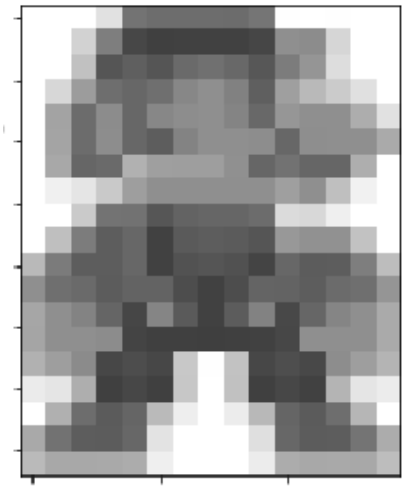
Homework 2

CAP 4453

Fall 2023

1. Considers the image below. What is the dimension of the matrix that represents the image? [10%]



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[255 255 255 225 115 109 109 109 108 117 254 255 254 255 255]
[255 255 210 126  71  64  65  65  65  70 143 140 214 255 255]
[255 255 193  85  95  85 106 112 104  87 124 149 221 255 255]
[255 215 156 110 103 111 135 143 129  95 159 185 203 224 255]
[255 163 108 140 103 135 141 143 132 104 150 143 143 173 225]
[255 164 108 141 103  93 131 143 143 141 103 142 143 143 170]
[255 168 102 106 177 159 158 158 144 102 114 102 102 174 255]
[255 240 229 201 158 143 143 143 143 143 159 143 191 241 255]
[255 255 202 115 114  86  99 102 102 108 219 216 239 255 255]
[255 191 124  93 103  65  90  93  91  85 152 145 145 194 255]
[185 123  93  93 103  65  82  85  81  68 103  92  93 126 189]
[143 111 106  92  99  99  77  65  77 100  99  93 107 111 149]
[166 143 131 101  70 133  90  65  92 130  72 102 133 143 170]
[165 143 143 140  68  64  64  63  64  65  71 143 143 143 170]
[177 158 139  71  77  71 200 254 194  71  78  72 141 158 180]
[235 229 174  64  71  64 199 255 193  64  71  65 180 229 236]
[255 176 109  91 101 186 238 255 236 184 101  91 110 182 255]
[169 114  93  92 103 227 255 255 255 223 103  92  93 116 172]
[186 168 168 169 172 239 255 255 255 236 172 168 169 168 189]
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2. If we filter using a 3x3 kernel to convolve previous image, and do not perform any padding/mirroring/flipping/copy on the borders of the image (convolution in the regions where filter and image are fully intersected), what will be the dimension of the previous image after filtering? [10%]

3. What will be the output dimension if the kernel is 5x5 (convolution in the regions where filter and image are fully intersected)? [10%]

4. Assuming the dimensions of the image are M x N . can you come out with a general formula that tells you the dimension of the image after filtering if only consider 'valid' regions (where filter and image fully intersect)? [10%]

5. Compute the output of applying the filter $\begin{matrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{matrix}$ on the first 5 valid columns of row 2 of "mario" (row 2 is the one that start with values 255 255 210 ..). Show your computations and write the obtained output. [10%]

Kernel estimation

6. Use the formula of 1D Gaussian function $$G(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}}$$ to find coefficients of a kernel of size 7 when σ=1.4. Hint: x is evaluated in interval [-3 -2 -1 0 1 2 3] [10%]

7. The size of a gaussian kernel is usually chosen to have values in the order of 2 or 3 sigmas, since after that the values of the function are almost zero. In the extreme parts of this kernel (when x is either -3 or 3) how many sigmas it corresponds to? Is the chosen size of 7 a good value? [10%]

8. Approximate the obtained kernel as a fraction of integer numbers. Hint: use 64 as the denominator. [10%]

9. Compute a 7x7 Gaussian kernel using the 1D estimated kernel you estimated in the previous exercise. Remember, this is a separable filter and can be obtained using matrix multiplication. [10%]

$$G = K_{7x1} * K_{1x7}$$

10. In class we build a sharpen filter as the sum of original filter + detail. The detail part was built with the original function and a box filter. Create a new kernel for sharpening but this time uses a gaussian filter. [10%]