CAP 4453
Robot Vision

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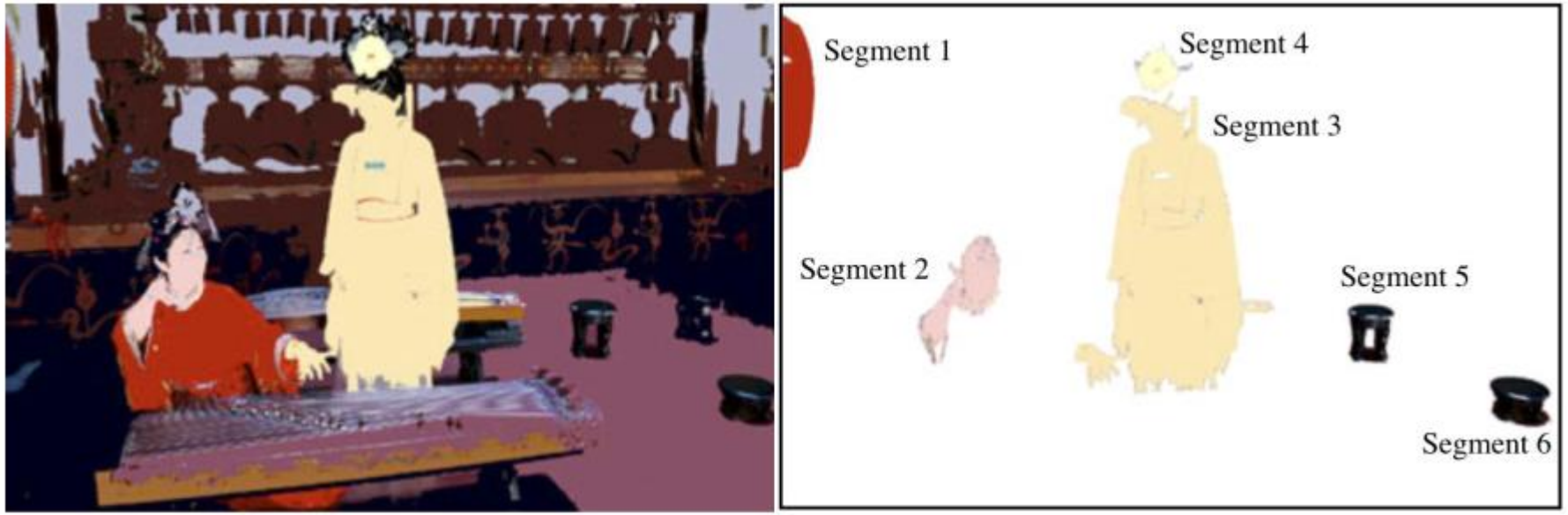
Credits

- Some slides comes directly from:
  - Yogesh S Rawat (UCF)
  - Noah Snavely (Cornell)
  - Ioannis (Yannis) Gkioulekas (CMU)
  - Mubarak Shah (UCF)
  - S. Seitz
  - James Tompkin
  - Ulas Bagci
  - L. Lazebnik

Short Review from last class

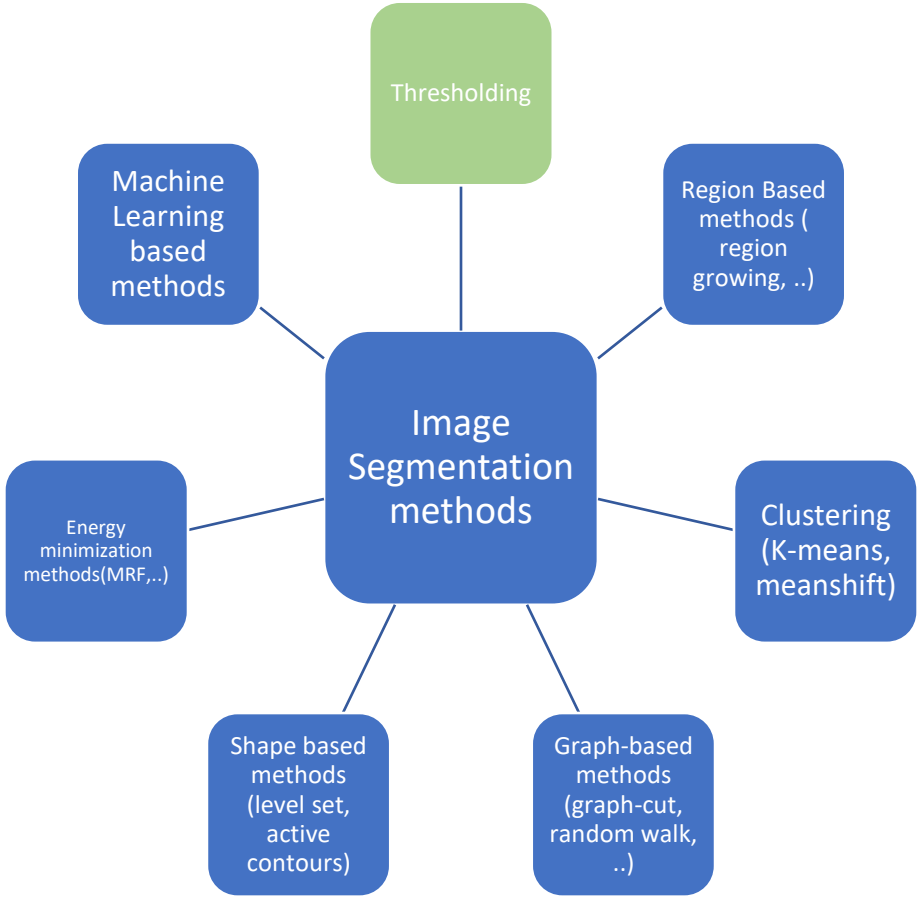
Image segmentation

- Image segmentation partitions an image into regions called segments.



- Image segmentation creates segments of connected pixels by analyzing some similarity criteria:
  - intensity, color, texture, histogram, features

Image segmentation methods



Otsu thresholding

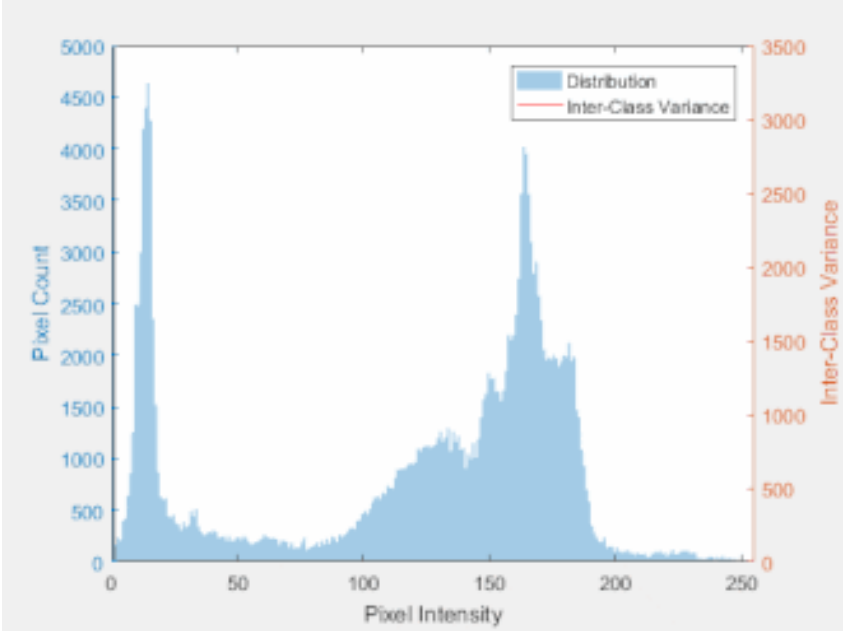
- Definition: The method uses grey-value histogram of the given image I as input and aims at providing the best threshold (foreground/background)
- Otsu's algorithm selects a threshold that maximizes the between-class variance σ_b^2 or minimize within-class variance σ_w^2

Option 1: maximum of:

$$\sigma_b^2(t) = w_1(t)w_2(t)[\mu_1(t) - \mu_2(t)]^2$$

$$\mu_1(t) = \sum_{i=1}^{t} \frac{iP(i)}{w_1(t)} \qquad w_1(t) = \sum_{i=1}^{t} P(i)$$

$$\mu_2(t) = \sum_{i=t+1}^{I} \frac{iP(i)}{w_2(t)} \qquad w_2(t) = \sum_{i=t+1}^{I} P(i)$$



Otsu thresholding

- Definition: The method uses grey-value histogram of the given image I as input and aims at providing the best threshold (foreground/background)
- Otsu's algorithm selects a threshold that maximizes the between-class variance σ_b^2.

$$\sigma_{total}^2 = \mathrm{w}_1(t)\sigma_1^2(t) + \mathrm{w}_2(t)\sigma_2^2(t) + w_1(t)w_2(t)(\mu_2^2(t) - \mu_1^2(t))$$

within-class variance — Minimize

between-class variance — Maximize

Option 2: minimum of:

$$\sigma_w^2(t) = w_1(t)\sigma_1^2(t) + w_2(t)\sigma_2^2(t)$$

$$w_1(t) = \sum_{i=1}^{t} P(i) \qquad P(i) = \frac{n_i}{n}$$

$$w_2(t) = \sum_{i=t+1}^{I} P(i)$$

$$\sigma_1^2(t) = \sum_{i=1}^{t} [i - \mu_1(t)]^2 \frac{P(i)}{w_1(t)}$$

$$\sigma_2^2(t) = \sum_{i=t+1}^{I} [i - \mu_2(t)]^2 \frac{P(i)}{w_2(t)}$$

Robot Vision

8. Segmentation II

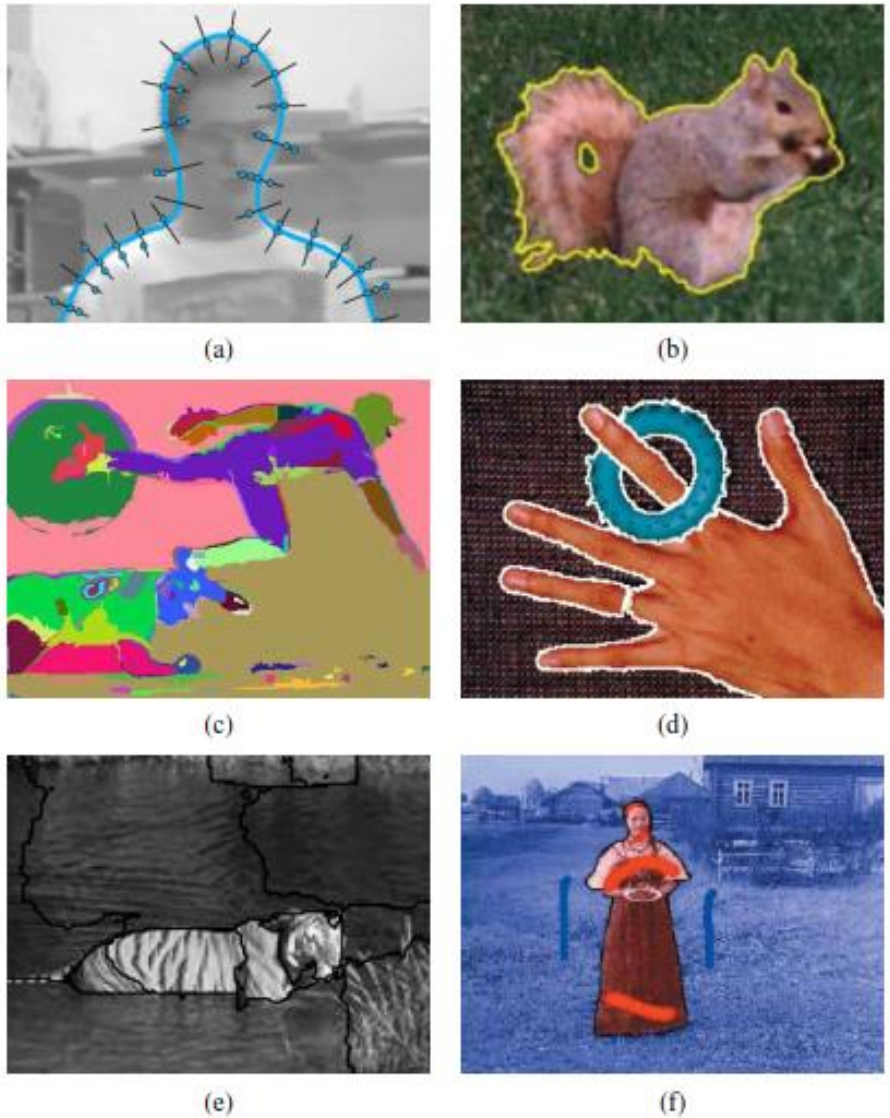
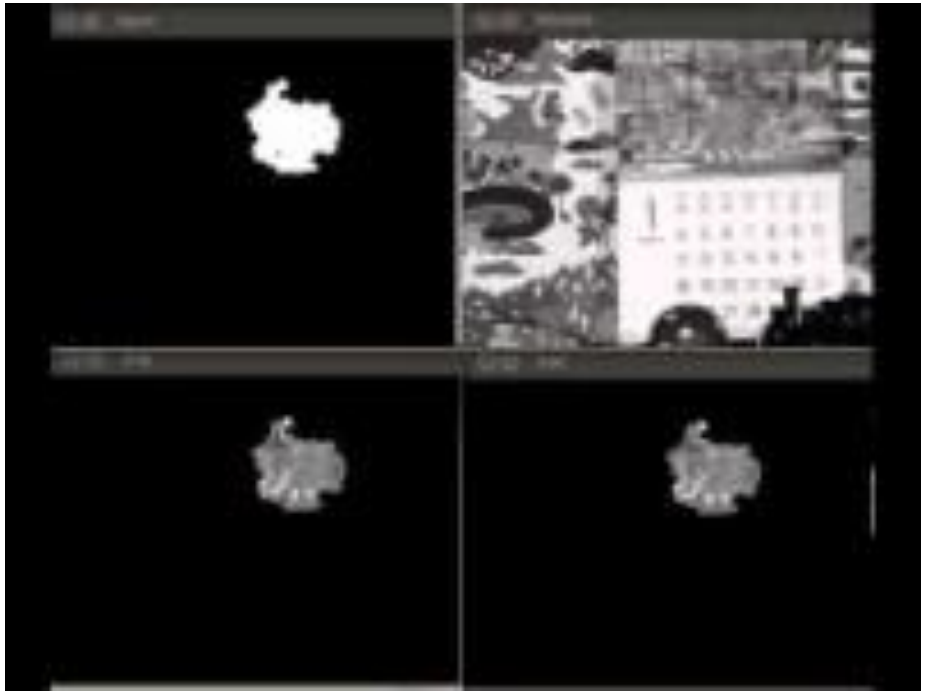
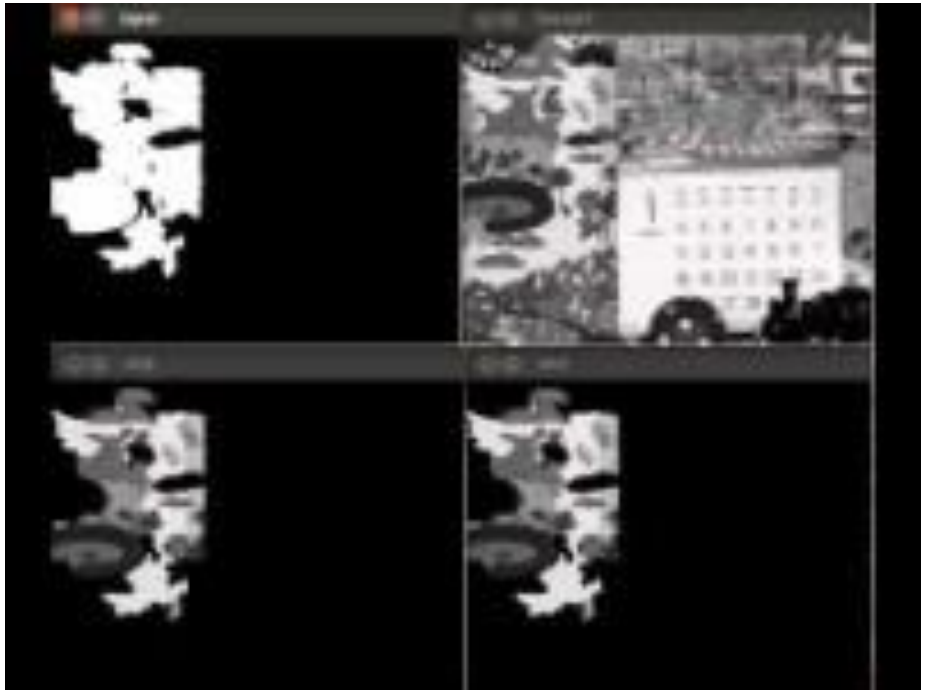
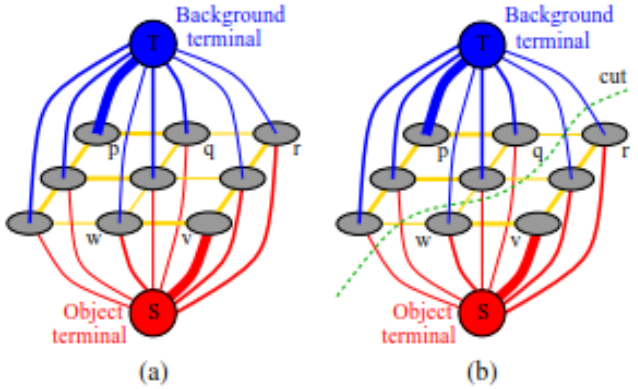


Figure 5.1 Some popular image segmentation techniques: (a) active contours (Isard and Blake 1998) © 1998 Springer; (b) level sets (Cremers, Rousson, and Deriche 2007) © 2007 Springer; (c) graph-based merging (Felzenszwalb and Huttenlocher 2004b) © 2004 Springer; (d) mean shift (Comaniciu and Meer 2002) © 2002 IEEE; (e) texture and intervening contour-based normalized cuts (Malik, Belongie, Leung *et al.* 2001) © 2001 Springer; (f) binary MRF solved using graph cuts (Boykov and Funka-Lea 2006) © 2006 Springer.





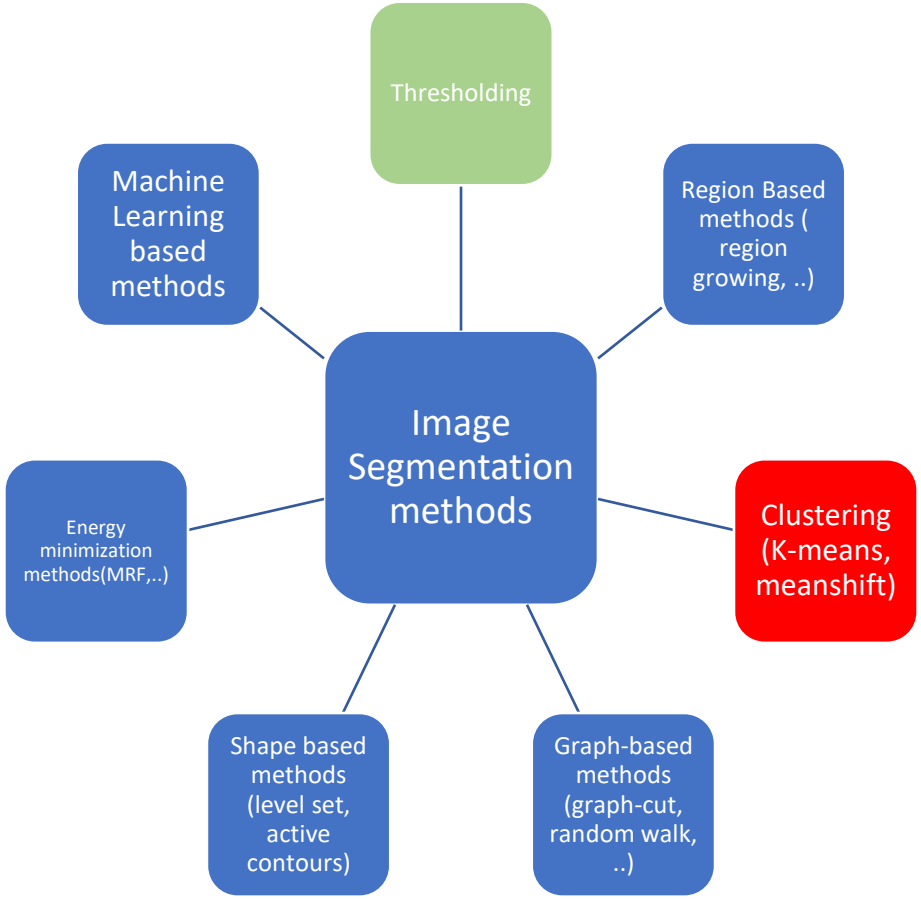
Energy-Based methods



Outline

- Image segmentation basics
- Thresholding based
  - Binarization
  - Otsu
- **Clustering based**
  - **K-means (SLIC)**
- Region based
  - Merging
  - Splitting

Image segmentation methods

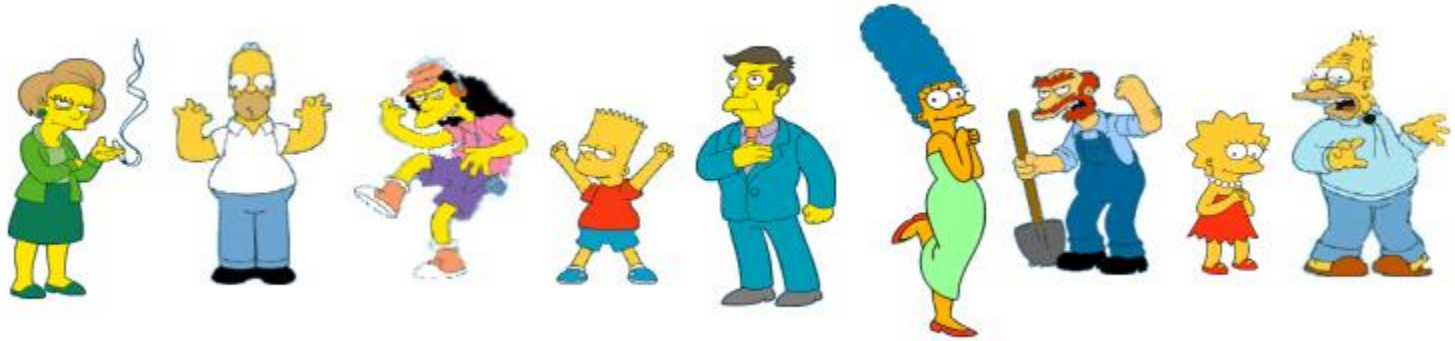


What is Clustering?

- Organizing data into classes such that:
  - High intra-class similarity
  - Low inter-class similarity

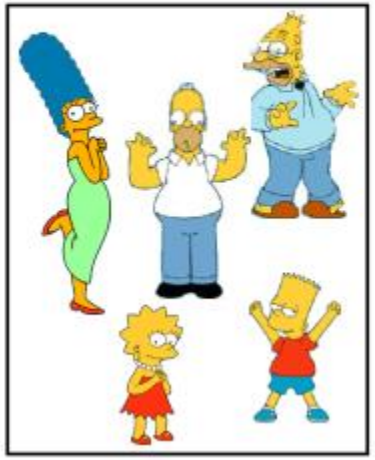
- Finding the class labels and the number of classes directly from the data (as opposed to classification tasks)

What is a natural grouping ?



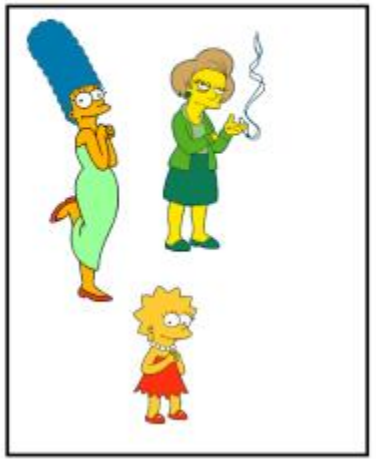
What is a natural grouping ?

Clustering is subjective



Simpson's family

School employees



Females

Males

What is similarity ?

What is similarity ?

