



1. (10 points) [Concepts] [UseModels] In Erlang, using `lists:foldr/3`, write a function `lengths/1`, whose type is given by the following.

```
-spec lengths(LL :: [[any()]]) -> [non_neg_integer()].
```

This function takes a list of lists, `LL`, as an argument and returns a list of the lengths of each of the sublists of `LL`, in order. The following are examples written using the testing module.

```
% $Id: lengths_tests.erl,v 1.1 2015/04/16 11:25:27 leavens Exp leavens $
-module(lengths_tests).
-import(lengths,[lengths/1]).
-import(testing,[dotests/2,eqTest/3]).
-export([main/0]).
main() -> compile:file(lengths),
        dotests("lengths_tests $Revision: 1.1 $", tests()).
tests() ->
    [eqTest(lengths([]),"==",[1]),
      eqTest(lengths([[]]),"==",[0]),
      eqTest(lengths([[4020]]),"==",[1]),
      eqTest(lengths([[], [7], [9,2], [5,1,5], [9,6,3,2]]),"==",[0,1,2,3,4]),
      eqTest(lengths([[9,8,7,6,4,5,3,2,1],[6,6,7,8,3]]),"==",[9,5]),
      eqTest(lengths([[2,4,6,8],[who,do,we,apprec,i,ate],[erlang]]),"==",[4,6,1]),
      eqTest(lengths([[today,all,'of',us,who,know],[go,to,the],
                    [store,for,shellfish],[its,good]]),"==",[6,3,3,2])    ].
```

Your solution must use `lists:foldr/3` in an essential way, so you must write it by filling in the remainder of the following. You must not use explicit recursion or a list comprehension in your solution. However, you may use helping functions and built-in functions.

```
lengths(LL) ->
    lists:foldr(
```

2. (10 points) [Concepts] [UseModels] In Erlang, using `lists:foldr/3`, write a function `count/2`, whose type is given by the following.

```
-spec count(What :: atom(), LOA :: [atom()]) -> pos_integer().
```

This function takes an atom, `What`, and a list of atoms, `LOA`, and returns the number of times that `What` occurs within `LOA`. The following are examples written using the testing module.

```
% $Id: count_tests.erl,v 1.1 2015/04/16 11:25:27 leavens Exp leavens $
-module(count_tests).
-import(count,[count/2]).
-import(testing,[dotests/2,eqTest/3]).
-export([main/0]).
main() -> compile:file(count),
        dotests("count_tests $Revision: 1.1 $", tests()).
tests() ->
    [eqTest(count(maple, []),"=",0),
     eqTest(count(maple, [the,maple,tree,makes,maple,syrup]),"=",2),
     eqTest(count(a, [a,ah,the,ah,a,fine,thing,to,do,is,to,get,an,a,eh,a]),"=",4),
     eqTest(count(secret, [secret,house,secret,storm,secret,secret,secret]),"=",5),
     eqTest(count(ucf, [ucf,second,largest,hitt,pres,ucf,at,ucf,ok]),"=",3) ].
```

Your solution must use `lists:foldr/3` in an essential way, so you must write it by filling in the remainder of the following. You must not use explicit recursion or a list comprehension in your solution. However, you may use helping functions and built-in functions.

```
count(What, LOA) ->
    lists:foldr(
```

## Text Data Type Problems

The next few problems are work with the type `text()` and the type `formatted()` exported by the module `text` found in Figure 1 below.

```
-module(text).
-export_type([face/0, formatted/0, text/0]).
-export([fs/1, ts/1]).
% A face is the way a charater looks
-type face() :: roman | bold | italic.
% A formatted is a pair of a face and a string (= list of characters)
-type formatted() :: {face(), string()}.
% A text can be ...
-type text() :: {fs, [formatted()]}      % a list of formatted strings
              | {ts, [text()]}.        % or a list of texts.

% constructor functions that you can use if you wish
-spec fs([formatted()]) -> text().
fs(LS) -> {fs, LS}.
-spec ts([text()]) -> text().
ts(LS) -> {ts, LS}.
```

Figure 1: The module `text`, used in the next few problems.

