Optimization of brushless direct current motor design using an intelligent technique

Alireza Shabanian a, Armin Amini Poustchi Tousiwas b, Massoud Pourmandi c, Aminollah Khormali d, Abdolhay Ataei e,*

a Chamran Ahvaz University, Ahvaz, Iran
b University of Industry and Mining, Iran
c Ferdowsi University of Mashhad, Iran
d K.N. Toosi University of Technology, Iran
e Birjand University, Iran

ABSTRACT

This paper presents a method for the optimal design of a slotless permanent magnet brushless DC (BLDC) motor with surface mounted magnets using an improved bee algorithm (IBA). The characteristics of the motor are expressed as functions of motor geometries. The objective function is a combination of losses, volume and cost to be minimized simultaneously. This method is based on the capability of swarm-based algorithms in finding the optimal solution. One sample case is used to illustrate the performance of the design approach and optimization technique. The IBA has a better performance and speed of convergence compared with bee algorithm (BA). Simulation results show that the proposed method has a very high/efficient performance.

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1. Introduction

The permanent magnet brushless DC (BLDC) motor is increasingly being used for industrial, domestic and medical applications, especially in fractional horse power range due to its excellent features, such as small size and weight, high torque density, large power to volume ratio, high efficiency, low noise, low vibration, simple maintenance, high dynamic response, longevity, improved reliability and good control characteristics in a wide range of speeds. The stator of a BLDC motor is like that of a conventional DC motor and the rotor has permanent magnets. The BLDC motor is also referred to as an electronically commutated motor as the commutation is performed electronically at certain rotor positions and this is the main difference of this motor with conventional DC motors that uses brushes and mechanical commutators. Applications of BLDC motors include electromagnetic actuators, electric vehicle propulsion systems, electric power steering of small and medium-size vehicles, extruders, printing presses, roll formers, pump's motive part, CD-ROMs, robotic, propulsion system for aircraft and underwater vehicles [1–9] but not limited to them.

The modern BLDC motors have been designed with slotless configuration in their stator, in spite of preliminary ones that have slotted stator. The slotless machine design advantages over slotted structure include reduction in manufacturing costs, simplicity in production, zero cogging torque, reduction in slot harmonic effect, smooth running, low winding inductance, high speed capability, lower vibration, lower audible noise, fast current response, high reliability, low electrical resistance, low static friction, operates in hostile environment and no sparking, and high thermal efficiency. The slotless BLDC motor mainly is used as a medical device and factory automation, such as high speed medical drills, surgical robot systems, prosthetic limb drive systems, high speed miniature spindles, etc. [1,9–11]. In [11], a simplified analytical method has been presented to design a slotless BLDC motor. This method consists of a system of equations with many approximations and beside that, this method is limited and just can be applied to small and two-pole BLDC motors. In [12], electromagnetic field analysis based on the finite element
method has been used to design a high power density and high efficiency outer rotor BLDC motor for the applications of electric vehicles. In [13], detailed and comprehensive formulations which are necessary and needed in the design of slotless BLDC motors with surface mounted magnet configuration have been presented considering both torque and speed as mechanical requirements. Then, GA has been used to find the optimal geometries of the motor. An objective function has been proposed covering the losses, cost and volume of the motor besides the mechanical and electrical requirements and constraints. In [14], a GA based optimal design of a permanent magnet BLDC motor has been proposed considering efficiency as the objective function and motor weight and temperature rise as the constraints. A computational study of a brushless DC motor is presented in [15], to determine the thermo-flow characteristics in the windings and bearings under the effects of heat generation. Thermal characteristics under the effects of heat generation in a BLDC motor using 3D finite element analysis in order to evaluate the motor cooling performance that has been done. The impact and the effect of various design parameters, including the inlet location, geometry, and bearing groove geometry, on the performance of a BLDC motor are considered and optimized. What is optimized? It is something to improve the motor’s cooling performance based on optimal design. In [16], a design method for a small-sized slotless BLDC motor with distributed hexagonal windings has been employed and the objective is maximizing torque density. This new design method is semi-analytical whereas, numerical approaches based on finite element analysis with an analytic model are used to analyze the magnetic and output characteristics of the motor. The designed motor is fabricated, and the experimental results are compared with the results of the simulation. In [17], a simulation method for BLDC motors has been presented. The purpose is to provide a solution for the initial design process, where the simulation speed is important due to the numerous variables that have to be considered. The method requires a reduced number of parameters in order to simplify and facilitate the design process. The experimental results from a 7-phase BLDC motor with one uncontrolled phase validate the modeling method. The precision and efficiency of the simulation method have been validated by the experimental tests on several prototypes, so it can be readily used as a design tool for this kind of motor. In [18], optimal design of a BLDC wheel motor has been done. For this purpose, an analytical model for the optimal design is detailed and each equation that was used for the sizing is explained. A specific procedure has been presented to order all equations in order to ease their resolution. Then, three optimization problems with an increasing number of parameters and constraints have been proposed. Also, a multimodal way has been introduced to promote the development of hybrid methods and special heuristics.

