypes, potentially requiring respecification and reverifi-
cation of the function [12]. Respecification and reverification go against the
ease of extension promised by proponents of OO software. Specification and
verification techniques which evolve with software, that is which do not re-
quire respecifying or reverifying whenever new components are added to the
system, are called modular.

Behavioral subtyping, subtyping based on the behavior of types, can be
used for modular specification and verification of OO software. A set of con-
ditions for behavioral subtyping has been proposed both proof-theoretically
[1, 14], and model-theoretically [12, 6]. It has been shown that with the
addition of new behavioral subtypes, existing unchanged software does not
have surprising behavior [12, 6]. Leavens and Weihl [13] present a technique
for modular verification of OO programs. But to use such a technique one
needs to verify that each specified subtype relation constitutes a behavioral
subtype.

In this paper we present a modular specification technique, which auto-
matically forces behavioral subtyping (and thus also avoids reverification).
We also define a new, weaker notion of behavioral subtyping that permits
more behavioral subtype relations than previous work [14]. Though we use
C++ [20] as an OO language and Larch/C++ [4, 10] as a specification lan-
guage, the techniques we present can also be applied to other programming
languages and with other specification languages.

2 Background on Larch/C++

Larch [7] is a family of specification languages with a two-level approach to
specification. One level of specification, the interface language, describes the
interface and behavior of the modules of a programming language like C++
and Modula-3. Larch/C++ [11] plays this role in this paper. The other com-
ponent, the Larch Shared Language (LSL), describes the underlying model
and the vocabulary that can be used in the interface language. For lack of
space we sometimes omit details of the LSL traits.

Figures 1 and 2 give the specification of a C++ class, BankAccount in
Larch/C++. The interface specification is given in Figure 1. The first uses
clause in Figure 1 says that the abstract values of BankAccount objects and
class BankAccount {
    public:
        uses BankAccountTrait;
        constraint self\'.owner = self\'.owner;

        BankAccount(long int cts, const char *name); // constructor
        behavior {
            uses cpp\_const\_char\_string;
            requires 0 < dollars(cts) ∧ nullTerminated(name, pre);
            modifies self;
            ensures self' = [dollars(cts), uptoNull(name, pre)];
        }

        virtual long int balance() const;
        behavior {
            ensures approx\_equal(dollars(result), self\'.credit);
        }

        virtual void withdraw(long int cts);
        behavior {
            requires 0≤dollars(cts) ∧ dollars(cts)≤self\'.credit;
            modifies self;
            ensures self' = [self\'.credit - dollars(cts),
                             self\'.owner];
        }

        // ... pay\_interest and deposit are omitted
};

Figure 1: Interface Specification for BankAccount
BankAccountTrait: trait
   includes long, Rational(long for Int),
   String(char for E, String[char] for C),
   NoContainedObjects(BankAccount)
BankAccount tuple of credit:Q, owner:String[char]
   introduces
   dollars: long → Q
   approx_equal: Q, Q → Bool
   asserts ∀ q1,q2:Q, c: long
      dollars(c) == c/100;
      approx_equal(q1, q2) == abs(q1 - q2) < (1/200)

Figure 2: Specification of BankAccountTrait

the vocabulary used to specify them are given by the trait BankAccountTrait
(in Figure 2). The abstract values are defined as tuples with a credit com-
ponent and an owner component. The constraint clause in Figure 1 states
that the owner of BankAccount cannot be changed. Following this history
constraint [14] in Figure 1 are the specifications of the C++ member func-
tions.

The pre-condition for the constructor BankAccount follows the keyword
requires. It requires that the amount be positive and that the name be
a valid C++ string. (The trait function nullTerminated is specified in the
trait listed by the uses clause in the constructor.) The modifies clause
is sugar for conjoining a frame axiom to the postcondition. It states that
only self, the receiver of the message, can be changed by the function. The
postcondition, given by the ensures clause, forces the value of the self in
the post-state to be the tuple with cts converted to dollars and the string
denoted by name in the pre-state. Specification for the member function
balance ensures that the result is approximately equal to the amount in
credit converted into dollars. The member function withdraw modifies
self by decreasing its credit component by dollars(cts), provided cts is
non-negative and is less than the credit of self in the pre-state.

After specifying and implementing BankAccount, a programmer might
#include "BankAccount.lcc"

void transfer(BankAccount& source, BankAccount& sink, 
               long int cts);

behavior {
    let amt: Q be dollars(cts);
    requires source ≠ sink ∧ source^.credit ≥ amt
    ∧ amt ≥ 0;
    modifies source, sink;
    ensures sink' = [sink^.credit + amt, sink^.owner]
    ∧ source' = [source^.credit - amt, source^.owner];
}

Figure 3: Specification of the function transfer

define functions to manipulate BankAccount objects. Figure 3 gives the 
Larch/C++ specification of such a function. The pre-condition prevents the 
two accounts from being the same object and requires the amount being 
transferred to be non-negative. The post-condition describes the transfer 
from source to sink. The specification uses the trait BankAccountTrait for 
its model and vocabulary, by importing the interface for BankAccount.