The type `formatted` is intended to model formatted text, that can appear in a roman, **bold**, or *italic* typeface. The type `text` can either be a list of formatted strings or a list of texts.

It may be helpful to recall that a string in Erlang is a list of characters.

3. (15 points) [UseModels] This is a problem about the `text:text()` type in Figure 1 on the previous page. In Erlang, write a function, `chars`, whose type is given by the following:

```
-spec chars(Txt :: text:text()) -> string().
```

This function takes a text, `Txt`, as an input and returns the string of all the characters that appear in the strings of `Txt`, in the left to right order in which they appear in the argument. Be sure to follow the grammar given by the types in Figure 1 on the preceding page! The following are tests.

```
-module(chars_tests).
-import(testing,[dotests/2,eqTest/3]).
-export([main/0, tests/1]).
main() -> compile:file(chars),
    dotests("chars_tests $Revision: 1.1 $", tests(fun chars:chars/1)).
-spec tests(CFun :: fun((text:text()) -> string()))
    -> testing:testCase([string()]).
tests(CFun) ->
    [eqTest(CFun({fs, []}), "==", ""),
     eqTest(CFun({ts, []}), "==", ""),
     eqTest(CFun({fs, [{bold,"are changin'"}]}), "==", "are changin'"),
     eqTest(CFun({fs, [{roman, "the times"},{bold," are changin'"}]}), "==",
        "the times are changin'"),
     eqTest(CFun({ts, [{fs, [{italic, "the times"}]},
        {fs, [{bold, " try "}, {italic, "our souls"}]}]}),
        "==", "the times try our souls"),
     eqTest(CFun({ts, [{ts, [{fs, [{italic, "the times"}]},
        {fs, [{bold, " try "}, {italic, "our souls"}]}]},
        {ts, [{fs, [{roman, " and these tests"}]},
        {fs, [{italic, " try "}, {roman, "our patience"}]}]}]}),
        "==", "the times try our souls and these tests try our patience"),
     eqTest(CFun({ts, [{ts, [{ts, [{ts, [{ts, [{fs, [{italic, "deep!"}]}]}]}]}]}]}),
        "==", "deep!").
```

4. (15 points) [UseModels] This is a problem about the `text:text()` type in Figure 1 on page 4. In Erlang, write a function, `embolden`, whose type is given by the following:

```
-spec embolden(T :: text:text()) -> text:text().
```

This function takes a text, `T`, as an input and returns a text that is just like `T` but in which each formatted string occurring in it is made bold. Be sure to follow the grammar given by the types in Figure 1 on page 4! The following are tests.

```
-module(embolden_tests).
-import(testing,[dotests/2,eqTest/3]).
-export([main/0, tests/1]).
main() -> compile:file(embolden),
  dotests("embolden_tests $Revision: 1.1 $" , tests(fun embolden:embolden/1)).
-spec tests(EBFun :: fun((text:text()) -> text:text()))
  -> testing:testCase([text:text()]).
tests(EBFun) ->
  [eqTest(EBFun({fs, []}), "==", {fs, []}),
   eqTest(EBFun({ts, []}), "==", {ts, []}),
   eqTest(EBFun({fs, [{italic,"are changin'"}]}),
     "==", {fs, [{bold, "are changin'"}]}),
   eqTest(EBFun({fs, [{roman, "the times"},{italic," are changin'"}]}),
     "==", {fs, [{bold, "the times"}, {bold," are changin'"}]}),
   eqTest(EBFun({ts, [{fs, [{italic, "the times"}]},
     {fs, [{bold, " try "}, {italic, "our souls"}]}]}),
     "==", {ts, [{fs, [{bold, "the times"}]},
     {fs, [{bold, " try "}, {bold, "our souls"}]}]}),
   eqTest(EBFun({ts, [{ts, [{fs, [{italic, "the times"}]},
     {fs, [{bold, " try "}, {italic, "our souls"}]}]}],
     {ts, [{fs, [{roman, " and these tests"}]},
     {fs, [{italic, " try "}, {roman, "our patience"}]}]}]}),
     "==", {ts, [{ts, [{fs, [{bold, "the times"}]},
     {fs, [{bold, " try "}, {bold, "our souls"}]}]}],
     {ts, [{fs, [{bold, " and these tests"}]},
     {fs, [{bold, " try "}, {bold, "our patience"}]}]}]}),
   eqTest(EBFun({ts, [{ts, [{ts, [{ts, [{ts, [{fs, [{italic, "deep!"}]}]}]}]}]}]}],
     "==", {ts, [{ts, [{ts, [{ts, [{ts, [{fs, [{bold, "deep!"}]}]}]}]}]}]}] ].
```