The effect of the required speed has been neglected in the optimization procedure in most of the published papers, and therefore the strength of the BLDC motor is not well defined. In this paper, a method for the optimal design of a slotless permanent magnet BLDC motor with surface mounted magnets using IBA has been presented by considering torque, power, maximum speed, voltage, losses and cost. An objective function has been proposed covering the power losses, material cost and volume of the motor simultaneously, besides the mechanical and electrical requirements. In addition to these criteria, other objectives may be considered such as cogging torque minimization, vibration reduction, magnetic flux leakage and leakage inductance minimization. Generally, the inclusion of some of these criteria into formulas is not regarded and thus a structural modification of the motor is required. For instance, cogging torque is almost eliminated in the slotless configuration against slotted structure of BLDC motors; or increase in the accuracy of fabrication procedure reduces vibration. So, the most important and common criterions are loss, cost and mass which were considered in this paper.

The cross-section area of a typical slotless BLDC motor and its geometric parameters are shown in Fig. 1. Three main regions are depicted in this figure as stator/rotor yoke, winding, and surface mounted permanent magnets (poles).

The proposed objective function is formed by a set of geometrical variables such as, number of pole pairs (\(p\)), cross sectional area of the winding (\(A_{w}\)), pole-arc per pole-pitch ratio (\(\beta\)), magnet thickness (\(l_m\)), stator/rotor core thickness (\(l_p\)), winding thickness (\(l_w\)), mechanical air gap (\(l_g\)), rotor radius (\(r_r\)), current density (\(J_{cu}\)), wire gauge and stator/rotor axial length (\(l_s\)), which is usually represented by the machine form factor (\(\lambda\)).

\[
\lambda = \frac{d_b}{l_s} \\
d_b = 2(r_r + l_g)
\]

where \(d_b\) is the bore diameter.

Specifications that depend on materials such as winding filling factor (\(k_f\)), permanent magnet remanence (\(B_r\)) and stator/rotor core flux density at knee point of \(B-H\) curve (\(B_{knee}^{core}\)) should be given. Requirements of the motor consist of the rated value of electromagnetic torque (\(T_{em}\)) and the rated rotor rotational velocity (\(\omega_r\)).

![Fig. 1. The cross-sectional area of a slotless BLDC motor.](image-url)
If the conductor and the magnetic field are perpendicular to each other, the electromagnetic torque to \( N \) conductors located at distance \( r \) from the center of rotation is

\[
T = NIBr
\]

(3)

where \( l \) is conductor of length, \( I \) is carrying current and located in a magnetic field, \( B \). The electromagnetic torque for the slotless BLDC motor depicted in Fig. 1 is found by

\[
NL = \pi l_w (2r_t + 2l_g + l_w) k_f k_c
\]

(4)

\[
A_w = \pi l_w (2r_t + 2l_g + l_w)
\]

(5)

\( k_f \) is the winding filling factor, while \( A_w \) is the winding cross sectional area, and \( k_c \) is a correction factor which represents the ratio of the number of active coils to the number of total coils at each instant/moments.

The armature reaction and the reluctance of the stator/rotor core, the magnetic flux density at the winding surface (\( B_g \)) is calculated as [13]:

\[
B_g = \frac{F_m}{A_w \eta} = \frac{B_{1m}}{(r_t + l_g) \ln((r_t + l_g + l_w)/(r_r - l_m))}
\]

(6)

\[
A_g = \frac{l_1}{p} (r_r + l_g)
\]

(7)

\( F_m \) is the magneto-motive force (MMF) of each magnet, while \( A_g \) is the air gap area at the winding surface, and \( \eta \) is the sum of the reluctances of the winding, air gap and magnet regions for each pole.

The magnetic field leakage represented by \( k_l \) is as follows [13]:

\[
k_l = 1 - \frac{0.9(r_r/(\rho l_{pl}(l_g + l_w)))^2}{1 + 1}
\]

(8)

Also, the contributed span of active winding wires and magnets represented by \( k_p \) is as follows [13]:

\[
k_p = \min(\beta, k_r) \frac{(1 - k_r \tan h(\delta - k_r \gamma))}{k_r}
\]

(9)

where \( k_r < 1 \) and \( \delta \) are empirical constants. This expression is approximated by using numerical field analysis.

Therefore, the developed electromagnetic torque can be written as [10,13]

\[
T_{em} = \frac{\pi l_1 l_w B_{1m} k_b k_f k_c (2r_t + 2l_g + l_w)}{\ln((r_t + l_g + l_w)/(r_r - l_m))}
\]

(10)

In following, the above expression for the calculation of electromagnetic torque will be modified by considering magnetic losses.