Distance metrics

0.23

Peter Piotr

3

342.7

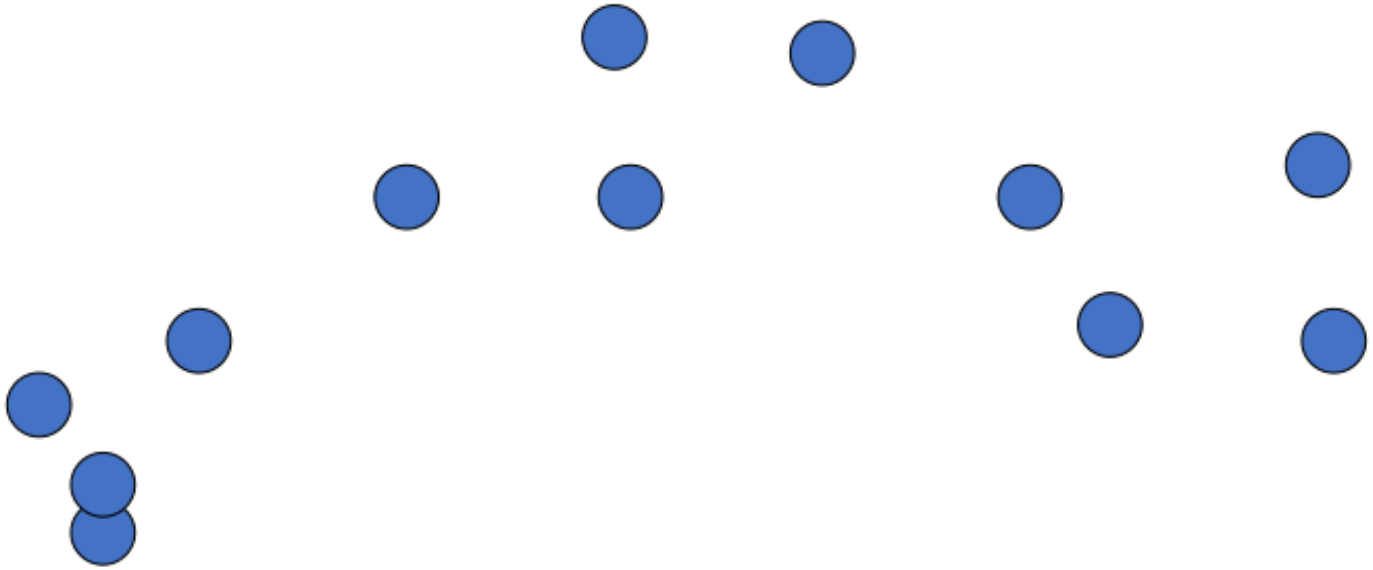
Outline

- ~~Image segmentation basics~~
- ~~Thresholding based~~
  - ~~Binarization~~
  - ~~Otsu~~
- ~~Region based~~
  - ~~Merging~~
  - ~~Splitting~~
- **Clustering based**
  - **K-means**
  - Superpixels (SLIC)

K-means

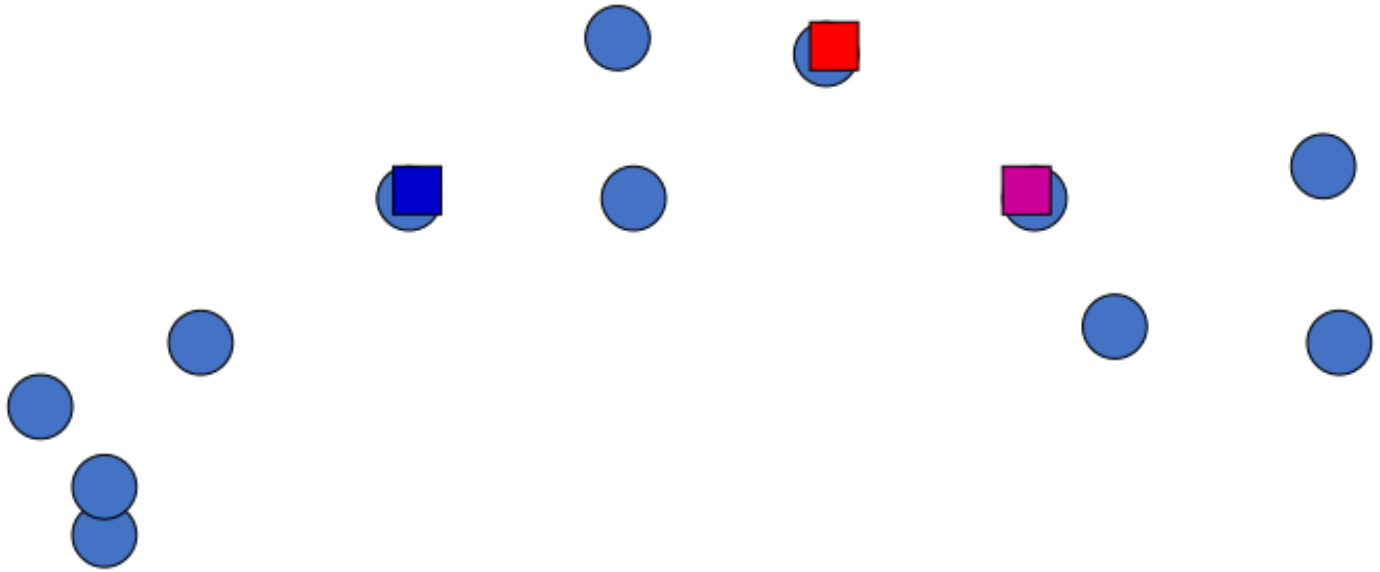
- Most well-known and popular clustering algorithm:
  - Start with some initial cluster centers
- Iterate:
  - Assign/cluster each example to closest center
  - Recalculate centers as the mean of the points in a cluster

K-means



K-means

Step 0:
- Pick number of classes
- Pick seeds for those classes

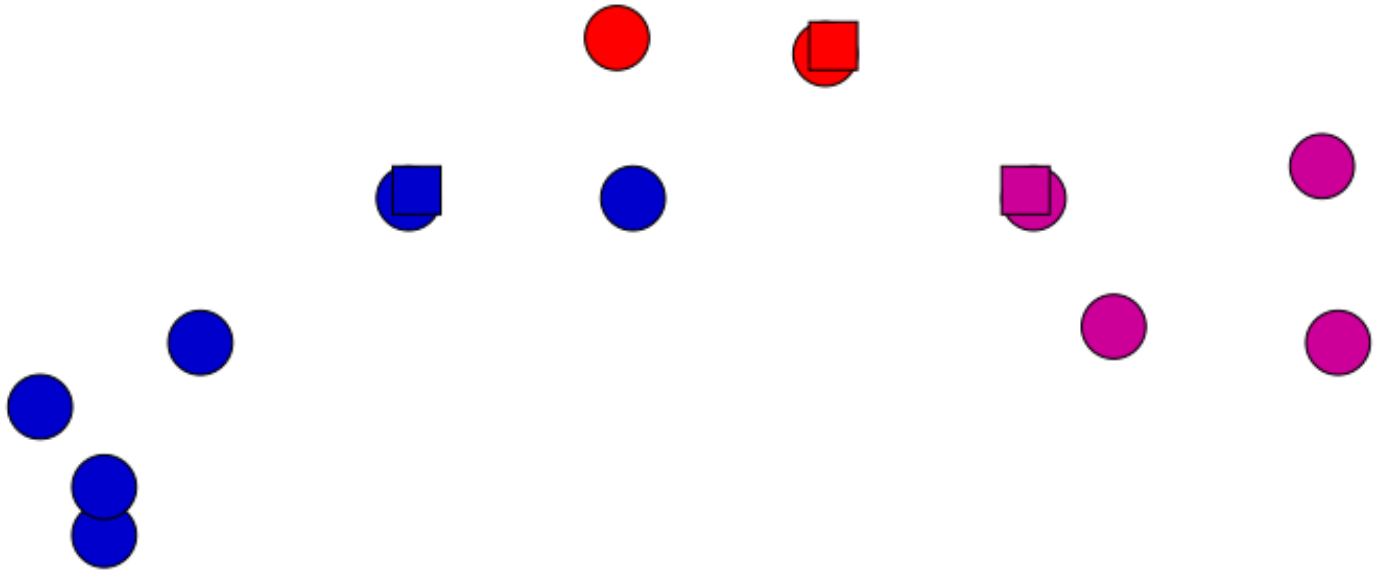


K-means

Iterate:

Assign/cluster each example to closest center

Recalculate centers as the mean of the points in a cluster



K-means

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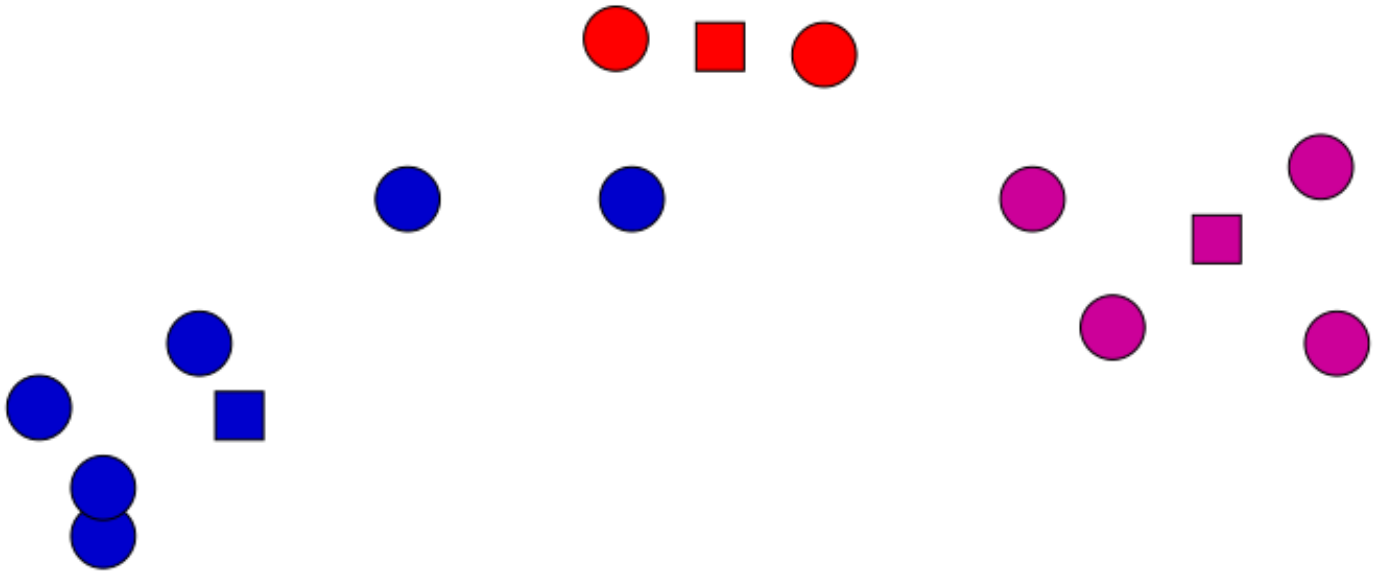

K-means

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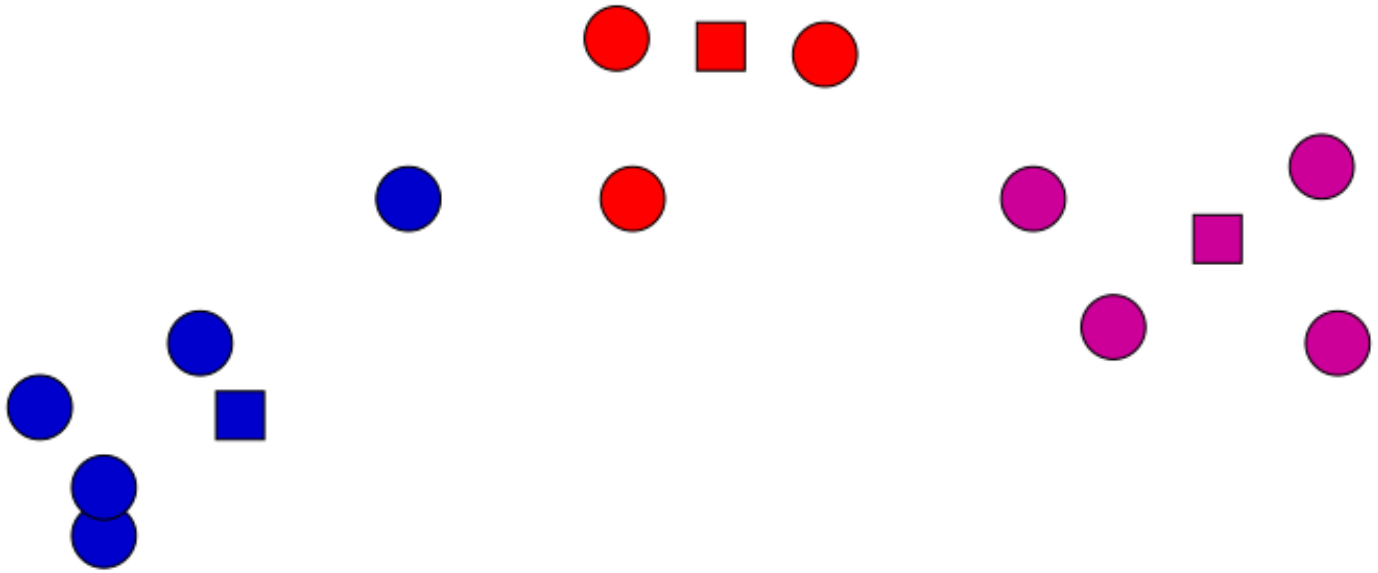