5. (15 points) [UseModels] This is a problem about the `text:text()` type in Figure 1 on page 4. In Erlang, write a function, `maptext/2`, whose type is given by the following.

```
-spec maptext(F::fun((text:formatted()) -> text:formatted()), Txt::text:text()) -> text:text().
```

This function takes a 1-argument function, `F`, and a text, `Txt`, and returns a text that is just like `T`, except that every formatted text `FT` that occurs inside `Txt` is replaced by `F(FT)`. Be sure to follow the grammar given by the types in Figure 1 on page 4! The following are tests.

```
-module(maptext_tests).
-import(testing,[dotests/2,eqTest/3]).
-export([main/0, tests/1]).
main() -> compile:file(maptext),
    dotests("maptext_tests $Revision: 1.1 $", tests(fun maptext:maptext/2)).
-spec tests(MTFun :: fun((fun((text:formatted()) -> text:formatted()),
    text:text()) -> text:text())) -> testing:testCase([text:text()]).
tests(MTFun) ->
    [eqTest(MTFun(fun id/1, {fs, []}), "==", {fs, []}),
    eqTest(MTFun(mk(bold), {ts, []}), "==", {ts, []}),
    eqTest(MTFun(mk(bold), {fs, [{italic,"are changin'"}]}), "==", {fs, [{bold, "are changin'"}]}),
    eqTest(MTFun(fun rev/1, {fs, [{roman, "the times"},{italic," are changin'"}]}),
    "==" , {fs, [{roman, "semit eht"},{italic,"'nignahc era "}]},
    eqTest(MTFun(mk(italic), {ts, [{fs, [{italic, "the times"}]},
    {fs, [{bold, " try "}, {italic, "our souls"}]}]}),
    "==" , {ts, [{fs, [{italic, "the times"}]},
    {fs, [{italic, " try "}, {italic, "our souls"}]}]}),
    eqTest(MTFun(fun bang/1,
    {ts, [{ts, [{fs, [{italic, "the times"}]},
    {fs, [{bold, " try "}, {italic, "our souls"}]}]},
    {ts, [{fs, [{roman, " and these tests"}]},
    {fs, [{italic, " try "}, {roman, "our patience"}]}]}]}),
    "==" , {ts, [{ts, [{ts, [{fs, [{italic, "the times!"}]},
    {fs, [{bold, " try !"}, {italic, "our souls!"}]}]},
    {ts, [{fs, [{roman, " and these tests!"}]},
    {fs, [{italic, " try !"}, {roman, "our patience!"}]}]}]}),
    eqTest(MTFun(fun rev/1,
    {ts, [{ts, [{ts, [{ts, [{fs, [{italic, "deep!"}]}]}]}]}]}),
    "==" , {ts, [{ts, [{ts, [{ts, [{ts, [{fs, [{italic, "!peed"}]}]}]}]}]}]}] ].
% Some functions for testing purposes, NOT FOR YOU TO IMPLEMENT!
id(FT) -> FT.
mk(FACE) -> fun({_Face,STR}) -> {FACE, STR} end.
rev({Face,STR}) -> {Face, lists:reverse(STR)}.
bang({Face,STR}) -> {Face, STR++!"}.