The terminal voltage and current of BLDC motor can be written as

\[
V = IR + E
\]

(11)

\[
I = A_1/E
\]

(12)

where \( R \) is the resistance of the active windings and \( E \) is induced voltage over the wire. The induced voltage is calculated as follows [10,13]:

\[
E = \frac{(r_r + l_g) k_b k_f k_c B_{1m} l_w (2r_t + 2l_g + l_w)}{\ln((r_t + l_g + l_w)/(r_r - l_m))}
\]

(13)

\[
V_1 l_1 k_b k_c A_w / A_c = \frac{1 + \pi r_t (r_r + l_g + l_w)/(\rho l_{pl} 1)}{1 + \pi r_t (r_r + l_g + l_w)/(\rho l_{pl} 1)} k_c B_{1m} l_w \nu_0 \nu_1 \ln((r_t + l_g + l_w)/(r_r - l_m))}
\]

(14)

where \( \rho \) is the resistivity of the wire and \( \gamma \) is the coil span factor. In addition, a correction factor to compensate for the end winding is considered in the above equation.

The input power is independent from \( A_c \), implying that the wire gauge has no effect on power of the motor as long as the winding cross sectional area is fixed. Therefore \( A_c \) can be used between current and voltage. The input power formula is

\[
P = VI
\]

(15)

The inductance of the winding (\( L \)) is [10]

\[
L = \frac{\pi l_1 l_w B_{1m} l_w}{54 A_1^2 \beta^2} \ln((r_t + l_g + l_w)/(r_r - l_m))
\]

(16)

where \( \beta = 4 \pi \times 10^{-7} \) H/m is free space permeability.

It should be noted that, electrical time constant (\( \tau \)) of the motor may restrict the maximum rotational velocity:

\[
\tau = L/R
\]

(17)

In order to employ the maximum electromagnetic torque capability of a BLDC motor, the following relationship between the rotational speed and the motor time constant is regarded:

\[
o_r \leq \frac{\pi}{4 \tau}
\]

(18)

If this inequality is not met, the maximum torque capability decreases.

Power losses of a BLDC motor can be divided into three major categories: electrical, magnetic and mechanical. Because of the resistance of the windings, the power loss is considered as the most important electrical loss, which can be represented by [10,13]

\[
P_{cu} = \rho V_s k_f k_c A_w l_w l_m
\]

(19)

Hysteresis and eddy current losses are the dominant magnetic losses of the core material. Assuming that the air gap magnetic flux is equal to the core magnetic flux, the stator core maximum flux density can be expressed as [13,20,21]

\[
B_{rj} = \frac{\pi k_s B_{1m} l_m}{2 \rho V_s \ln((r_t + l_g + l_w)/(r_r - l_m))}
\]

(20)

Therefore, the following expressions are obtained for hysteresis and eddy current losses:

\[
P_h = k_p \rho d V_{cu} B_{rj} f
\]

(21)

\[
P_e = k_e \rho d V_{cu} B_{rj}^2 f^2
\]

(22)

\[
f = p w r / \pi
\]

(23)

where \( d \) is material mass density and \( V_{cu} \) is the stator core volume.

The mechanical losses are categorized as windage and bearing friction. The friction of bearings can be calculated as [22]

\[
P_b = N_b \frac{f_d d o_r}{2}
\]

(24)

where \( f_d \) is radial load of the bearing, \( d \) is inner diameter of the bearing, \( o_r \) is friction coefficient of the bearing, and \( N_b \) is the number of bearings.

Although, windage losses depend on the rotor parameters, it is negligible compared with the other losses for a smooth cylindrical rotor [13,22]:

\[
P_w = k_s C_r \rho_{air} \omega^2 r^2 l_t
\]

(25)

where \( k_s \) is the roughness coefficient of the rotor, \( \rho_{air} \) is the air density and \( C_r \) is the friction coefficient, which is obtained as [21]

\[
C_f = \begin{cases} 
0.5150 \left(\frac{\mu t}{\mu_{air}}\right)^{0.5} & \text{for} \ 500 < Re < 10^4 \\
0.0325 \left(\frac{\mu t}{\mu_{air}}\right)^{0.5} & \text{for} \ 10^4 < Re
\end{cases}
\]

(26)

\[
Re = \frac{\rho_{air} W_r l_t}{\mu_{air}}
\]

(27)
where \( Re \) is the Couette–Reynolds number, and \( \mu_{air} \) is the dynamic viscosity of air.