K-means

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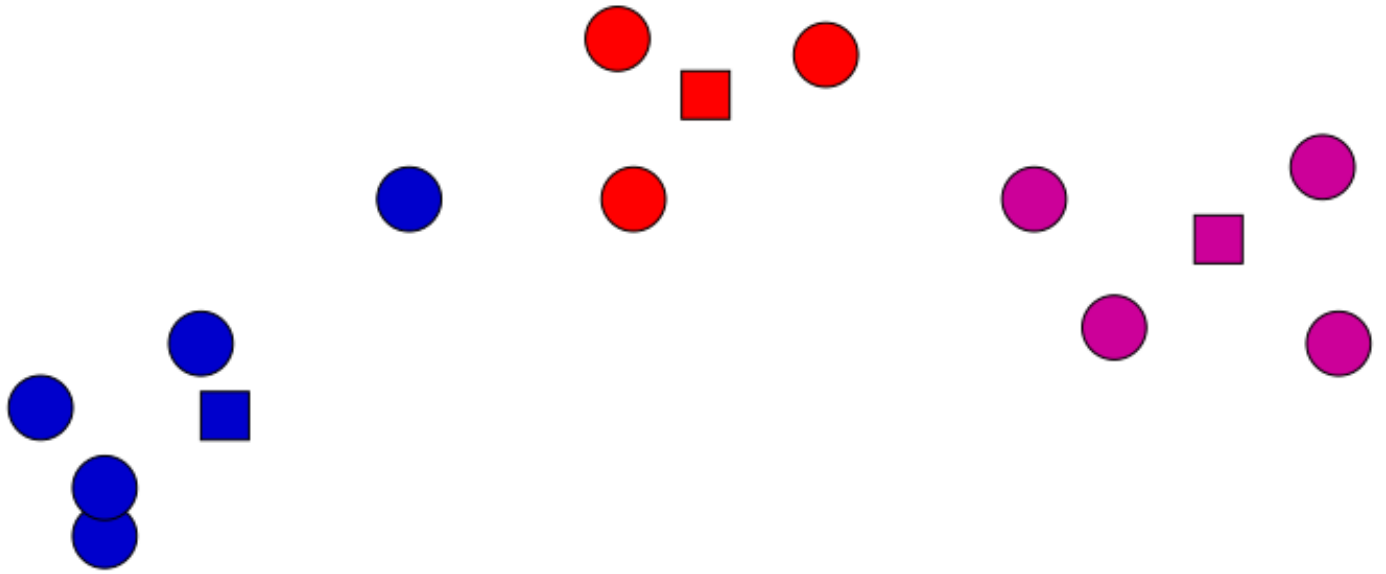

K-means

Iterate:

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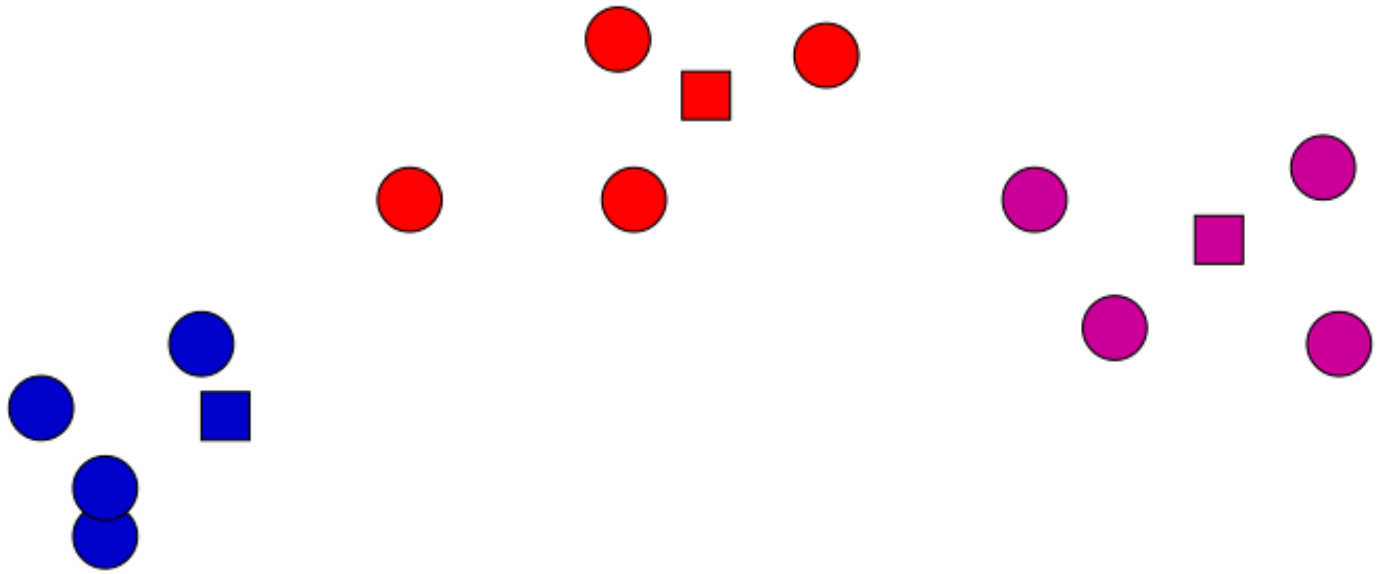
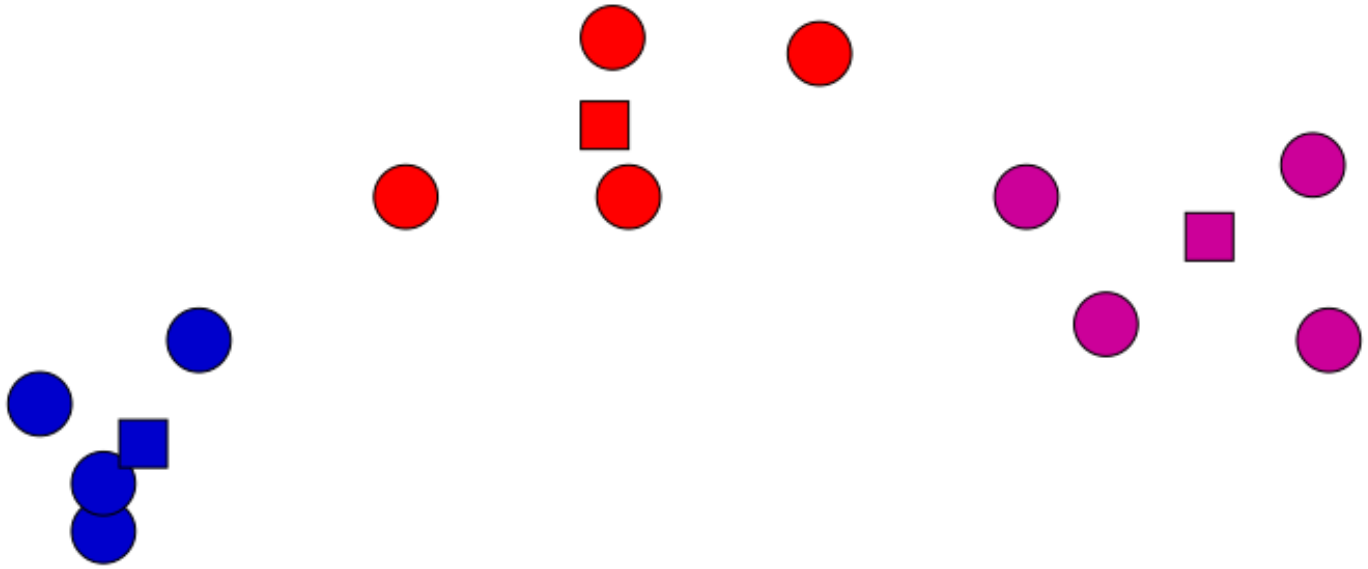


K-means

Iterate:

Assign/cluster each example to closest center

Recalculate centers as the mean of the points in a cluster

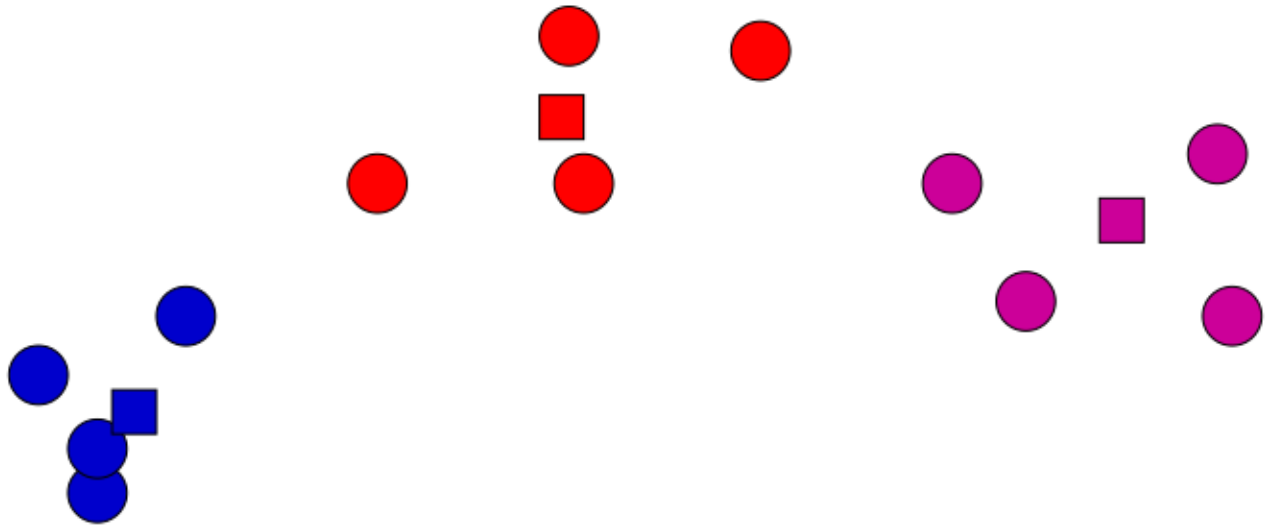


K-means

Iterate:

Assign/cluster each example to closest center

Recalculate centers as the mean of the points in a cluster

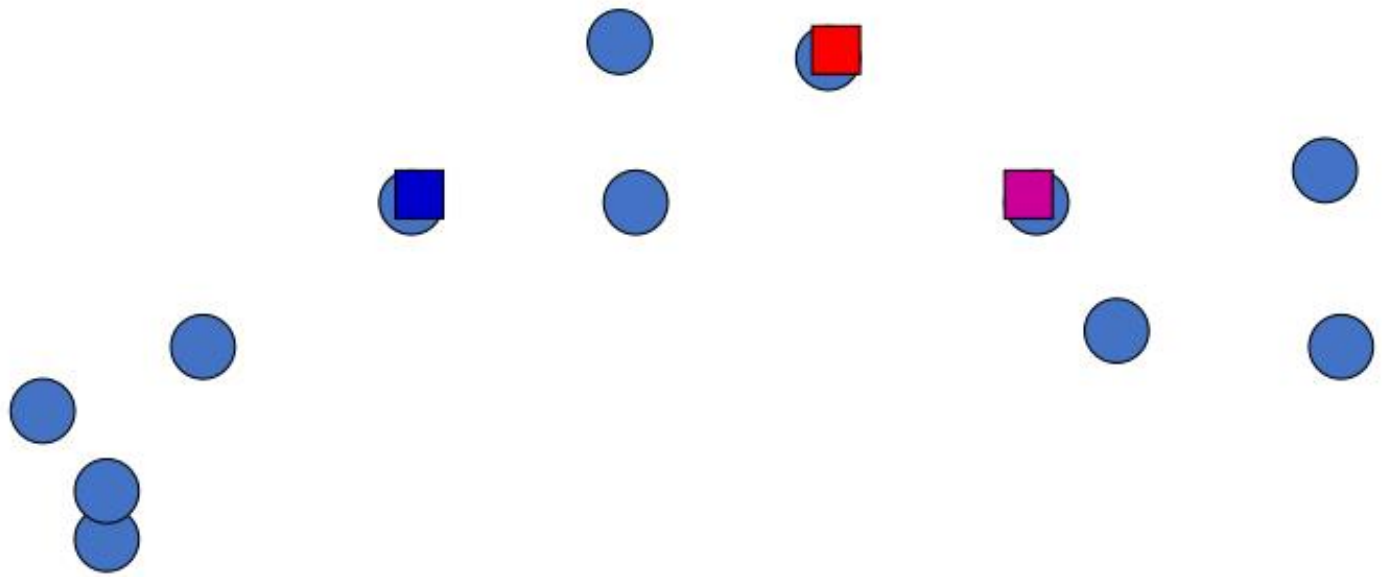


No changes: Done

K-means

Iterate:

- **Assign/cluster each example to closest center**
- Recalculate centers as the mean of the points in a cluster



How do we do this?