3 The Problem

One set of problems caused by OO programming techniques is illustrated by 
the following scenario. Suppose that, after using the type BankAccount, a 
new type of account, PlusAccount, is added to the program. A PlusAccount 
object has both a savings and a checking account. PlusAccount is intended 
to be a behavioral subtype of BankAccount. Such an extension does not 
change the existing code either for BankAccount or for transfer.
3.1 Specification and verification problems

Subtype polymorphism allows one to send \texttt{PlusAccount} objects in place of \texttt{BankAccount} objects as arguments to \texttt{transfer}. However, Figure 3 specifies \texttt{transfer} using the vocabulary for \texttt{BankAccount} values defined in \texttt{BankAccountTrait}. How can one interpret what the specification of \texttt{transfer} says about \texttt{PlusAccount} objects, which might have completely different abstract values? For example, what does the trait function \texttt{.credit} mean when applied to a \texttt{PlusAccount}? Do we need to resppecify \texttt{transfer} or do we need to use the same abstract values and trait functions when \texttt{PlusAccount} is added? Respecification goes against the modularity of OO software. Using the same abstract values and trait functions would not work in this example, because \texttt{PlusAccount} contain more information than \texttt{BankAccount} objects. One alternative would be to specify all abstract values as tuples, and treat subtype values in supertype contexts by projecting away the extra components, as is done in other OO specification languages (such as Object-Z, VDM++, MooZ, and Z++ [19, 9]). But then viewing \texttt{PlusAccount} objects as \texttt{BankAccount} objects whose balance is the sum of the two account balances would require the \texttt{PlusAccount} abstract values to have three components, the total balance, and amounts in savings and checking; one would also have to maintain an invariant that the total balance is always the sum of the savings and checking. Although this alternative works, it requires more cleverness than in a specification language that does not use tuples for all abstract values.

To see the verification problem, consider the case that \texttt{transfer} is verified, before \texttt{PlusAccount} is added, using the specification of \texttt{BankAccount}. When the subtype \texttt{PlusAccount} is added, the implementation of \texttt{transfer} is not changed, but the code it executes changes when \texttt{PlusAccount} objects are passed. In such cases, the code will execute methods with different specifications than those used during verification. Therefore the verification of \texttt{transfer} might no longer be valid. Reverifying \texttt{transfer} for such cases, using the new specifications of methods of \texttt{PlusAccount} would solve this problem. However, reverifying all unchanged functions whenever new subtypes are added is not practical or desirable. A modular verification technique is needed to avoid reverification.
3.2 Overview of the Solution

Our approach to solving the specification problem is to require that behavioral subtypes provide a way to interpret the mathematical vocabulary of their supertypes. In this paper the interpretation is given by specifying simulation function, which are mappings from the abstract values of a subtype to those of its supertypes. The use of such functions dates back to Hoare’s work [8], and has been used in the context of subtyping by other authors [17, 2, 1, 14].

If all the subtypes used in a program are behavioral subtypes, the technique of supertype abstraction can be used for modular verification of OO programs [12]. Supertype abstraction uses static types of variables to reason about code and restricts the runtime types of variables to be behavioral subtypes of the static types. Such verification is valid because of the semantic conditions imposed on behavioral subtypes, which constitute an additional proof obligation. When new behavioral subtypes are added such a technique does not require reverification, because subtype objects behave like supertype objects.

In this paper we illustrate how these two techniques are combined in Larch/C++ to give a semantics for specification inheritance that forces subtypes to be behavioral subtypes (following [21]). We also define a new, weaker notion of behavioral subtyping for mutable types, which has advantages over Liskov and Wing’s definitions [14].

In the next section we discuss different notions of behavioral subtyping. In Section 5 we describe semantics of specification inheritance and show how specification inheritance forces behavioral subtyping. Section 6 discusses our techniques, and the last section presents our conclusions.

4 Behavioral Subtyping

In contrast to structural subtyping [3], behavioral subtyping should be based on both the syntax and the semantics of types. That is, behavioral subtyping is a property that relates type specifications. Syntactic constraints, as in structural subtyping, ensure that an expression of a subtype can be used in place of an expression of its supertype without any type error. Semantic constraints ensure that subtype objects behave like supertype objects; that is,
the use of subtype objects in place of supertype objects does not produce any unexpected behavior. Expected behavior is characterized by the supertype’s specification.

To define behavioral subtyping, we use the following notation. We use strong behavioral subtyping, \( \leq_S \), to refer to a notion similar to Liskov and Wing’s constraint-based behavioral subtyping [14] and weak behavioral subtyping, \( \leq_W \), to refer to our new, weaker notion of behavioral subtyping. Type symbols are represented by \( S, T \) and type vectors by \( U, V \). A simulation function from type \( S \) to \( T \) is denoted by \( c_{S \rightarrow T} \), an invariant of a type \( T \), by \( I_T \), and a constraint of \( T \), by \( C_T \). The notation \( \text{pre}_T^p(\text{self}, \vec{x}) \) denotes the precondition of method \( m \) in \( T \), with receiver \( \text{self} \) and additional parameters \( \vec{x} \). Substituting \( z \) for \( y \) in predicate \( p(y) \) is written as \( p(z) \). We use \( x|s \) for the value of \( x \) in state \( s \), and \( x^\pre \) and \( x^\post \) for values of \( x \) in \( \pre \) and \( \post \) states respectively. (Hence \( x|\pre \) is equivalent to \( x^\pre \).)

We present the definitions of \( \leq_S \) and \( \leq_W \) in two steps. We first define pre-behavioral subtyping, which captures the common parts of these definitions. Then we define \( \leq_S \) and \( \leq_W \) using pre-behavioral subtyping, which highlights the differences between the two definitions. The definition of pre-behavioral subtyping uses ideas from [1, 14]. The definition is specialized for single dispatching languages like C++, Eiffel, and Smalltalk.

**Definition 4.1 (Pre-Behavioral Subtyping)** \( S \) is a pre-behavioral subtype of \( T \) with respect to a binary relation \( \leq \) on types if and only if the following properties are satisfied.

**Syntactic** For every method \( m \) in \( T \) there exists a method \( m \) in \( S \) such that the following hold.

- **Contravariance of arguments.** If the types of the additional arguments of \( m \) in \( S \) and \( T \) are \( U \) and \( V \) respectively, then \( |U| \leq |V| \land (\forall i. V_i \leq U_i) \).

- **Covariance of result.** If the result types of \( m \) in \( S \) and \( T \) are \( U_r \) and \( V_r \) respectively, then \( U_r \leq V_r \).

- **Exception rule** For each exception \( e : E_S \) of \( m \) in \( S \), \( m \) in \( T \) has an exception \( e : E_T \), and \( E_S \leq E_T \).

