Supplement to Homework 3: Data Flow Analysis

Here's an example that Neeraj and I discussed, which may be helpful in working the problems in section 1.7. I use the calculational style (as in the handouts we provided in class [1, Section 4.2] [2, Chapter 4], see also Gries's article in *CACM* [3]), to justify each step.

Consider the following equations among sets drawn from a universe U, where $a, b \in U$.

$$\begin{array}{lcl} X_1 & = & \{a\} \\ X_2 & = & \{b\} \cup X_1 \cup X_2 \end{array}$$

We can represent these equations as a function F as follows.

$$\begin{array}{lcl} F & : & (U \times U) \to (U \times U) \\ F(u_1, u_2) & = & (F_1(u_1, u_2), F_2(u_1, u_2)) \\ F_1(u_1, u_2) & = & \{a\} \\ F_2(u_1, u_2) & = & \{b\} \cup u_1 \cup u_2 \end{array}$$

We wish to solve these using Chaotic Iteration. Let us represent the steps of the Chaotic Iteration algorithm using the symbol $\leadsto$. Writing $\vec{u} = (x, y)$ to mean $u_1 = x \wedge u_2 = y$ (because when dealing with 12-tuples it will be convenient to have a smaller formula), we can calculate as follows.

$\vec{u} = (\{\}, \{\})$
$\leadsto$ $\langle$by definition of F_2, $u_2 \neq F_2(\{\}, \{\})\rangle$
$\vec{u} = (\{\}, \{b\})$
$\leadsto$ $\langle$by definition of $F_1\rangle$
$\vec{u} = (\{a\}, \{b\})$
$\leadsto$ $\langle$by definition of $F_2\rangle$
$\vec{u} = (\{a\}, \{a, b\})$

At this point no more steps are possible, so the fixed point of F is $\vec{u} = (\{a\}, \{a, b\})$, i.e.,

$$\begin{array}{lcl} X_1 & = & \{a\} \\ X_2 & = & \{a, b\} \end{array}$$

is a solution to the equations above.

References

[1] Ralph-Johan Back and Joakim von Wright. *Refinement Calculus: A Systematic Introduction*. Graduate Texts in Computer Science. Springer-Verlag, 1998.

[2] Edsger W. Dijkstra and Carel S. Scholten. *Predicate Calculus and program semantics*. Springer-Verlag, NY, 1990.

[3] David Gries. Teaching calculation and discrimination: A more effective curriculum. *Communications of the ACM*, 34(3):44–55, March 1991.