K-means

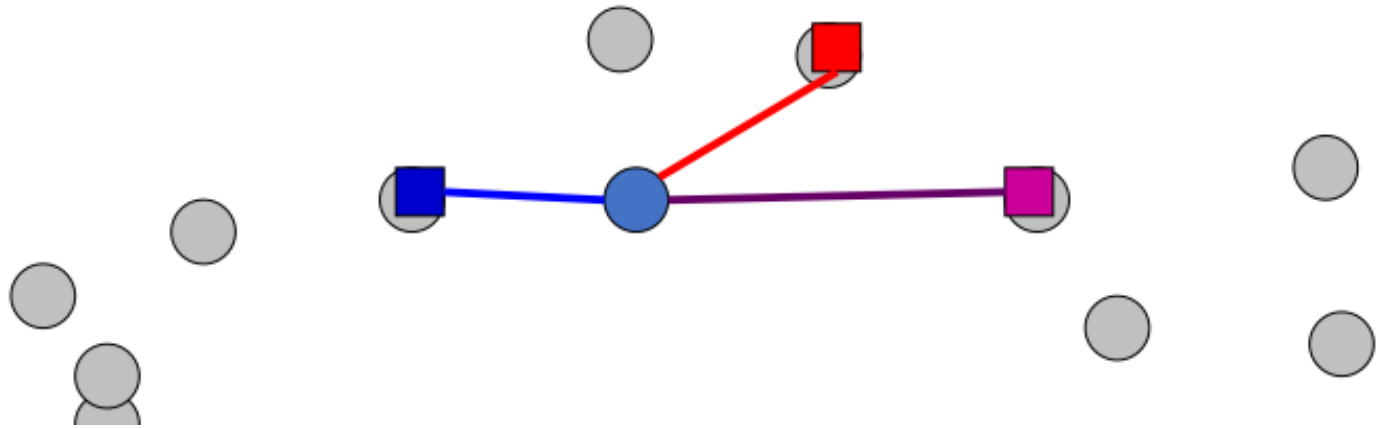
Iterate:

- **Assign/cluster each example to closest center**

  iterate over each point:

  - get distance to each cluster center
  - assign to closest center (hard cluster)

- Recalculate centers as the mean of the points in a cluster



K-means

Iterate:

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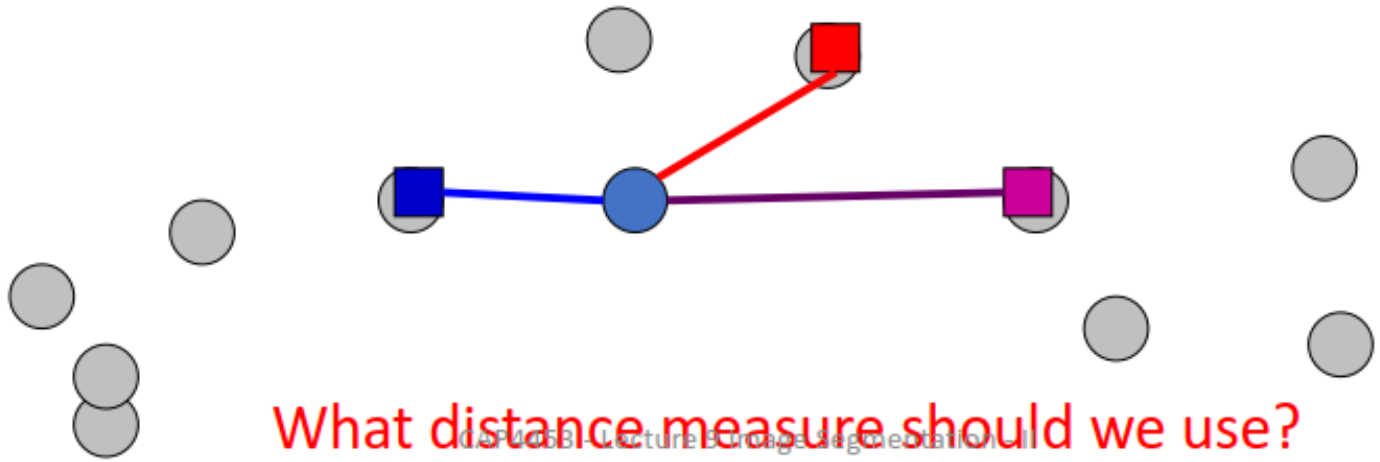
  iterate over each point:

  - get **distance** to each cluster center
  - assign to closest center (hard cluster)

- Recalculate centers as the mean of the points in a cluster



What distance measure should we use?

Distance measures

$$d = \sqrt{\sum_i (x_i^2 - y_i^2)}$$

- p-norm

$$\|x\|_p = \left(\sum_i |a_i|^p \right)^{\frac{1}{p}} \qquad p \geq 1$$

- Euclidean norm

$$\|x\|_2 = \left(\sum_i |a_i|^2 \right)^{1/2}$$

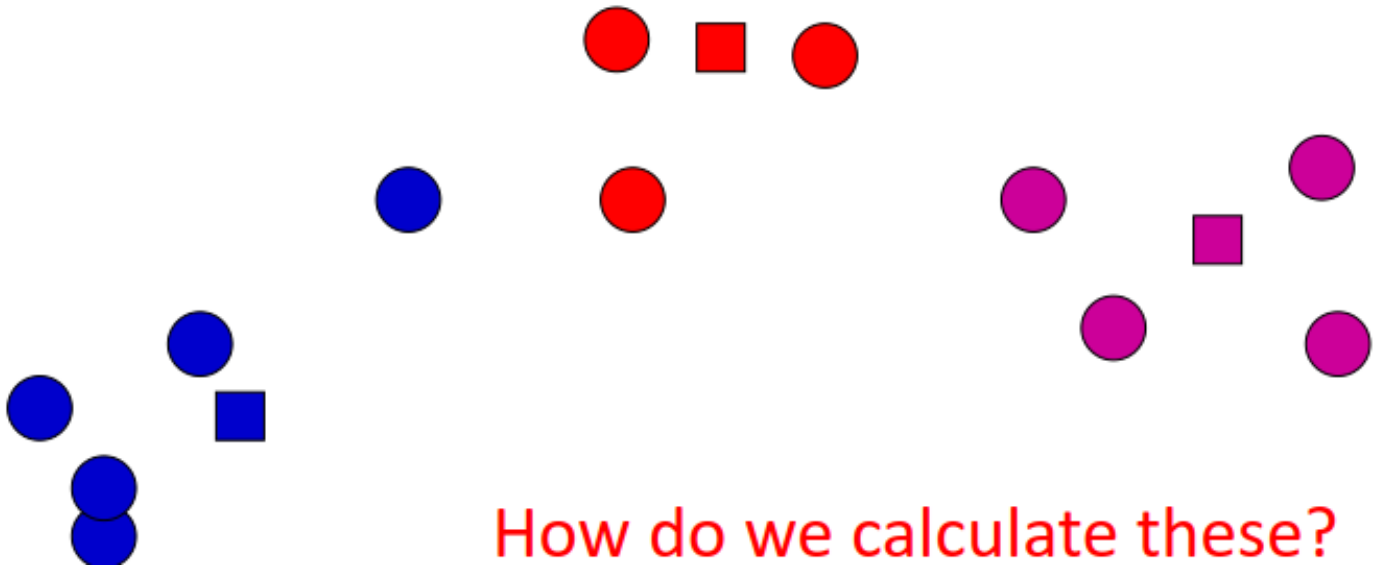
- L1-norm

$$\|x\|_1 = \left(\sum_i |a_i| \right)$$

K-means

Iterate:

- Assign/cluster each example to closest center
- Recalculate centers as the mean of the points in a cluster



How do we calculate these?

K-means

Iterate:

- Assign/cluster each example to closest center
- Recalculate centers as the mean of the points in a cluster

$$\mu_j = \frac{\sum_{i:y_i=j} x_i}{\sum_{i:y_i=j} 1}$$

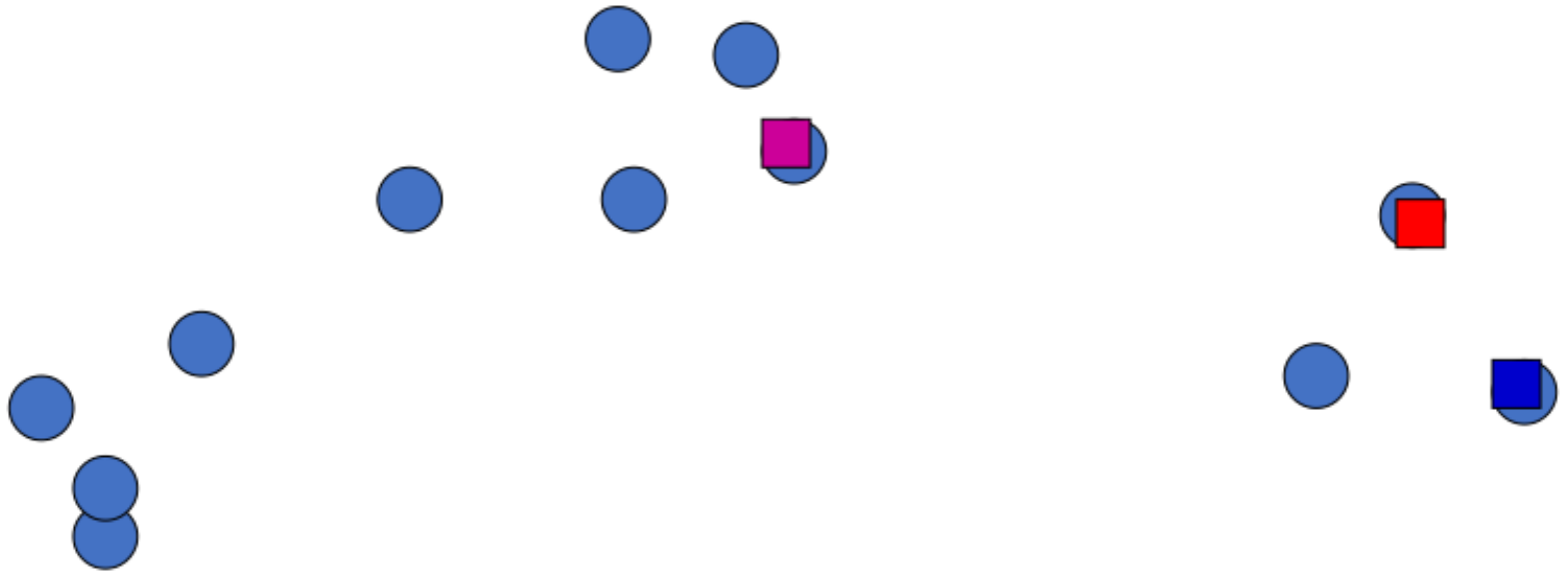
K-means loss function

K-means tries to minimize what is called the “k-means” loss function:

$$loss = \sum_{i=1}^{n} d^2(x_i, \mu_k), \qquad where\ \mu_k\ is\ the\ cluster\ center\ for\ x_i$$

that is, the sum of the squared distances from each point to the associated cluster center

K-means: initialization



What would happen here?