**Semantic** There exists a family of simulation functions, \( \{c_{X \rightarrow Y} : X \leq Y\} \), such that the following hold.
• Invariant rule. For all values $v_S$ of $S$, $I_S(v_S) \Rightarrow I_T(c_{S\rightarrow T}(v_S))$

• Methods rule. For all common methods $m$, if the types of the additional arguments types of $m$ in $S$ and $T$ are $\bar{U}$ and $\bar{V}$ respectively and the result types of $m$ in $S$ and $T$ are $U_r$ and $V_r$ respectively, then for all objects $self : S$, $\bar{y} : \bar{V}$, and $result : U_r$

  – Precondition rule.
    \[
    prec_T^m(c_{S\rightarrow T}(self^*), \bar{y}^*) \Rightarrow prec_S^m(self^*, c_{V\rightarrow U}(\bar{y}^*)),
    \]

  – Postcondition rule.
    \[
    (prec_S^m(self^*, c_{V\rightarrow U}(\bar{y}^*)) \\
    \Rightarrow post_S^m(self^*, self', c_{V\rightarrow U}(\bar{y}''), result')) \\
    \Rightarrow (prec_T^m(c_{S\rightarrow T}(self^*), \bar{y}^*) \\
    \Rightarrow post_T^m(c_{S\rightarrow T}(self^*), c_{S\rightarrow T}(self'), \bar{y}'', c_{V_r\rightarrow U}(result')))
    \]

In C++ the additional arguments of a method must have the same types as in the corresponding method of the supertype, otherwise overloading instead of inheritance results. (Also, in C++ one can think of $self$ as $*\text{this}$.) Therefore, for C++ we can state the methods rule more simply as follows (compare [14, Figure 4]).

For all common virtual functions $m$, if the types of the additional arguments of $m$ in $S$ and $T$ are $\bar{V}$ and the result types of $m$ in $S$ and $T$ are $U_r$ and $V_r$ respectively, then for all objects $self : S$, $\bar{y} : \bar{V}$, and $result : U_r$

• Precondition rule.
  \[
  prec_T^m(c_{S\rightarrow T}(self^*), \bar{y}^*) \Rightarrow prec_S^m(self^*, \bar{y}^*),
  \]

• Postcondition rule.
  \[
  (prec_S^m(self^*, \bar{y}^*) \\
  \Rightarrow post_S^m(self^*, self', \bar{y}'', \bar{y}'', result')) \\
  \Rightarrow (prec_T^m(c_{S\rightarrow T}(self^*), \bar{y}^*) \\
  \Rightarrow post_T^m(c_{S\rightarrow T}(self^*), c_{S\rightarrow T}(self'), \bar{y}'', \bar{y}'', c_{V_r\rightarrow U}(result')))
  \]
An additional semantic condition on the history constraints of the types distinguishes between strong and weak behavioral subtyping. History constraints are introduced by Liskov and Wing in order to capture the properties of objects that are true in any execution history (which Liskov and Wing call a computation) [15]. For example, the history constraint of BankAccount specified in Figure 1 states that the name of the owner does not change in any computation.

4.1 Strong behavioral subtyping

The following definition is a modified version of Liskov and Wing’s definition [14, Figure 4]. The exception rule and the methods rule are changed from the original definition (see the section on related work below for details on the differences).

Definition 4.2 (Strong Behavioral Subtyping, \( \leq_s \)) \( S \leq_s T \) if \( S \) is a pre-behavioral subtype of \( T \) with respect to \( \leq_s \), and the following constraint rule is satisfied.

Semantic

- Constraint rule. For all valid computations, \( c \), for all states \( \text{pre} \) and \( \text{post} \) in \( c \) such that \( \text{pre} \) precedes \( \text{post} \), and for all objects \( \text{self}: S \),

\[
C_S(\text{self}\text{\pre}, \text{self}\text{\post}) \Rightarrow C_T(c_{S\rightarrow T}(\text{self}\text{\pre}), c_{S\rightarrow T}(\text{self}\text{\post})).
\]

Unlike Liskov and Wing’s definition, the exception rule for \( \leq_s \) allows subtype methods to return objects of a behavioral subtype of the supertype’s exception type. This change is necessary for languages like C++ where exceptions are allowed to return subtype objects\(^1\). For lack of space, the postcondition rule ignores exception results and their coercions; technically this is justified because the semantic conditions on exceptions are captured in the postcondition, and because one can model methods with exceptions by methods that return a tagged union of the normal result and exception results.

The syntactic conditions can be checked by the type system of a language. But the semantic conditions need to be verified and are generally beyond the power of most type systems to check. Several examples

---

\(^1\)Technically, in C++ one has to use a pointer or a reference.
of \( \leq_s \), like \texttt{Bag} \( \leq_s \texttt{PriorityQueue} \), are given by Liskov and Wing [14]. The \texttt{PlusAccount} referred in Section 3 is a strong behavioral subtype of \texttt{BankAccount}. A detailed discussion of that example is provided in the next section.

### 4.2 Weak behavioral subtyping

The constraint rule in the definition of strong behavioral subtyping requires even the extra mutators in subtypes to satisfy the history constraints of the supertype. The only kinds of mutations permitted by this requirement are those that are possible in the supertype or those that mutate extra state (state lost in simulation functions) in the subtype objects. For example, a type of immutable arrays cannot have any strong behavioral subtypes that allow changing elements.

However, when subtype objects are passed in place of supertypes, extra mutators in the subtypes are not visible, because subtype objects are viewed from supertype’s methods. If appropriate restrictions are placed on certain forms of aliasing (discussed below) then one can allow extra mutations in the subtype and can still expect subtype objects to behave like supertype objects when viewed through the supertype’s methods [6]. Since this notion allows more behavioral subtypes by weakening the constraint rule, it is called weak behavioral subtyping.

**Definition 4.3 (Weak behavioral subtyping, \( \leq_w \))** \( S \leq_w T \) if \( S \) is a pre-behavioral subtype of \( T \) with respect to \( \leq_w \), and the following constraint rule is satisfied.