Now, considering the magnetic and mechanical losses, the modified electromagnetic torque and output torque can be written as

\[
T_{em} = \frac{\pi k_f k_l k_2 B_l I_{lm} I_{lw} (2r_t + 2l_g + l_w) f_{cut}}{\ln((r_t + l_g + l_w)/(r_t - l_m))} - \frac{P_e + P_b}{w_r}
\]

(28)

\[
T_{out} = T_{em} - (P_w + P_b)/\omega_r
\]

(29)

where \( T_{out} \) is output torque of the BLDC motor.

In addition, total power loss of this BLDC motor can be written as

\[
P_{total} = P_{cw} + P_h + P_e + P_b + P_w
\]

(30)

The volumes of permanent magnets, winding and stator/rotor core are depending on motor geometry, so, these can incorporate material costs, respectively.

The cost of magnet can be divided into two categories, raw materials price and terminating process price. Raw material cost is proportional to the needed magnet volume and also, terminating process price corresponds to the finishing process, coating and adhesion procedure, and is assumed to be linearly dependent on the number of pole pairs. Therefore, magnet cost expression can be written as

\[
C = C_m + C_w + C_y
\]

(31)

where \( C_m, C_w, \) and \( C_y \) are permanent magnet, winding and core material costs, respectively.

The cost of magnet can be divided into two categories, raw materials price and terminating process price. Raw material cost is proportional to the needed magnet volume and also, terminating process price corresponds to the finishing process, coating and adhesion procedure, and is assumed to be linearly dependent on the number of pole pairs. Therefore, magnet cost expression can be written as

\[
C_m = c_{m1} \rho_m V_m + c_{m2} P
\]

(32)

where \( \rho_m, c_{m1}, c_{m2}, \) and \( V_m \) are the mass density of permanent magnet, the cost per unit mass of magnet, the cost corresponding to terminating process, and the volume of the magnet, respectively.

Since the price of the wire for per unit mass is inversely proportional to its cross sectional area, the following equation is found based on the wire price using a curve fitting approach [10,13];

\[
C_w = c_{11} k_{rw} V_w + c_{12} k_{rw} V_w
\]

(33)

where \( \rho_{wire}, V_w, c_1, \) and \( c_2 \) are the mass density of winding, the volumes of the winding, and constants to be found using a curve fitting technique, respectively.

The cost of stator/rotor core can be easily taken by knowing the selected type and thickness of laminations for stator/rotor core material. However, due to the waste of materials during the punch press process, the volume of the consumed material for core should be redefined as

\[
V_t = \pi d (r_t + l_g + l_w + b_y)^2
\]

(34)

\[
C_y = c_{y1} \rho_y V_t
\]

(35)

where \( V_t, c_y, \) and \( \rho_y \) are the volume of the stator/rotor core, the cost of per unit mass of core, and the mass density of stator/rotor core, respectively.

The magnetic field distribution at the winding surface of a slotless BLDC motor with surface mounted PM can be presented by the following expression, based on partial differential equation technique involving the Laplacian/quasi-Poissonian field equations [23]:

\[
B_g(\theta) = \sum_{n=1,3,...}^{\infty} \frac{2B_l \beta}{\mu_{r1}} \frac{\sin(n\pi \beta / 2)}{(np\beta / 2)} \left[ \frac{r_e}{r_l} \right]^{np-1} \left[ \frac{r_l}{r_w} \right]^{np+1} \left[ \frac{r_w}{r_g} \right]^{np+1} \left[ \frac{(r_m+1)/(r_m)}{1-(r_m/r_l)^{2np}-(r_m/r_e)^{2np}}-(r_m/r_e)^{2np}-(r_m/r_l)^{2np}\right] \cos np\theta
\]

(36)

where \( \mu_{r1} \) is the magnet permeability, also

\[
r_s = r_t + l_g + l_w
\]

(37)

\[
r_y = r_t - l_m
\]

(38)

\[
r_g = r_t + l_g
\]

(39)

3. Optimization algorithm

3.1. Original bees algorithm (BA)

Bees algorithm is an optimization algorithm inspired by the natural foraging behavior of honey bees to find the optimal solution. Fig. 2 shows the pseudocode for the algorithm in its simplest form. The algorithm requires a number of parameters to be set, namely: number of scout bees (\( n_s \)), number of sites selected out of \( n \) visited sites (\( m \)), number of the best sites out of \( m \) selected sites (\( e \)), number of bees recruited for the best \( e \) sites (\( np \)), number of bees recruited for the other (\( m-e \)) selected sites (\( np_s \)), initial size of patches (\( ngh \)) which includes site and its neighborhood and stopping criterion. The algorithm starts with the \( n \) scout bees being placed randomly in the search space. The fitness of the sites visited by the scout bees is evaluated in step 2.