```

6. (10 points) [Concepts] [UseModels] Suppose we want to generalize the previous three problems involving the text type from Figure 1 on page 4. That is, suppose we want to have a function `foldtext/4` whose type is given by the following:

```
-spec foldtext(FL :: fun([R]) -> R), FF :: fun((text:formatted()) -> R),
          TF :: fun([R]) -> R, T :: text:text() -> R.
```

This function should be such that, for any desired result type `R`, it takes three function arguments, `FL`, of type `fun([R]->R)`, `FF`, of type `fun((text:formatted()) ->R)`, and `TF`, of type `fun([R]) ->R)`, and a text, `T`, and returns a value of type `R`. This function is an abstraction of the pattern of recursion over values of type `text:text()`. It should apply `FF` to all the formatted values in the `fs` case and apply `FL` to the list of those results, and should recurse over the list of texts in the `ts` case and apply `TF` to the list of those results. The following are test cases, written using the test cases in the previous 3 problems.

```
% Functions that exercise foldtests, NOT FOR YOU TO IMPLEMENT
chars(Txt) -> foldtext(fun lists:concat/1, fun({_Face,STR}) -> STR end, fun lists:concat/1, Txt).
embolden(Txt) -> foldtext(fun text:fs/1, fun({_Face,STR}) -> {bold, STR} end, fun text:ts/1, Txt).
maptext(F,Txt) -> foldtext(fun text:fs/1, fun({_Face,STR}) -> F({Face, STR}) end, fun text:ts/1, Txt).
main() -> compile:file(foldtext), io:format("foldtext_tests $Revision: 1.1 $~n",[]),
      dotests("chars_tests", chars_tests:tests(fun chars/1)),
      dotests("embolden_tests", embolden_tests:tests(fun embolden/1)),
      dotests("maptext_tests", maptext_tests:tests(fun maptext/2)).
```

Your task in this problem is to choose which one of the following, if any, is a declaration that correctly implements `foldtext`. The correct implementation should have the type and behavior described above and satisfy the test cases given above. (So don't ask us why some choice has a type error or is incorrect during the test — it's because it is the wrong answer!) Circle the letter of the correct choice.

- A. `foldtext(FL, FF, TF, LS) -> TF(FL(lists:map(FF, LS)))`.
- B. `foldtext(FL, FF, _TF, {fs, LOF}) -> text:fs(FL(lists:map(FF, LOF)))`;  
`foldtext(FL, FF, TF, {ts, LOT}) ->`  
`text:ts(TF(lists:map(fun(T) -> foldtext(FL, FF, TF, T) end, LOT)))`.
- C. `foldtext(FL, FF, _TF, {fs, LOF}) -> text:fs(lists:map(FF, LOF))`;  
`foldtext(FL, FF, TF, {ts, LOT}) ->`  
`text:ts(lists:map(fun(T) -> foldtext(FL, FF, TF, T) end, LOT))`.
- D. `foldtext(FL, FF, TF, {fs, LOF}) -> foldtext(FL, FF, TF, {fs, LOF})`;  
`foldtext(FL, FF, TF, {ts, LOT}) -> foldtext(FL, FF, TF, {ts, LOT})`.
- E. `foldtext(_FL, FF, _TF, {fs, LOF}) -> lists:map(FF, LOF)`;  
`foldtext(FL, FF, TF, {ts, LOT}) ->`  
`lists:map(fun(T) -> foldtext(FL, FF, TF, T) end, LOT)`.
- F. `foldtext(FL, FF, _TF, {fs, LOF}) -> FL(lists:map(FF, LOF))`;  
`foldtext(FL, FF, TF, {ts, LOT}) ->`  
`TF(lists:map(fun(T) -> foldtext(FL, FF, TF, T) end, LOT))`.
- G. `foldtext(FL, FF, _TF, {fs, LOF}) -> FL(lists:map(FF,LOF))`;  
`foldtext(FL, FF, TF, {ts, LOT}) -> foldtext(FL, FF, TF, LOT)`.
- H. `foldtext(FL, FF, _TF, {fs, LOF}) -> FL(FF(LOF))`;  
`foldtext(FL, FF, TF, {ts, LOT}) -> foldtext(FL, FF, TF, LOT)`.
- I. None of the above purported solutions are correct.