**Semantic**

- Constraint rule. For all valid computations, \( c \), which do not invoke extra methods of \( S \), for all states \( \text{pre} \) and \( \text{post} \) in \( c \) such that \( \text{pre} \) precedes \( \text{post} \), and for all objects \( \text{self}: S \),

\[
C_S(\text{self}\backslash\text{pre}, \text{self}\backslash\text{post}) \Rightarrow C_T(c_{S\rightarrow T}(\text{self}\backslash\text{pre}), c_{S\rightarrow T}(\text{self}\backslash\text{post})).
\]

Another way of interpreting weak behavioral subtyping is to view the supertype’s history constraint as part of the postcondition of each of its methods. In such a case, when weak behavioral subtypes are specified, the
postconditions of the extra methods in the subtypes need not include the supertype’s history constraint.

The main drawback of weak behavioral subtyping is that a restriction on certain kinds of aliasing is required for modular verification. Since the supertype’s constraints might not be satisfied by the extra mutators of the subtype, manipulating an object simultaneously both from a supertype’s point of view and from a subtype’s point of view will result in an unexpected behavior. To avoid this, one must restrict direct aliasing between variables or objects of different types. For a detailed discussion on techniques to restrict such forms of aliasing and for a model theoretic definition of weak behavioral subtyping see [6]. Other forms of aliasing such as direct aliasing between variables and objects of the same type and indirect aliasing (that is aliasing of components) between variables and objects of different types are allowed. Whether such restrictions are too painful for OO programmers is an open problem.

Weak behavioral subtyping captures several useful behavioral subtype relationships not captured by strong behavioral subtyping, including mutable types that are weak behavioral subtypes of immutable types. One such example is that a mutable array type can be a subtype of an immutable array type, which allows one to pass a mutable array as an argument to a function, like finding the maximum array element, that expects an immutable array. Similarly, a mutable record type can be specified as a weak behavioral subtype of an immutable record type with fewer fields. One can have a hierarchy of weak behavioral subtypes with varying degrees of mutability [6].

5 Specification Inheritance

To prove a strong or a weak behavioral subtyping relation between two types, one needs to prove that the conditions of the appropriate definitions are met. In this section we show how specification inheritance can be used to force behavioral subtyping, eliminating the need for users of a specification language to verify behavioral subtyping by hand\(^2\). This idea is due to Wills [21] although apparently he allows one to escape the mechanism and still specify

\(^2\)If the specification given in the subtype contradicts a supertypes’ specification then the type may not be implementable, so some verification of implementability might still be useful.
#include "BankAccount.lcc"

class PlusAccount : public BankAccount {
public:
    uses PlusAccountTrait;
    simulates BankAccount by toBA;

    // constructor, pay_interest, deposit,
    // and credit_check are omitted.
    // specification of balance is inherited.
    virtual void withdraw(long int cts);

    behavior {
        let amt:Q be dollars(cts);
        requires 0 ≤ amt \land amt ≤ self^.savings;
        modifies self;
        ensures self^.checking = self^.checking;
    }
};

Figure 4: Interface Specification of PlusAccount

subtypes that are not behavioral subtypes.

5.1 Inheritance for strong behavioral subtyping

The specification of PlusAccount in Figures 4 and 5 gives an example of
specification inheritance. Figure 4 gives the interface specification. The C++
syntax :public BankAccount in the declaration of PlusAccount states that
it is a subtype of BankAccount. From the uses clause and the trait in Fig-
ure 5 one can see that the abstract values of PlusAccount objects are tuples
with savings, checking and owner components. The simulates clause states
that PlusAccount is a strong behavioral subtype of BankAccount (because
it is not written weakly simulates). It also says that the simulation func-
tion toBA is used to view PlusAccount values as BankAccount values. The
simulation function toBA is defined in PlusAccountTrait in Figure 5.
PlusAccountTrait: trait
    includes BankAccountTrait,
        NoContainedObjects(PlusAccount)
    PlusAccount tuple of savings:Q, checking:Q,
        owner:String[char]
    introduces
toBA: PlusAccount → BankAccount
    asserts ∀ q1,q2: Q, o: String[char]
    toBA([q1, q2, o]) == [q1+q2, o]

Figure 5: Trait for PlusAccountTrait

The specification in Figure 4 inherits its invariant, history constraint, and parts of method specifications from the specification in Figure 1. For example, the specification of withdraw is partly inherited and partly given in Figure 4. The specification of withdraw in Figure 4 only states that when the savings part is greater than the amount, then the checking part does not change. The inherited part of the specification states that the amount, if less than the PlusAccount's credit, is deducted from the PlusAccount's credit.

As an aid to explaining the semantics of specification inheritance, consider the completed specification of the withdraw method. Its precondition is formed as a disjunction of the added precondition in the subtype with the supertype's precondition, after coercing subtype values to supertype values. That is, the completed precondition for withdraw is:

\[
\text{requires } (0 \leq \text{dollars}(cts) \\
\quad \quad \wedge \text{dollars}(cts) \leq \text{toBA(self').credit}) \\
\quad \quad \vee (0 \leq \text{amt} \wedge \text{amt} \leq \text{self'.savings});
\]

The first disjunct is inherited from BankAccount, and the second disjunct is the added precondition in PlusAccount. The modifies clause of the subtype's method lists all the objects in the supertype's and the subtype's modifies clauses. The completed modifies clause of withdraw is:

\[
\text{modifies self};
\]
Its postcondition is formed as a conjunction of two implications. The first is that the subtype’s added precondition implies the subtype’s added postcondition. The second is that, after coercion, the supertype’s precondition implies the supertype’s precondition. That is, the completed postcondition for *withdraw* is:

\[
\text{ensures } (0 < \text{dollars}(\text{cts}) \\
\quad \quad \land \text{dollars}(\text{cts}) \leq \text{toBA}(\text{self}').\text{credit}) \\
\quad \quad \Rightarrow (\text{toBA}(\text{self}')) = \\
\quad \quad \quad [\text{toBA}(\text{self}').\text{credit} - \text{dollars}(\text{cts}), \\
\quad \quad \quad \text{toBA}(\text{self}').\text{owner}]) \\
\quad \quad \land (0 \leq \text{dollars}(\text{cts}) \land \text{amt} \leq \text{self}'.\text{savings}) \\
\quad \quad \Rightarrow (\text{self}'.\text{checking} = \text{self}'.\text{checking});
\]

In general, the modifies clause of the completed specification is larger than the modifies clause of the specification being inherited. In such cases the consequent of each implication in the completed specification’s postcondition asserts that the objects that are not in the corresponding modifies clause are unchanged. (See [11] for details.)