In step 4, bees which have the highest fitness are chosen as “selected bees”, and sites visited by them are chosen for neighborhood search. Then, in steps 5 and 6, the algorithm conducts searches in the neighborhood of the selected sites, assigning more bees to search near the best \( e \) sites. The bees can be chosen directly according to the fitness associated with the sites they are visiting. Alternatively, the fitness values are used to determine the probability of the bees being selected. Searches in the neighborhood of the best \( e \) sites which represent more promising solutions are made more detailed by recruiting more bees to follow them than the other selected bees. Together with scouting, this differential recruitment is a key operation of the bees algorithm.

However, in step 6, for each patch only the bee with the highest fitness will be selected to form the next bee population. In nature, there is no such restriction. This restriction is introduced here to reduce the number of points to be explored. In step 7, the remaining bees in the population are assigned randomly around
the search space scouting for new potential solutions. These steps are repeated until a stopping criterion is met. At the end of each iteration, the colony will have two parts to its new population representatives from each selected patch and other scout bees assigned to conduct random searches [24].

**Table 1**

List of constant parameters and their values.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_f$</td>
<td>0.7</td>
</tr>
<tr>
<td>$k_r$</td>
<td>0.666</td>
</tr>
<tr>
<td>$k_t$</td>
<td>0.95</td>
</tr>
<tr>
<td>$k_b$</td>
<td>1</td>
</tr>
<tr>
<td>$\delta$</td>
<td>5</td>
</tr>
<tr>
<td>$B_r$ (T)</td>
<td>1</td>
</tr>
<tr>
<td>$B_{max}$ (T)</td>
<td>1.5</td>
</tr>
<tr>
<td>$\alpha$ (m$^{-1}$)</td>
<td>$10^{11}$</td>
</tr>
<tr>
<td>$\rho$ (g/m$^3$)</td>
<td>$1.8 \times 10^{-8}$</td>
</tr>
<tr>
<td>$k_i$ (Ws$^{-1}$kg$^{-1}$)</td>
<td>0.018</td>
</tr>
<tr>
<td>$k_e$ (Wst$^{-1}$kg$^{-1}$)</td>
<td>0.00088</td>
</tr>
<tr>
<td>$n$</td>
<td>1.92e</td>
</tr>
<tr>
<td>$W_p$</td>
<td>1</td>
</tr>
<tr>
<td>$W_r$</td>
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</tr>
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</table>

**Table 2**

Upper and lower limits of the optimization variables.

<table>
<thead>
<tr>
<th>No</th>
<th>Parameters</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
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<td>$p$</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>$\beta$</td>
<td>0.5</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>$l_p$</td>
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<td>0.015</td>
</tr>
<tr>
<td>4</td>
<td>$l_f$</td>
<td>0.002</td>
<td>0.01</td>
</tr>
<tr>
<td>5</td>
<td>$l_r$</td>
<td>0.001</td>
<td>0.0055</td>
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<tr>
<td>6</td>
<td>$l_e$</td>
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<td>0.004</td>
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<td>7</td>
<td>$r_f$</td>
<td>0.005</td>
<td>0.1</td>
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<tr>
<td>8</td>
<td>$l_c$</td>
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<td>0.6933</td>
</tr>
<tr>
<td>9</td>
<td>$A_c$</td>
<td>0.1</td>
<td>2</td>
</tr>
<tr>
<td>10</td>
<td>$J_{cm}$</td>
<td>$3 \times 10^6$</td>
<td>$6 \times 10^6$</td>
</tr>
</tbody>
</table>

**Table 3**

Parameters used in the IBA.

| Number of scout bees, $n$ | 100 |
| Number of sites selected for neighborhood search, $m$ | 20 |
| Number of best "elite" sites out of $m$ selected sites, $e$ | 10 |
| Number of bees recruited for best $e$ sites, nep | 8 |
| Number of bees recruited for the other $(m - e)$ selected sites, nsp | 8 |
| Number of iterations, $R$ | 100 |

**Table 4**

Optimum value of BLDC parameters using IBA.

<table>
<thead>
<tr>
<th>No</th>
<th>Parameters</th>
<th>IBA-BLDC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$p$</td>
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<tr>
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<td>$\beta$</td>
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<td>$l_p$</td>
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<tr>
<td>6</td>
<td>$l_e$</td>
<td>0.0010</td>
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<td>$r_f$</td>
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<td>9</td>
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<tr>
<td>10</td>
<td>$J_{cm}$</td>
<td>5.8198e+06</td>
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**Table 5**

Obtained values for optimized BLDC motor parameters.