Seed selection ideas?

Seed choice

Results can vary drastically based on random seed selection

Some seeds can result in poor convergence rate, or convergence to sub-optimal clusterings

Common heuristics

- Random centers in the space
- Randomly pick examples
- Points least similar to any existing center (furthest centers heuristic)
- **Try out multiple starting points**
- Initialize with the results of another clustering method

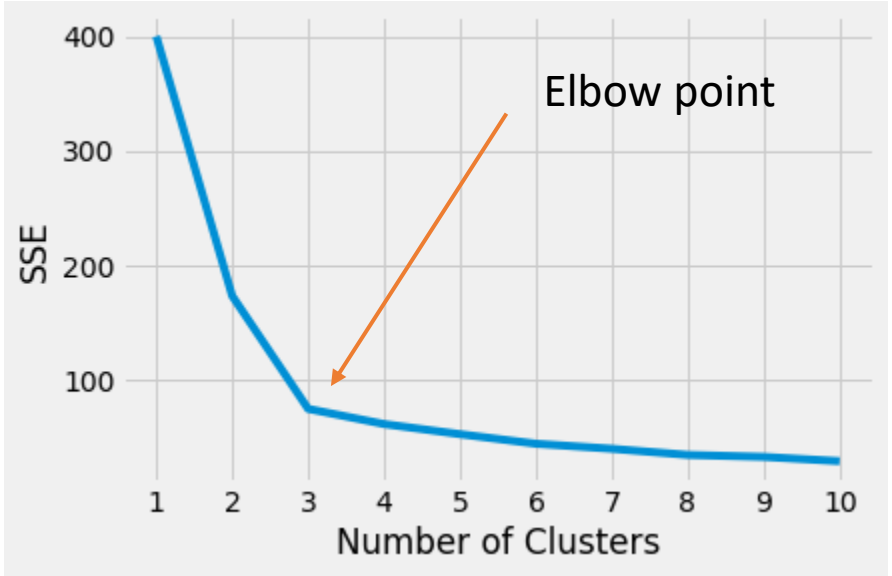
Choosing the Appropriate Number of Clusters

1. The **elbow method**
2. The **silhouette coefficient**

K-Means Clustering in Python: A Practical Guide – Real Python

Choosing the Appropriate Number of Clusters

- run several k-means,
- increment k with each iteration
- record the **sum of the squared error (SSE)**
  - The SSE is defined as the sum of the squared Euclidean distances of each point to its closest centroid



1. The **elbow method**

Choosing the Appropriate Number of Clusters

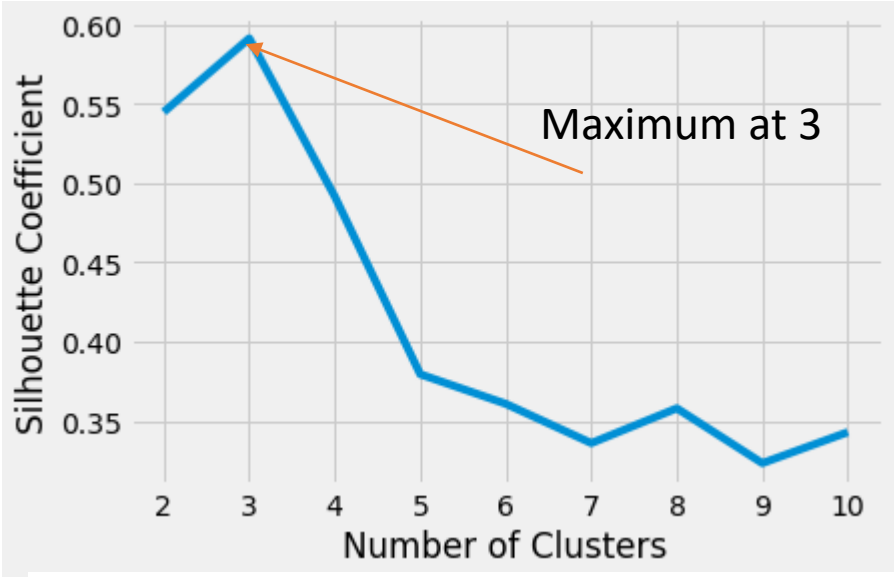
- run several k-means,
- increment k with each iteration
- Pick max [silhouette coefficient]

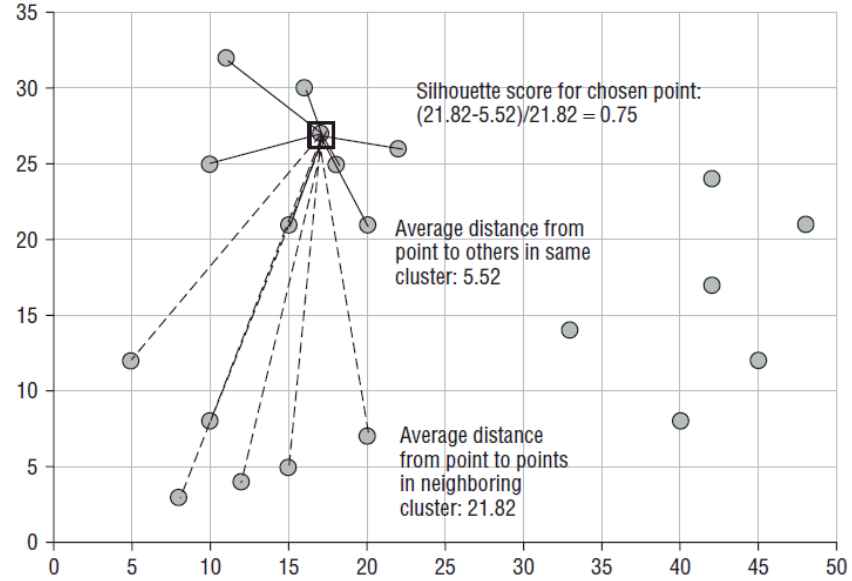
1. How close the data point is to other points in the cluster
2. How far away the data point is from points in other clusters

- (b - a) / max(a, b). Where,

  a: intra-cluster distance

  b: distance between a sample and the nearest cluster that the sample is not a part of.





2. The **silhouette coefficient**

Segmenting an image with K-means

- Example:
  - Vector: (coordinates i, coordinate j , Color L, Color a, Color b) : 5 dims
  - Distance: Euclidean distance
  - Number of clusters: 10
  - Seeds selected randomly

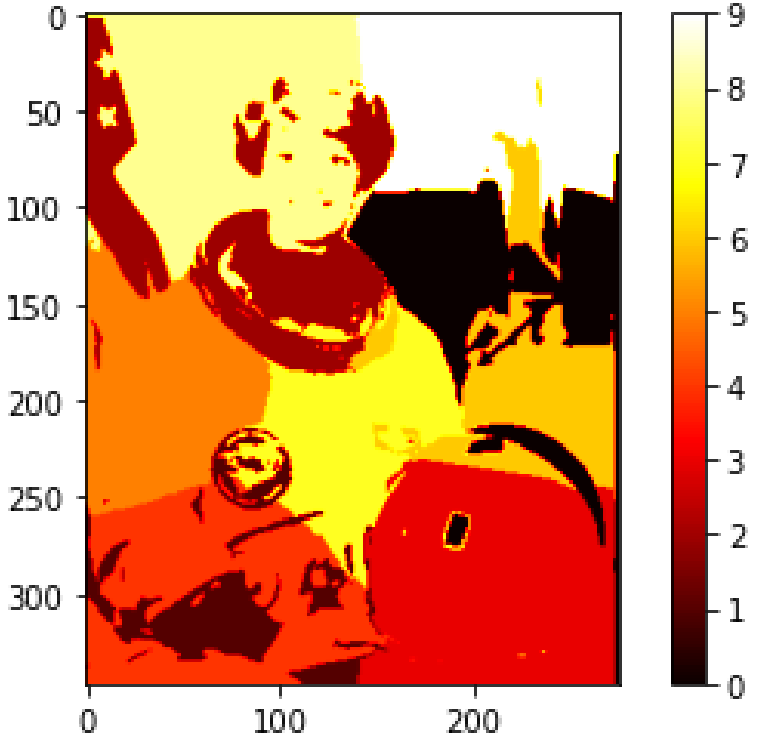
Kmeans function from scratch

```python
import numpy as np
from scipy.spatial.distance import cdist
import matplotlib.pyplot as plt
import cv2

#Defining our function
def kmeans(x,k, no_of_iterations):
    idx = np.random.choice(len(x), k, replace=False)
    #Randomly choosing Centroids
    centroids = x[idx, :] #Step 1

    #finding the distance between centroids and all the data points
    distances = cdist(x, centroids ,'euclidean') #Step 2

    #Centroid with the minimum Distance
    points = np.array([np.argmin(i) for i in distances]) #Step 3

    #Repeating the above steps for a defined number of iterations
    #Step 4
    for _ in range(no_of_iterations):
        centroids = []
        for idx in range(k):
            #Updating Centroids by taking mean of Cluster it belongs to
            temp_cent = x[points==idx].mean(axis=0)
            centroids.append(temp_cent)

        centroids = np.vstack(centroids) #Updated Centroids

        distances = cdist(x, centroids ,'euclidean')
        points = np.array([np.argmin(i) for i in distances])

    return points
```

Calling Kmeans

```
45  imFile = 'C:\\Users\\gonza\\OneDrive\\Teaching\\CAP4453\\class12\\img.png'
46
47  im = cv2.imread(imFile)
48  LabImg = cv2.cvtColor(im,cv2.COLOR_BGR2LAB)
49
50
51  i = np.linspace(0, LabImg.shape[0]-1, LabImg.shape[0]).astype(int)
52  j = np.linspace(0, LabImg.shape[1]-1, LabImg.shape[1]).astype(int)
53  xv, yv = np.meshgrid(i, j)
54
55  numpoints = xv.ravel().shape[0]
56
57  L = LabImg[xv.ravel(),yv.ravel(),0].reshape((numpoints,1))
58  a = LabImg[xv.ravel(),yv.ravel(),1].reshape((numpoints,1))
59  b = LabImg[xv.ravel(),yv.ravel(),2].reshape((numpoints,1))
60
61
62  #X=np.concatenate((xv.ravel().reshape((numpoints,1)), yv.ravel().reshape((numpoints,1)),L,a,b))
63  X=np.concatenate((xv.ravel().reshape((numpoints,1)), yv.ravel().reshape((numpoints,1)),L,a,b), axis=1)
64
65
66  points = kmeans(X,10,50)
67
68  newImg = np.zeros((im.shape[0],im.shape[1]))
69  newImg[xv.ravel(),yv.ravel()]=points
70  im1=plt.imshow(newImg, cmap='jet'); plt.colorbar(im1, cmap='jet'); plt.show()
71
```

Results



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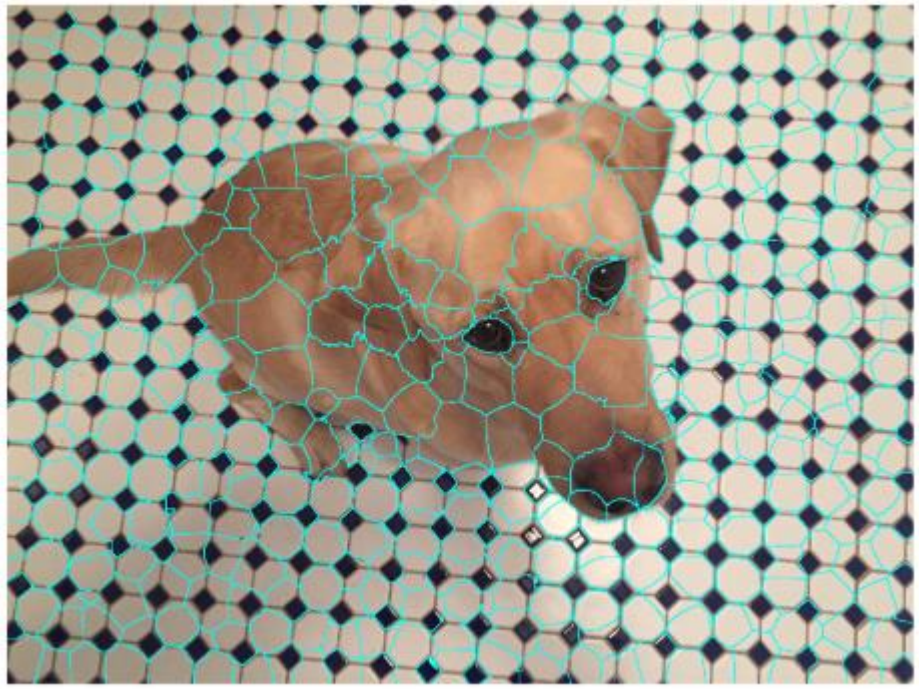
  - ~~Binarization~~
  - ~~Otsu~~
- ~~Region based~~
  - ~~Merging~~
  - ~~Splitting~~
- **Clustering based**
  - K-means
  - **Superpixels (SLIC)**

Superpixels

- They carry more information than pixels.
- Superpixels have a perceptual meaning since pixels belonging to a given superpixel share similar visual properties.
- They provide a convenient and compact representation of images that can be very useful for computationally demanding problems.