The completed specification’s invariant is formed as a conjunction of the supertype’s invariant (with appropriate coercions) and subtype’s invariant. The completed specification’s history constraint is a conjunction of the supertype’s constraint (with appropriate coercions) and the added constraint on the subtype. For example, the completed constraint for *PlusAccount* is:

\[
\text{constraint } \text{toBA}(\text{self}').\text{owner} = \text{toBA}(\text{self}').\text{owner};
\]

The following definition generalizes these notions for multiple supertypes. The notation used below is as follows. The set of all supertypes of a type *S* is given by *Sups(S)* and the set of all methods of a type *T* is given by *meths(T)*. The predicates *added_sm*, *added_pr_em*, and *added_ps_em* are, respectively, the added predicates for invariant of *S* and the pre- and postconditions of a method *m* in *S* respectively.

**Definition 5.1 (Specification inheritance)** Let *S* be specified as a subtype of all the types in *Sups(S)*. Let \( \langle e_{X \rightarrow Y} : X \leq Y \rangle \) be the specified family of simulation functions. The completed specification of *S* is:
Invariant $I_S(v)$ is:

$$added_I_S(v) \land \left( \bigwedge_{T \in \text{Sups}(S)} I_T(c_{S\rightarrow T}(v)) \right).$$

Precondition for all methods $m$ of $S$, $pre^m_S(self^*, \bar{x}^*)$ is:

$$added_{pre}^m_S(self^*, \bar{x}^*)$$

$$\bigvee \left( \bigvee_{T \in \text{Sups}(S), \ m \in \text{meths}(T)} (\exists \bar{z}^*: \bar{V}.c^m_{T\rightarrow V}(\bar{z}^*) = \bar{x}^* \land pre^m_T(c_{S\rightarrow T}(self^*), \bar{z}^*)) \right).$$

Postcondition for all methods $m$ of $S$, $post^m_S(self^*, self^\prime, \bar{x}^*, \bar{x}^\prime, \text{result}^\prime)$ is:

$$(added_{pre}^m_S(self^*, \bar{x}^*) \Rightarrow added_{post}^m_S(self^*, self^\prime, \bar{x}^*, \bar{x}^\prime, \text{result}^\prime))$$

$$\land \left( \bigwedge_{T \in \text{Sups}(S), \ m \in \text{meths}(T)} \begin{cases} (\forall \bar{z}^*: \bar{V}.c^m_{T\rightarrow V}(\bar{z}^*) = \bar{x}^* \land c^m_{T\rightarrow V}(\bar{z}^\prime) = \bar{x}^\prime) \\ \Rightarrow (pre^m_T(c_{S\rightarrow T}(self^*), \bar{z}^*)) \Rightarrow (post^m_T(c_{S\rightarrow T}(self^*), c_{S\rightarrow T}(self^\prime), \bar{z}^*, \bar{z}^\prime, c_{U\rightarrow V}(\text{result}^\prime)))) \end{cases} \right).$$

Constraint for all states pre, post, $C_S$ is: for all objects $self: S$

$$added_{C_S}(self\backslash pre, self\backslash post)$$

$$\land \left( \bigwedge_{T \in \text{Sups}(S)} C_T(c_{S\rightarrow T}(self\backslash pre), c_{S\rightarrow T}(self\backslash post)) \right).$$

We can simplify the precondition and the postcondition rules for C++, because additional arguments to inherited methods cannot have different types. The precondition rule for specification inheritance in Larch/C++ is given below.
for all common virtual member functions \( m \) of \( S \), \( \text{pre}^m_S (\text{self}', \vec{x}') \) is:

\[
\text{added-pre}^m_S (\text{self}', \vec{x}') \lor \bigg( \bigvee_{T \in \text{Supps}(S), \ m \in \text{meths}(T)} \text{pre}^m_T (c_{S-T}(\text{self}'), \vec{x}') \bigg).
\]

The postcondition rule is:

for all common virtual member functions \( m \) of \( S \), \( \text{post}^m_S (\text{self}', \text{self}', \vec{x}', \vec{x}', \text{result}') \) is:

\[
(\text{added-pre}^m_S (\text{self}', \vec{x}') \Rightarrow \text{added-post}^m_S (\text{self}', \text{self}', \vec{x}', \vec{x}', \text{result}'))
\]

\[
\land \bigg( \bigwedge_{T \in \text{Supps}(S), \ m \in \text{meths}(T)} \text{pre}^m_T (c_{S-T}(\text{self}'), \vec{x}') \Rightarrow (\text{post}^m_T (c_{S-T}(\text{self}'), c_{S-T}(\text{self}'), \vec{x}', \vec{x}', c_{U-V}(\text{result}'))) \bigg).
\]

Since message passing in C++[20] is dynamic only for the virtual member functions, the semantics of specification inheritance in Larch/C++ applies the above rules only for the virtual member functions of \( S \). As an example, the inherited specification of balance in PlusAccount is as follows:

```plaintext
virtual long int balance() const;
behavior {
   ensures approx_equal(dollars(results), toBA(self').credit);
}
```