<table>
<thead>
<tr>
<th>No</th>
<th>$V_r$</th>
<th>$C$</th>
<th>$P_i$</th>
<th>$P_w$</th>
<th>$P_b$</th>
<th>$P_r$</th>
<th>$P_p$</th>
<th>$W_{r1}$</th>
</tr>
</thead>
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<td>51.2446</td>
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<td>4.4115</td>
<td>2.4500</td>
<td>2.1195</td>
<td>0.0783</td>
</tr>
<tr>
<td>2</td>
<td>0.8205</td>
<td>1.0248</td>
<td>2.6345</td>
<td>2.6401</td>
<td>0.9661</td>
<td>0.03</td>
<td>1.3034e+02</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>11.6115</td>
<td>1513</td>
<td>1462</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

### 3.2. Improved bees algorithm (IBA)

In order to improve the convergence velocity and accuracy of the BA, this paper recommends an improved bee algorithm (IBA). In bees algorithm ngh the initial size of the neighborhood is defined in which follower bees are placed. For example, if $X$ is the position of an elite bee in the $i$th dimension, follower bees will be placed randomly in the interval $X_{i,p} \pm \text{ngh}$ in that dimension at the beginning of the optimization process. As the optimization advances, the size of the neighborhood search gradually decreases to facilitate fine tuning of the solution.

For each of the $m$ selected sites, the recruited bees are randomly placed with uniform probability in a neighborhood of the high fitness location marked by the scout bee. This neighborhood (flower patch) is defined as an $n$-dimensional hyper box of sides $a_1, \ldots, a_n$ that is centered on the scout bee. For each flower patch, the fitness of the locations visited by the recruited bees is evaluated. If one of the recruited bees lands in a position of higher fitness than the scout bee, that recruited bee is chosen as the new scout bee. At the end, only the fittest bee of each patch is retained. The fittest solution visited so-far is thus taken as a representative of the whole flower patch. This bee becomes the best once back at the hive.

In BA, the size of a patch is kept unchanged as long as the local search procedure yields higher points of fitness. If the local search fails to bring any improvement in fitness, the size of $i$ is decreased. The updating of the neighborhood size follows the following heuristic formula:

$$\text{ngh}(t + 1) = 0.8 \times \text{ngh}(t)$$

(40)

where $t$ denotes the $t$th iteration of the bees algorithm main loop.

Thus, following this strategy, the local search is initially defined over a large neighborhood, and has a largely explorative character. As the algorithm progresses, a more detailed search is needed to refine the current local optimum. Hence, the search is made increasingly and the area around the optimum is searched more thoroughly.

Since the search process of BA is nonlinear and highly complicated patch with no feedback taken from the elite bees fitness cannot truly reflect the actual search process. In the beginning of the search process, the bees are far away from the optimum point and hence a big patch size is needed to globally search the solution space. Conversely, when the best solution found by the population improves greatly after some iteration, i.e., the bees find a near optimum solution, only small movements are needed and patch size must be set to small values. Based on this, in this study, we
have proposed improved bees algorithm (IBA) in which the patch size is set as a function of elite bees fitness during search process of BA, and is as follows:

$$ngh_i^t = \frac{1}{1 + \exp(-F(\text{elite}_i^t))}$$

(41)

where $F(\text{elite}_i^t)$ is the fitness of $i$th elite bee in $t$th iteration and $ngh_i^t$ is the $i$th elite bees size of the neighborhood in $t$th iteration. In this case, patch size changes according to the rate of elite bee fitness improvement.

According to Eq. (41), during the search of IBA, while the fitness of an elite bee is far away from the real global optimal, the value of patch size will be large resulting in strong global search abilities and locating the promising search areas. Meanwhile, when the

<table>
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<tr>
<th>Run. no</th>
<th>Fitness</th>
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<th>$\beta$</th>
<th>$l_m$</th>
<th>$l_y$</th>
<th>$l_w$</th>
<th>$l_g$</th>
<th>$r_i$</th>
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<th>$A_i$</th>
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</table>

Standard deviation $\pm 0.03$ –

**Fig. 3.** The best fitness evolution of the fitness function for different runs.

**Fig. 4.** Variation of each optimization parameters during IBA run.
fitness of an elite bee is achieved near the real global optimal, the patch size will be set small, depending on the nearness of its fitness to the optimal value, to facilitate a finer local explorations and hence to accelerate convergence.

The main difference between BA and IBA is in the patch size definition. First, in IBA, the patch size is associated with fitness value (Eq. (41)). Second, in BA patch size is same for all elite bees; meanwhile in IBA every elite bee has its own patch size [19].