[Superpixels and SLIC. What is a Superpixel? | by Darshita Jain | Medium]

SLIC (Anchanta et. al. TPAMI 2012)

Input:

- a desired number of approximately equally-sized superpixels K

N	Number of pixels in the input image
K	Number of Superpixels used to segment the input image
N/K	Approximate size of each superpixel
$S = \sqrt{N/K}$	For roughly equally sized superpixels there would be a superpixel centre at every grid interval S

Features:

five-dimensional [labxy] space,

- [lab] is the pixel color vector in CIELAB color space
- xy is the pixel position.

Distances:

$$d_{lab} = \sqrt{(l_k - l_i)^2 + (a_k - a_i)^2 + (b_k - b_i)^2}$$

$$d_{xy} = \sqrt{(x_k - x_i)^2 + (y_k - y_i)^2}$$

$$D_s = d_{lab} + \frac{m}{S} d_{xy} \ ,$$

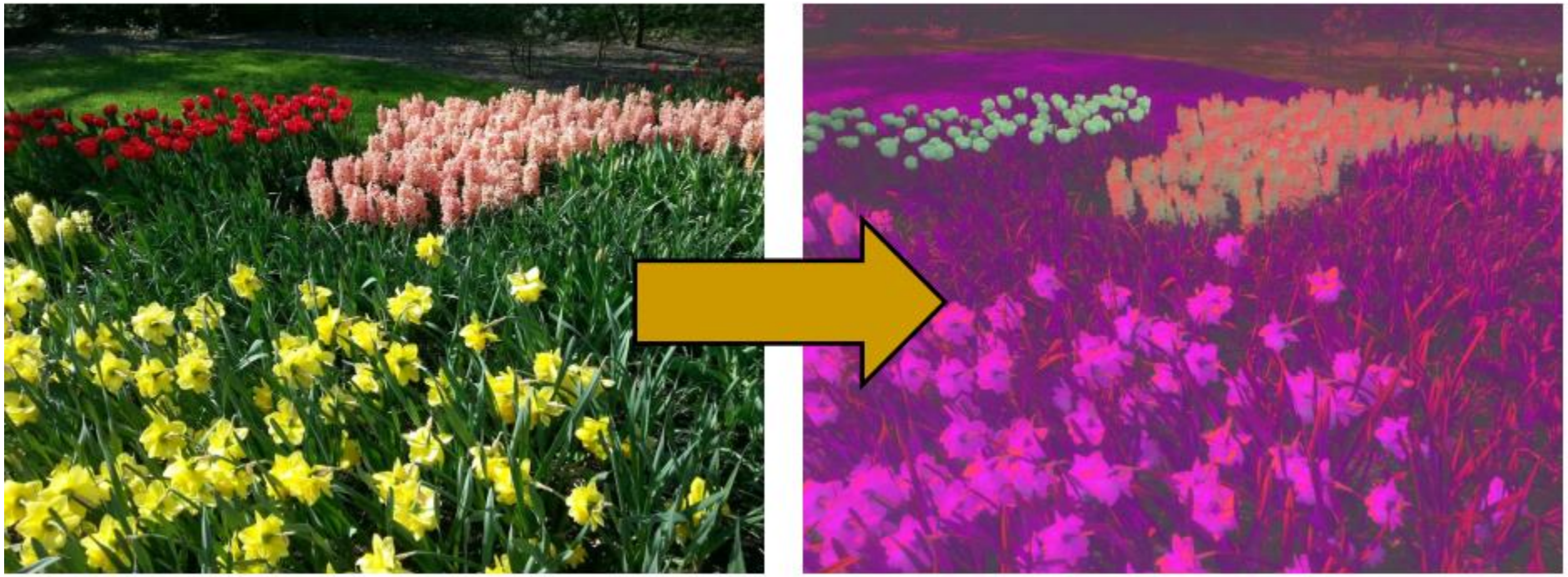
SLIC (Anchanta et. al. TPAMI 2012)

1. Get Features: Lab color, x-y position
2. Initialize cluster centers on pixel grid in steps S
3. Move centers to position in 3x3 window with smallest gradient
4. Compare each pixel to cluster center within 2S pixel distance and assign to nearest
5. Recompute cluster centers as mean color/position of pixels belonging to each cluster
6. Stop when residual error is small

+ Fast 0.36s for 320x240
+ Regular superpixels
+ Superpixels fit boundaries
- May miss thin objects
- Large number of superpixels

SLIC Example

1. Convert the RGB image to CIELAB color space.



http://www.ele.puc-rio.br/~raul/ImageAnalysis/Superpixels.pdf

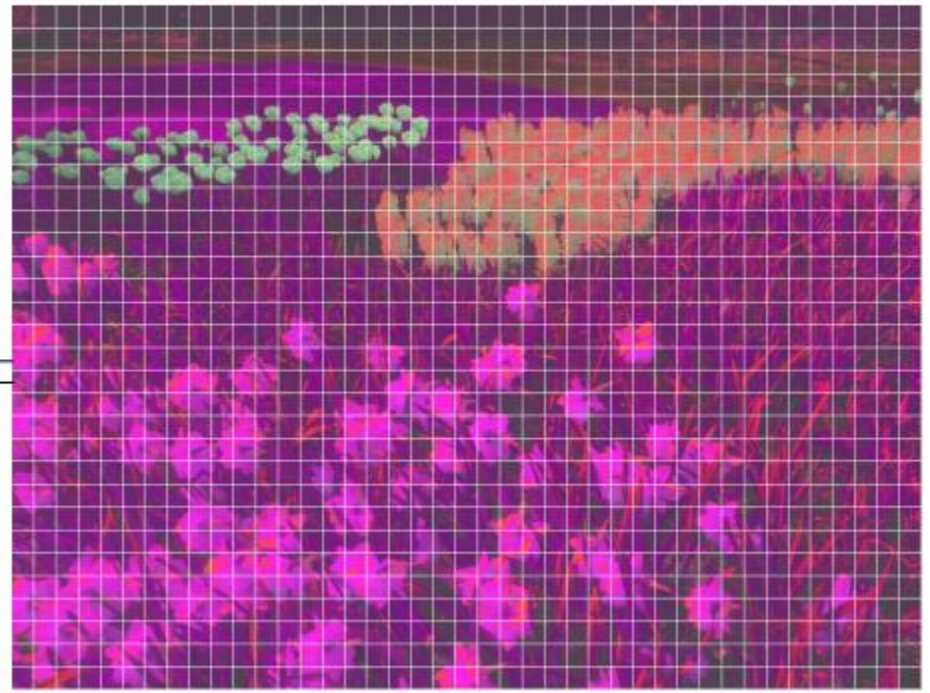
SLIC Example

2. Initialize cluster centers $C_k = [l_k; a_k; b_k; x_k; y_k]^T$ by sampling pixels at regular grid steps S.

number of pixels in the image

$$\sqrt{N/K} = S$$

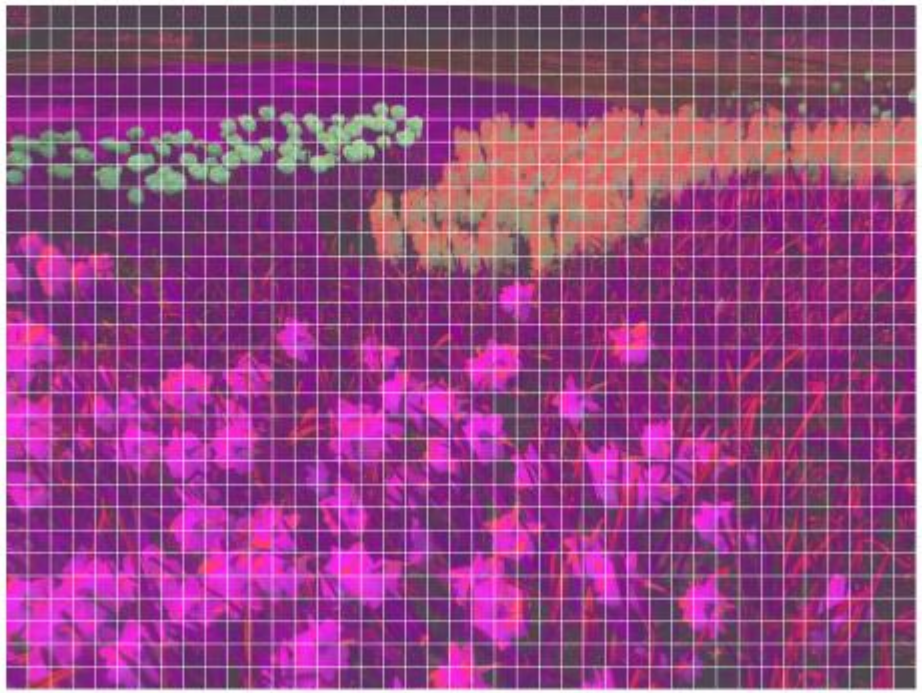
desired number of superpixels



http://www.ele.puc-rio.br/~raul/ImageAnalysis/Superpixels.pdf

SLIC Example

3. Move cluster centers to the lowest gradient position in a 3×3 neighborhood.

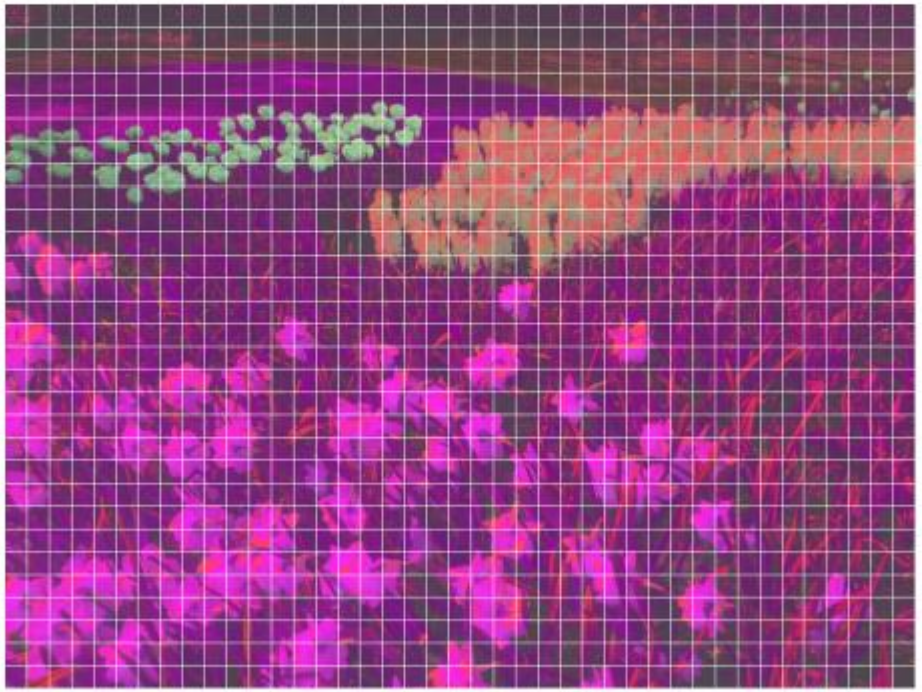


This is done to avoid placing them at an edge and to reduce the chances of choosing a noisy pixel

http://www.ele.puc-rio.br/~raul/ImageAnalysis/Superpixels.pdf

SLIC Example

3. Move cluster centers to the lowest gradient position in a 3×3 neighborhood.



$$G(x,y) = \|I(x+1,y)-I(x-1,y)\|^2+\|I(x,y+1)-I(x,y-1)\|^2$$

- I(x,y) is the lab vector corresponding to the pixel at position (x,y),
- ‖.‖ is the L2 norm.

http://www.ele.puc-rio.br/~raul/ImageAnalysis/Superpixels.pdf

SLIC Example

4. A 2D label matrix L as large as the input image will contain the superpixel each pixel belongs to. L is initialized with -1 for all pixels.