For some methods, such as withdraw, the completed specification obtained by using specification inheritance is difficult to understand (see above). Separating the inherited part from the added part, we believe, enhances the readability of completed specifications and serves as an aid to understanding. This separation can be achieved by using case-analysis [21]. Case-analysis is a syntactic sugar used in method specifications. For example, Figure 6 gives the completed specification of withdraw in PlusAccount using case-analysis.
virtual void withdraw(long int cts);
behavior {
let amt:Q be dollars(cts);
requires 0 \leq \text{amt} \land \text{amt} \leq \text{self}.\text{savings};
modifies self;
ensures \text{self}'.\text{checking} = \text{self}'.\text{checking};
ensures 0 \leq \text{dollars(cts)}
\land \text{dollars(cts)} \leq \text{toBA(self').credit};
modifies self;
requires \text{self}' =
\begin{array}{l}
\text{toBA(self').credit} - \text{dollars(cts)},
\text{toBA(self').owner};
\end{array}
}

Figure 6: Completed specification of withdraw in PlusAccount

The body of the specification contains two parts. The first is the added specification and the second is the inherited specification. The semantics is that an implementation must satisfy both parts. Therefore, the meaning is the same as before. However, this form of specification clearly shows that behavioral subtypes must satisfy their supertypes’ specifications. Thus, we believe this form of specification would be useful in specification browsers.

The following theorem ties specification inheritance to strong behavioral subtyping.

**Theorem 5.2** Each type specified as a subtype using specification inheritance is a strong behavioral subtype of its supertypes.

*Proof:* Suppose $S$ is specified as a subtype of $T$. Since the syntactic hold because $S$ is a subtype of $T$, we check the semantic conditions as follows.

**Invariant** For all $v : S$ we calculate as follows.
\[
I_S(v)
= \{ \text{by specification inheritance and definition of invariant} \}
\]
\[
\text{added}_I_S(v) \land I_T(c_{S \to T}(v))
\]
\[
\Rightarrow \{ \text{by } A \land B \Rightarrow B \}
I_T(c_{S \to T}(v))
\]

**Constraint** For all valid computations \( c \), for all states \( \text{pre} \) and \( \text{post} \) in \( c \) such that \( \text{pre} \) precedes \( \text{post} \), and for all objects \( \text{self} : S \),

\[
C_S(\text{self} \setminus \text{pre}, \text{self} \setminus \text{post})
= \{ \text{by definition of specification inheritance} \}
\]
\[
\text{added}_C_S(\text{self} \setminus \text{pre}, \text{self} \setminus \text{post})
\land C_T(c_{S \to T}(\text{self} \setminus \text{pre}), c_{S \to T}(\text{self} \setminus \text{post}))
\]
\[
\Rightarrow \{ \text{by } A \land B \Rightarrow B \}
C_T(c_{S \to T}(\text{self} \setminus \text{pre}), c_{S \to T}(\text{self} \setminus \text{post}))
\]

**Methods** The following calculation uses the precondition rule of specification inheritance to show the precondition rule of behavioral subtyping. Let \( m \) be a common method in \( S \) and \( T \), for all objects \( \text{self} : S \), and \( \vec{y} : \vec{V} \), and \( \text{result} : U_r \).

\[
\text{pre}_T^m(c_{S \to T}(\text{self}^r), \vec{y}^r)
= \{ \text{by } A \equiv (\text{true} \land A) \}
\]
\[
c_{\vec{V} \to \overrightarrow{U}}(\vec{y}^r) = c_{\vec{V} \to \overrightarrow{U}}(\vec{y}^r) \land \text{pre}_T^m(c_{S \to T}(\text{self}^r), \vec{y}^r)
\]
\[
\Rightarrow \{ \exists \vec{z} : \vec{V}, c_{\vec{V} \to \overrightarrow{U}}(\vec{z}^r) = c_{\vec{V} \to \overrightarrow{U}}(\vec{y}^r) \land \text{pre}_T^m(c_{S \to T}(\text{self}^r), \vec{z}^r) \}
\]
\[
\Rightarrow \{ \text{by } B \Rightarrow (A \lor B) \}
\text{added}_C^m(\text{self}^r, c_{\vec{V} \to \overrightarrow{U}}(\vec{y}^r))
\]
\[
\lor \{ \exists \vec{z} : \vec{V}, c_{\vec{V} \to \overrightarrow{U}}(\vec{z}^r) = c_{\vec{V} \to \overrightarrow{U}}(\vec{y}^r) \land \text{pre}_T^m(c_{S \to T}(\text{self}^r), \vec{z}^r) \}
\]
\[
\Rightarrow \{ \text{by definition of specification inheritance} \}
\text{pre}_S^m(\text{self}^r, c_{\vec{V} \to \overrightarrow{U}}(\vec{y}^r))
\]

The following calculation uses postcondition rule of specification inheritance to complete the proof of methods rule. Let \( m \) be a common method in \( S \) and \( T \), for all objects \( \text{self} : S \) and, \( \vec{y} : \vec{V} \), and \( \text{result} : U_r \).