4. Simulation results and discussion

4.1. Optimized BLDC

The first stage in the optimization of a permanent magnet BLDC motor design is choosing a proper objective function. First, the optimization variables, i.e. those motor parameters that need to be optimally found, should be represented as a vector, $x$:

$$x = [p \quad \beta \quad l_m \quad l_y \quad l_s \quad r_t \quad \lambda \quad A_{Jcu}^T]$$

The form of an objective function depends on the application and the required quantity of the motor. In this study the objective function consists of losses, cost and volume (mass) all of which should be minimized simultaneously. Three weighting factors are considered in order to bring all the objectives in a comparable scale and to control the importance of each individual objective. Therefore the objective function is written as

$$f_0(x) = w_P V_t(x) + w_R P_{total}(x) + w_C C(x)$$

$$+ \frac{1}{e^{\epsilon}} f_s(1 - (T_{em}/T_{em}^*) + f_u (1 - (\omega_{r_{max}}^{\omega_r}/\omega_{r_{max}}^*) + f_d (h_{max}/h_{max}^*)) - 1)$$

where $w_P$, $w_C$ and $w_R$ are weighting factors, $P_{total}$ is total power loss of motor, and $C$ is the total cost of the materials used in the motor design. Also, $\epsilon$ is a small constant and

$$f_s(x) = \frac{1}{1 + e^{-\epsilon x}}$$

where $\epsilon$ is a very large constant.

The constant parameters of the motor, upper and lower limits of variables have been shown in Tables 1 and 2, respectively. Table 3 shows the IBA parameters.

The electrical requirement is the level of voltage source for the motor, $V^*$. This constraint is satisfied by a proper choice of the wire gauge for the BLDC motor windings. Moreover, the mechanical specifications can be represented by a set of inequality constraints, such as

$$T_{em} > T_{em}^*$$

$$\omega_{r_{max}} \geq \omega_{r_{max}}^*$$

where $T_{em}$, $\omega_{r_{max}}$ and $\omega_{r_{max}}^*$ are required torque, required rotational speed, and the maximum speed at the required torque, respectively.

In addition, heat and saturation constraints can be written as

$$K \geq k_f I_{sat}^2$$

$$B_{sat} \geq B_{sat}$$

where $K$ is the maximum permissible heat in winding, and $B_{sat}$ is flux density of the stator/rotor yoke at the knee point of $B-H$ curve.

Table 7

<table>
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<tr>
<th>No</th>
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<td>5</td>
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Table 8

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<td>1462</td>
<td>1462</td>
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Optimum values after optimization are listed in Table 4. Also, other characteristics of the optimized motor are listed in Table 5. Additionally, to evaluating the performance of the proposed algorithm (IBA-BLDC), five different runs have been performed. The IBA finds the best parameter of BLDC motor to gain the fitness function minimum. Fig. 3 shows a typical increase of the fitness of the best bees obtained from IBA-BLDC for the five different runs. As indicated in Fig. 3, its fitness curves gradually improved from iteration 0 to 100, and exhibited no significant improvements after iteration 70 for the five different runs. The optimal stopping iteration to get the best performance for the five different runs was around iteration 60–70. In Table 6, the obtained fitnesses are shown. It can be seen that the proposed IBA-BLDC method has low standard deviation (±0.04) and it is robust and reliable method for motor design problems.

The variation of each optimization parameters during IBA run is shown in Fig. 4. It can be observed that after 60 iterations, all the parameters are stable and reach to their optimum value. Therefore, it is shown that the IBA has a good speed for convergence. This fact can be seen from Fig. 5. In Fig. 5, the accuracy and the speed of
bees algorithm and improved bees algorithm are compared. The achieved diagrams show the mean of 5 different runs for both algorithms. As depicted in this figure, the improved bees algorithm has higher accuracy and speed of convergence compared with bees algorithm.

4.2. Comparison with different optimization algorithms

In order to compare the performance of IBA with another nature inspired algorithms, we have used genetic algorithm (GA) [13], chaotic bat algorithm (CBA) [28] and Social Spider Optimization (SSO) [29] to evolve the proposed method. The GA, CBA and SSO results have been shown in Table 7. The standard deviation is calculated for 5 different runs. According to results in Tables 7 and 8, the best performance obtained by IBA-BLDC is 2.64. It can be seen that the success rates of IBA is higher than the performance of other methods.

4.3. Sensitivity analysis

According to the proposed objective function, high number of pole pairs is desirable as illustrated in Fig. 6(a); however, it is observed that more pole pairs increase flux leakage and degrade amount of developed torque. Similarly, total volume is not affected by the pole-arc per pole-pitch ratio, $\beta$. Smaller $\beta$ implies lower materials cost, magnetic losses and flux leakage; however, it reduces the developed torque due to lower air gap flux density, as shown in Fig. 6(b). It is also shown that larger $\beta$ causes higher flux leakage and consequently lower torque.

Decrease in magnet thickness $l_m$ improves cost, power loss and total volume; however as indicated in Fig. 6(c), it diminishes the amount of developed torque and maximum speed. There should be a compromise between cost and magnetic losses to find the optimum value of the stator and rotor core thickness $l_r$. As shown in Fig. 6(d), increase in $l_r$ would reduce the magnetic losses and would increase the cost and total volume. Also saturation may occur if $l_r$ is too small. More winding space has adverse effect on
cost and volume, but improves the efficiency for a constant current density.