## Infinite Bag Data Type Problem

7. [Concepts] [UseModels] In this problem you will implement three functions on infinite bags in Erlang. An infinite bag, or multiset, can hold any number of items; for example 6 Cokes and 2 Pepsis. Infinite bags of type T are to be represented as pairs containing the atom bag and a function that takes a T value and returns a non-negative integer.

```
-type infbag(T) :: {bag, fun((T) -> non_neg_integer())}.
```

The three functions you will program are described below.

- (a) (9 points) The function `fromRule/1` takes a function, F, and returns an infinite bag (using the representation above).

```
-spec fromRule(F::fun((T) -> non_neg_integer())) -> infbag(T).
```

The resulting infinite bag contains a value X exactly F(X) times.

- (b) (6 points) The function `how_many/2` takes an infinite bag, B, and a value What, and returns the number of times that What is contained in B.

```
-spec how_many(B::infbag(T), What::T) -> non_neg_integer().
```

- (c) (10 points) The function `union/2` takes two infinite bags, B1 and B2, and returns an infinite bag that contains every value X as the number of times that X is contained in B1 plus the number of times that X is contained in B2.

```
-spec union(B1::infbag(T), B2::infbag(T)) -> infbag(T).
```

The following are tests.

```
-module(infbag_tests).
-import(testing,[dotests/2, eqTest/3]).
-import(infbag,[fromRule/1,how_many/2,union/2]).
-export([main/0, tests/0]).
main() -> compile:file(infbag),
    dotests("infbag_tests $Revision: 1.2 $", tests()).
-spec tests() -> [testing:testCase(non_neg_integer())].
tests() ->
    EverythingOnce = fromRule(fun(_) -> 1 end),
    SixCokeTwoPepsi = fromRule(fun(What) -> case What of
        coke -> 6;
        pepsi -> 2;
        _ -> 0
    end),
    E06C2P = union(EverythingOnce, SixCokeTwoPepsi),
    [eqTest(how_many(SixCokeTwoPepsi, coke), "=", 6),
    eqTest(how_many(SixCokeTwoPepsi, pepsi), "=", 2),
    eqTest(how_many(EverythingOnce, bmw), "=", 1),
    eqTest(how_many(EverythingOnce, ford), "=", 1),
    eqTest(how_many(EverythingOnce, coke), "=", 1),
    eqTest(how_many(E06C2P, coke), "=", 7),
    eqTest(how_many(E06C2P, pepsi), "=", 3),
    eqTest(how_many(E06C2P, chevy), "=", 1),
    eqTest(how_many(E06C2P, knitting), "=", 1),
    eqTest(how_many(E06C2P, warblers), "=", 1),
    eqTest(how_many(fromRule(fun(_) -> 42 end), hhgg), "=", 42) ].
```

There is space for your answer on the next page.

For your answer, complete the following module.

```
-module(infbag).  
-export([fromRule/1, how_many/2, union/2]).  
-export_type([infbag/1]).  
-type infbag(T) :: {bag, fun((T) -> non_neg_integer())}.  
  
-spec fromRule(F::fun((T) -> non_neg_integer())) -> infbag(T).  
-spec how_many(B::infbag(T), What::T) -> non_neg_integer().  
-spec union(B1::infbag(T), B2::infbag(T)) -> infbag(T).
```