\[
\text{pre}_S^m(\text{self}^r, c_{\vec{V} \to \overrightarrow{U}}(\vec{y}^r))
\Rightarrow \text{post}_S^m(\text{self}^r, \text{self}^r, c_{\vec{V} \to \overrightarrow{U}}(\vec{y}^r), c_{\vec{V} \to \overrightarrow{U}}(\vec{y}^r), \text{result}^r)
\[= \{ \text{by definition of specification inheritance} \}
\]
\[
(\text{added}_{\preceq}^m (self^*, c_{\varphi - \ell}(\bar{y}^*)))
\]
\[
\vee \ (\exists z : \tilde{V} : c_{\varphi - \ell}(\bar{z}^*) = c_{\varphi - \ell}(\bar{y}^*) \land \text{pre}_{T}^m (c_{S - T}(self^*), \bar{z}^*))
\]
\[
\Rightarrow (\text{added}_{\preceq}^m (self^*, c_{\varphi - \ell}(\bar{y}^*)))
\]
\[
\Rightarrow \text{added}_{\post}^m (self^*, self', c_{\varphi - \ell}(\bar{y}^*), c_{\varphi - \ell}(\bar{y}'), \text{result}')
\]
\[
\land \ (\forall z : \tilde{V} : c_{\varphi - \ell}(\bar{z}^*) = c_{\varphi - \ell}(\bar{y}^*) \land c_{\varphi - \ell}(\bar{z}') = c_{\varphi - \ell}(\bar{y}')
\]
\[
\Rightarrow (\text{pre}_{T}^m (c_{S - T}(self^*), \bar{z}^*))
\]
\[
\Rightarrow (\text{post}_{T}^m (c_{S - T}(self^*), c_{S - T}(self'), \bar{z}^*, \bar{z}', c_{V - V}(\text{result}')))
\]
\[
\Rightarrow \{ \text{by } A \land B \Rightarrow B \}
\]
\[
(\text{added}_{\preceq}^m (self^*, c_{\varphi - \ell}(\bar{y}^*)))
\]
\[
\vee \ (\exists z : \tilde{V} : c_{\varphi - \ell}(\bar{z}^*) = c_{\varphi - \ell}(\bar{y}^*) \land \text{pre}_{T}^m (c_{S - T}(self^*), \bar{z}^*))
\]
\[
\Rightarrow (\forall z : \tilde{V} : c_{\varphi - \ell}(\bar{z}^*) = c_{\varphi - \ell}(\bar{y}^*) \land c_{\varphi - \ell}(\bar{z}') = c_{\varphi - \ell}(\bar{y}')
\]
\[
\Rightarrow (\text{pre}_{T}^m (c_{S - T}(self^*), \bar{z}^*))
\]
\[
\Rightarrow (\text{post}_{T}^m (c_{S - T}(self^*), c_{S - T}(self'), \bar{z}^*, \bar{z}', c_{V - V}(\text{result}')))
\]
\[
\Rightarrow \{ \text{by } (A \Rightarrow B) \land (B \Rightarrow C) \Rightarrow (A \Rightarrow C) \}, \text{where } A \text{ is the first line below, } B \text{ is the first line in the formula above, and } A \Rightarrow B \text{ by } \exists-1\}
\]
\[
(c_{\varphi - \ell}(\bar{y}^*) = c_{\varphi - \ell}(\bar{y}^*) \land \text{pre}_{T}^m (c_{S - T}(self^*), \bar{y}^*))
\]
\[
\Rightarrow (\forall z : \tilde{V} : c_{\varphi - \ell}(\bar{z}^*) = c_{\varphi - \ell}(\bar{y}^*) \land c_{\varphi - \ell}(\bar{z}') = c_{\varphi - \ell}(\bar{y}')
\]
\[
\Rightarrow (\text{pre}_{T}^m (c_{S - T}(self^*), \bar{z}^*))
\]
\[
\Rightarrow (\text{post}_{T}^m (c_{S - T}(self^*), c_{S - T}(self'), \bar{z}^*, \bar{z}', c_{V - V}(\text{result}')))
\]
\[
= \{ \text{by } (\text{true } \land A) \equiv A \}
\]
\[
\text{pre}_{T}^m (c_{S - T}(self^*), \bar{y}^*)
\]
\[
\Rightarrow (\forall z : \tilde{V} : c_{\varphi - \ell}(\bar{z}^*) = c_{\varphi - \ell}(\bar{y}^*) \land c_{\varphi - \ell}(\bar{z}') = c_{\varphi - \ell}(\bar{y}')
\]
\[
\Rightarrow (\text{pre}_{T}^m (c_{S - T}(self^*), \bar{z}^*))
\]
\[
\Rightarrow (\text{post}_{T}^m (c_{S - T}(self^*), c_{S - T}(self'), \bar{z}^*, \bar{z}', c_{V - V}(\text{result}')))
\]
The significance of this theorem is that strong behavioral subtyping is automatic for types specified using specification inheritance. From this theorem we can conclude that `PlusAccount` is a strong behavioral subtype of `BankAccount`.

### 5.2 Inheritance for weak behavioral subtyping

Figure 7 gives the specification (using specification inheritance) of a type `MutableAccount`. The `weakly simulates` clause states that `MutableAccount` is intended to be a weak behavioral subtype of `BankAccount`. If we use the specification inheritance rules discussed above we would inherit the history constraint of `BankAccount`, and would apply it for all the methods of `MutableAccount`. Since `changeOwner` violates this history constraint, a different rule is needed for inheritance of history constraints to make weak behavioral subtypes.

For weak behavioral subtypes the inherited history constraint is applied only to the common methods. This condition allows the extra methods in the subtype, such as `changeName`, to mutate the state in a way that is not possible in the supertype. For `MutableAccount` the inherited constraint is given below. (The syntax "for ...." indicates to which methods the constraint applies.)
#include "BankAccount.lcc"

class MutableAccount : public BankAccount {
public:
    uses BankAccountTrait(MutableAccount for BankAccount);
    weakly simulates BankAccount by toBAwithoutChange;

    // constructor omitted
    // balance and withdraw are inherited

    virtual void change_name(const char *name);

    behavior {
        uses cpp_const_char_string;
        requires nullTerminated(name, pre);
        modifies self;
        ensures self' = [self^credit, uptoNull(name, pre)];
    }
};

Figure 7: MutableAccount as a weak behavioral subtype of BankAccount
constraint identity(self).name = identity(self').name
for virtual long int balance(),
  virtual void withdraw(long int cts),
  virtual void pay_interest(double rate),
  virtual deposit(long int cts);

The proof of the following theorem is essentially the same as for the previous theorem.

**Theorem 5.3** Each type specified as a weak behavioral subtype using specification inheritance is a weak behavioral subtype of its supertypes.

6 Discussion

In this section we compare our work on behavioral subtyping and specification with other related work and also discuss issues in specification inheritance.