The winding thickness cannot be less than a specified amount in which the required torque may not be produced and should not be too large to reduce the maximum speed. Rotor radius is one of the most effective parameters in the motor design problems. All three criteria are minimized simultaneously by reducing the rotor radius as shown in Fig. 6(f); however, too small rotor radius may not generate the required torque and too large radius may adversely affect the maximum speed. Fig. 6(g) shows clearly that higher machine form factor is desirable; however an excessive form factor will decrease the developed torque. Mechanical air gap can be excluded from the optimization variables since it is generally set to its minimum possible value, as depicted in Fig. 6(h). For a constant winding cross sectional area, developed torque and copper loss are proportional to current density and its square, respectively. Although, increase in current density will improve the material cost and total volume, the adverse effect on copper loss dominates the other two criteria, as shown in Fig. 6(i).

4.4. Validation of results

Two different methods have been used in order to determine the optimization results for BLDC geometrical design: FEA and analytical methods. Both of them demonstrate the accuracy of the optimization method and its design.

4.4.1. Finite element analysis

Finite element method (FEM) is a very useful tool that is widely used by engineers and researchers to solve engineering problems arising from various physical fields such as electromagnetic, thermal, structural, fluid flow, acoustic and others. Currently the FEM is definitely the dominant numerical analysis method for the simulation of physical field distributions in electrical machinery design, without being paralleled by any other numerical technique [25,26].

This paper uses Ansoft-MAXWELL v.13 software, which is a FEM-based software, to run the simulation for BLDC motor design. The magnetic field inside the motor is directed by the following nonlinear partial differential equations [25–27]:

\[ \nabla \times \mathbf{A} = \mathbf{B} \]  \hspace{1cm} (49)

\[ \nabla \times (\nu \nabla \times \mathbf{A}) = \mathbf{j} \]  \hspace{1cm} (50)

\[ \nabla \cdot \sigma \left( \frac{\partial \mathbf{A}}{\partial t} + \nabla \mathbf{V} \right) = 0 \]  \hspace{1cm} (51)

where \( \mathbf{A} \) is the magnetic potential vector, \( \mathbf{j} \) is the total current density, \( \nu \) is the magnetic reluctivity, \( \mathbf{V} \) is the electric scalar potential and \( \sigma \) is the electric conductivity.

In order to model BLDC motor, magnetic core and windings are primarily simulated in 2D form, according to structural formation of the studied BLDC. For this purpose, the number of core’s lamination, magnetization curve of magnetic core, turns number of windings,
internal and external diagonal, exact height of windings, and even the material used in each element should be modeled exactly. The materials of winding and core laminations have been considered as Copper and M19, respectively. Moreover, the space between windings and core has been filled with air. In addition, the introduced magnetizing curve or B–H curve of magnetic core to software is shown in Fig. 7. This curve has been derived from manufacturer’s data. It should be noted that, residual flux should be considered accurately, because of its critical impact on the magnetic field density and distribution of flux lines. Moreover, 2D cross section of the motor has been shown in Fig. 8.

After simulation steps, the flux line distribution and magnetic flux density of core have been shown in Figs. 9 and 10, respectively. The magnetic flux density variation in the air gap of motor is shown in Fig. 11.

4.4.2. Analytical method

The magnetic field distribution at the winding surface of a slotless surface mounted PM BLDC motor is a criterion for validity check. This expression is introduced in Eq. (36) and after optimization stage; the designed parameters can be included in this equation to find the Laplacian/quasi-Poissonian magnetic field. This value should be similar to the flux density value of Eq. (6). So, using Eq. (36) and values of Table 4, the magnetic flux density at the winding surface is calculated. Fig. 12 shows the open-circuit magnetic flux distribution of the designed motor, its fundamental component and the approximate maximum calculated in this study. Clearly the presented maximum flux density is in perfect agreement with the PDE-based solution.

5. Conclusion

A new technique for optimization of slotless BLDC motors design with surface magnet structure has been presented considering torque, speed, voltage, losses and cost. It is used as the search method. This method is based on the population-based optimization algorithms in finding the optimal solution. For this purpose we have considered GA, BA and IBA.

A proper optimization algorithm should have both accuracy in local and global search and also, high convergence speed. For this purpose, it should find and trace the optimum solution to have a response. The used IBA has these characteristics. In addition, the proposed IBA has a straight and simple structure, and from this point of view the application of IBA is very reasonable in comparison with GA. In addition, based on obtained simulation results, the IBA converged to the optimum solution in low iterations. Moreover, one of the obvious and distinguished characteristics of this algorithm is low standard derivation. This algorithm reaches to the optimum results in different runs.

Concerning the computational efforts, IBA was very fast, requiring a few seconds to find the optimum. The results have been analyzed which has shown the efficacy of the proposed technique for designing of the electrical machineries designing.

References