6.1 Related work

The important difference between our work and Liskov and Wing’s work [14] is the new definition of weak behavioral subtyping. However we also refined their definition of strong behavioral subtyping. These refinements include changes to the exception rule, handling additional arguments in the methods rule, and generalizing the post-condition rule. The change in the exception rule is necessary to handle the case when the subtype objects are passed as exception results. But the change in the post-condition rule allows subtype methods to operate outside the domain of the supertype methods. For example consider the specification of a method given in both a supertype and a subtype.

```c
virtual int foo(int x);  // supertype’s specification
behavior {
  requires x > 0;
  ensures result > 0;
}

virtual int foo(int x);  // subtype’s specification
behavior {
```

23
requires x \leq 0;
enhances result = -3;
}

By our semantics of specification inheritance, the completed specification has the following postcondition.

\[(x > 0) \Rightarrow (result > 0) \land (x \leq 0) \Rightarrow (result = -3)\]

This does not imply the supertype’s postcondition. However, when one reasons about an invocation of \texttt{foo} on an object whose static type is the supertype, the subtype’s \texttt{foo} performs adequately (without surprises). Hence their original rule is needlessly strong. Our definition of strong behavioral subtyping permits more strong behavioral subtype relationships, and gives the specifier more flexibility. Therefore although such a method specification could be used in a strong behavioral subtype according to our definition, it would not yield a strong behavioral subtype according to Liskov and Wing’s original definition.

Cusack [5] defines specialization, which is like behavioral subtyping. Specialization does not handle subtyping on additional arguments and does not have any notion of history constraints. As in our work, her definition uses a simulation function and does not constrain a subtype’s method outside the domain of the corresponding method of a supertype. Her technique for deriving one specification from another is more restrictive than ours, in that she requires both subtype and supertype to be specified with the same Z schema. Her technique for inheritance of specifications does not allow one to add behavior incrementally.

Eiffel also attempts to force behavioral subtyping through specification inheritance [16, Section 10.15]. Invariants, pre- and postconditions are inherited. (Eiffel’s assertion sublanguage has no support for frame axioms like the modifies clause in Larch/C++.) The keyword “else” is used at the start of a precondition to indicate that the completed precondition is the disjunction of the supertype’s precondition and the one stated. Similarly the keyword “then” is used in the postcondition to indicate that the completed postcondition is the conjunction of the supertype’s postcondition and the one stated. This rule for inheriting postconditions is the source of the postcondition rule in Liskov and Wing’s definition. As such our rule is more general. Further,
by allowing covariant arguments to methods, Eiffel violates the contravariant rule for behavioral subtyping. Therefore, although Eiffel contains the basic idea, it does not force behavioral subtyping. In Eiffel there is no need for simulation functions in inherited specifications, as the assertion sublanguage is polymorphic by virtue of using Eiffel subexpressions. Eiffel's syntax provides no support for case-analysis in method specifications.

The work of Wills in Fresco [21] is most closely related to ours. Capsules in Fresco support the idea of case-analysis - all the specification capsules for a given method must be satisfied by that method. Wills has no way to write his “retrieval relations” into specifications of subtypes, however, making it difficult to apply the specification of supertypes to subtypes unless the subtype has the same instance variables. Wills also does not force behavioral subtyping, as he allows users to escape from specification inheritance if desired.

6.2 Specification and verification

To solve the specification problem for the transfer function discussed in Section 3, one can use a technique similar to the one used in specification inheritance. Whenever a subtype of BankAccount is passed as an argument to transfer, the object’s abstract value is coerced to a BankAccount abstract value using a simulation function. The vocabulary of BankAccount is then used to interpret the specification. For example, when PlusAccount object is passed to transfer one would coerce its value using toBA. However, there remains a problem of information loss with this technique. That is, the specification of transfer does not say how the amount transferred is distributed between checking and savings.

Since PlusAccount is a strong behavioral subtype of BankAccount, all the properties that are true for BankAccount objects are true for PlusAccount objects. The verification of transfer (done before PlusAccount was added) is valid even for PlusAccount objects passed to transfer. Reverification is not required.

However, in the case of weak behavioral subtypes, all the properties of the supertype are satisfied by weak behavioral subtype objects only when viewed as a supertype object. To avoid reverification, the programming method or verification logic must prevent aliases that allow a weak behavioral subtype object to be viewed both as a subtype and as a supertype. If that is done,
then reverification for *MutableAccount* arguments is not needed.

A problematic feature of OO programming is the pervasiveness of objects. One can have abstract values that contain objects. Therefore, in general, simulation functions on values alone are not sufficient for an OO setting. One needs, at least, to give simulation functions access to the state functions, which map objects to their values (and so model a computer’s memory). As an example, consider refined specifications of *PlusAccount* and *BankAccount* where one uses variables to document a design decision. Since the variable can be mutated, the abstract values contain objects that model them. Such abstract values for *BankAccount* and *PlusAccount* might look like the following (where `Obj[int]` represents an integer variable).

```plaintext
BankAccount tuple of dollars: Obj[int], cents: Obj[int],
    owner: String[char]
PlusAccount tuple of svgs_dlr: Obj[int], svgs_cents: Obj[int],
    chkg_dlr: Obj[int], chkg_cents: Obj[int],
    owner: String[char]
```

A simulation function from the abstract values of *BankAccount* to the abstract values of *PlusAccount* is not enough, because a state is required to get the values inside objects such as `svgs_dlr`. Thus one needs simulation functions to map states with subtype values to states with supertype values. Such a function should preserve aliasing. However, given the incremental nature of OO software development, specifying simulation functions from states to states is not practical. Techniques are needed to construct such simulation functions.

In this paper we have used simulation functions, which are convenient in formulas. However, in general, one needs relations instead of functions [18].

7 Conclusions

The main contributions of this paper are a modular specification technique which forces behavioral subtyping and a new, weaker notion of behavioral subtyping. While the semantics of behavioral subtyping may seem somewhat intricate, the basic idea is that the subtype must satisfy the supertype’s specifications. This is enforced by our semantics of specification inheritance, and made visible by the case-analysis form of the completed specification.
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References