   (meaning that each pixel belongs to no superpixel in the beginning)

$L=$

-1	-1	-1	-1	-1	-1
-1	-1	-1	-1	-1	
-1	-1	-1			
-1	-1				

http://www.ele.puc-rio.br/~raul/ImageAnalysis/Superpixels.pdf

SLIC Example

5. A 2D distance matrix d as large as the input image will contain the distance of each pixel to the centroid of its superpixel. d is initialized with ∞ for all pixels.

(distance to superpixel centroid in the beginning)

$$d = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \\ \infty & \infty & \infty & & \\ \infty & \infty & & & \\ & & & & \end{bmatrix}$$

$$d_s = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2}$$

$$d_c = \sqrt{(l_j - l_i)^2 + (a_j - a_i)^2 + (b_j - b_i)^2}$$

$$D' = \sqrt{\left(\frac{d_c}{m}\right)^2 + \left(\frac{d_s}{S}\right)^2}$$

http://www.ele.puc-rio.br/~raul/ImageAnalysis/Superpixels.pdf

SLIC Example

5. A 2D distance matrix d as large as the input image will contain the distance of each pixel to the centroid of its superpixel. d is initialized with ∞ for all pixels.

(distance to superpixel centroid in the beginning)

$d =$

∞	∞	∞	∞	∞	∞
∞	∞	∞	∞	∞	
∞	∞	∞			
∞					

$$D = \sqrt{(d_c)^2 + \left(\frac{d_s}{S}\right)^2 m^2}$$

controls the relative importance of shape and color

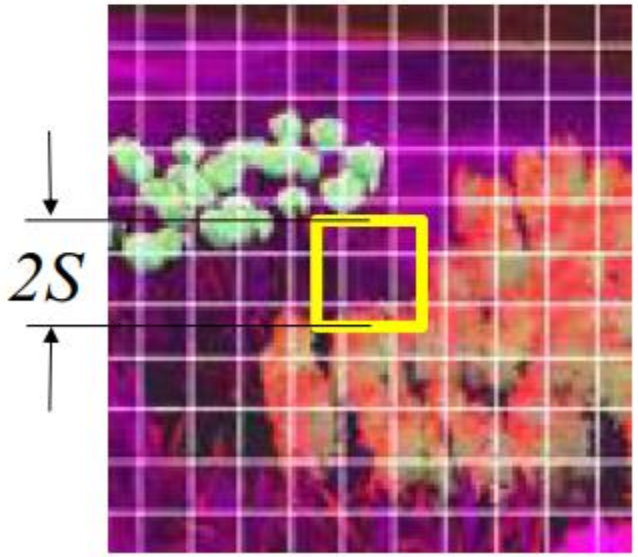
m large → favors more compact (lower area to perimeter ratio) superpixels.

m small → favors more adherence to edges.

http://www.ele.puc-rio.br/~raul/ImageAnalysis/Superpixels.pdf

SLIC Example

6. **repeat**
  for each cluster center C_k **do**
    for each pixel i in a $2S \times 2S$ region around C_k do
      Compute the distance D between C_k and i.
      if $D < d(i)$ **then**
        set $d(i) = D$
        set $L(i) = k$
      end if
    end for
  end for
  compute new cluster centers.
  compute residual error E.
until $E <$ threshold.



http://www.ele.puc-rio.br/~raul/ImageAnalysis/Superpixels.pdf

Example 1: image size = 735×980 pixels

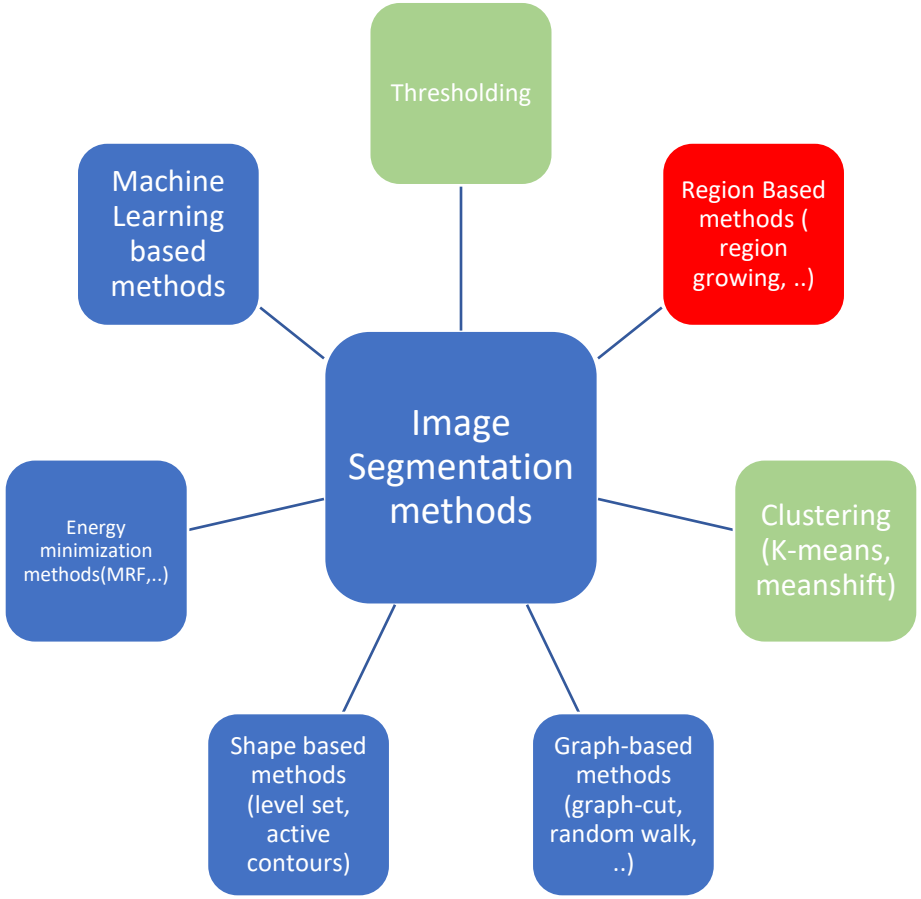
K = 1333 superpixels; m = 40

More examples

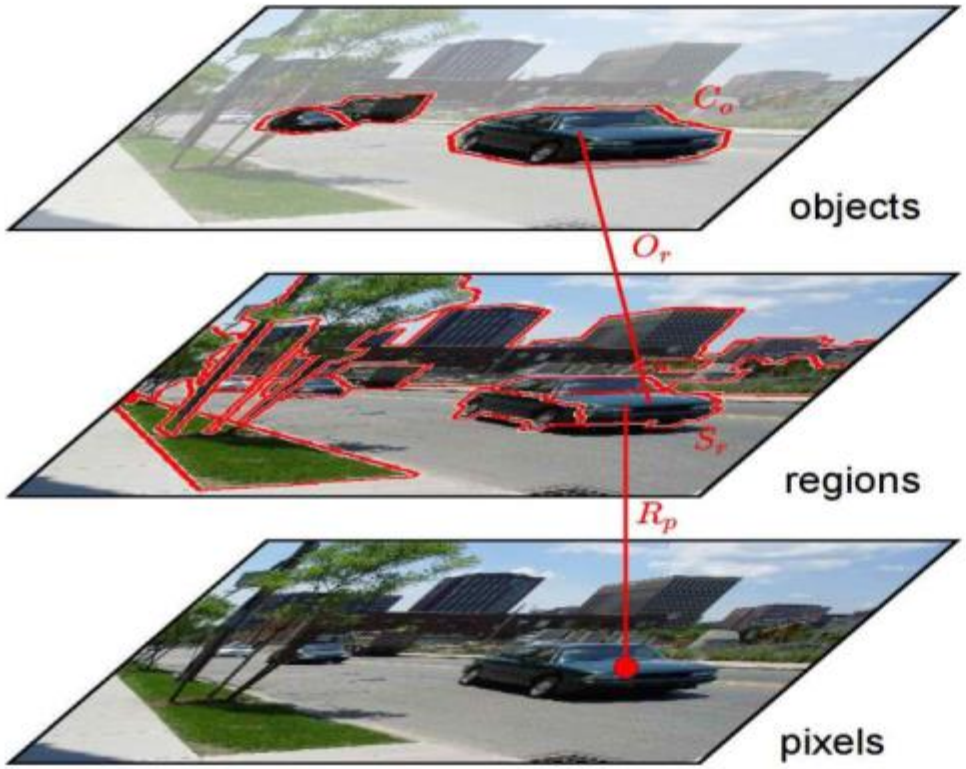
Outline

- Image segmentation basics
- Thresholding based
  - Binarization
  - Otsu
- Clustering based
  - K-means (SLIC)
- **Region based**
  - **Merging**
  - Splitting

Image segmentation methods



Region based segmentation



Region:

A group of connected pixels with similar properties

Closed boundaries

Computation of regions is based on similarity

Regions may correspond to Objects in a scene or parts of objects

Spatial proximity + similarity

Region growing

- For segment generation in grey-level or color images, we may start at one seed pixel (x,y,I(x,y)) and add recursively adjacent pixels that satisfy a “similarity criterion” with pixels contained in the so-far grown region around the seed pixel.

- Defining similarity criteria alone is not an effective basis for segmentation
- It is necessary to consider the adjacency spatial relationship between pixels

Region growing

- Algorithm

1. The absolute intensity difference between candidate pixel and the seed pixel must lie within a specified range

2. The absolute intensity difference between a candidate pixel and the running average intensity of the growing region must lie within a specified range;

3. The difference between the standard deviation in intensity over a specified local neighborhood of the candidate pixel and that over a local neighborhood of the candidate pixel must (or must not) exceed a certain threshold

http://cs.brown.edu/people/pfelzens/papers/seg-ijcv.pdf

Efficient Graph-Based Image Segmentation.

Pedro F. Felzenszwalb, Daniel P. Huttenlocher

International Journal of Computer Vision 59(2), 167–181, 2004

Efficient Graph-Based Image Segmentation

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Abstract. This paper addresses the problem of segmenting an image into regions. We define a predicate for measuring the evidence for a boundary between two regions using a graph-based representation of the image. We then develop an efficient segmentation algorithm based on this predicate, and show that although this algorithm makes greedy decisions it produces segmentations that satisfy global properties. We apply the algorithm to image segmentation using two different kinds of local neighborhoods in constructing the graph, and illustrate the results with both real and synthetic images. The algorithm runs in time nearly linear in the number of graph edges and is also fast in practice. An important characteristic of the method is its ability to preserve detail in low-variability image regions while ignoring detail in high-variability regions.

Keywords: image segmentation, clustering, perceptual organization, graph algorithm

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Efficient Graph-Based Image Segmentation.

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Original Image | Incorrect Segmentation | Correct Segmentation

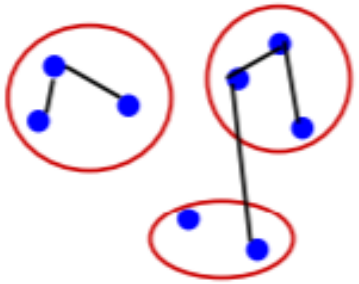
Image Segmentation With Felzenszwalb's Algorithm ! - Analytics Vidhya

http://cs.brown.edu/people/pfelzens/papers/seg-ijcv.pdf

Efficient Graph-Based Image Segmentation.

Pedro F. Felzenszwalb, Daniel P. Huttenlocher

- **Grid Graph:** Each pixel is only connected with surrounding neighbors (8 other cells in total). The edge weight is the absolute difference between the intensity values of the pixels.
- **Nearest Neighbor Graph**: Each pixel is a point in the feature space (x, y, r, g, b), in which (x, y) is the pixel location and (r, g, b) is the color values in RGB. The weight is the Euclidean distance between two pixels' feature vectors.



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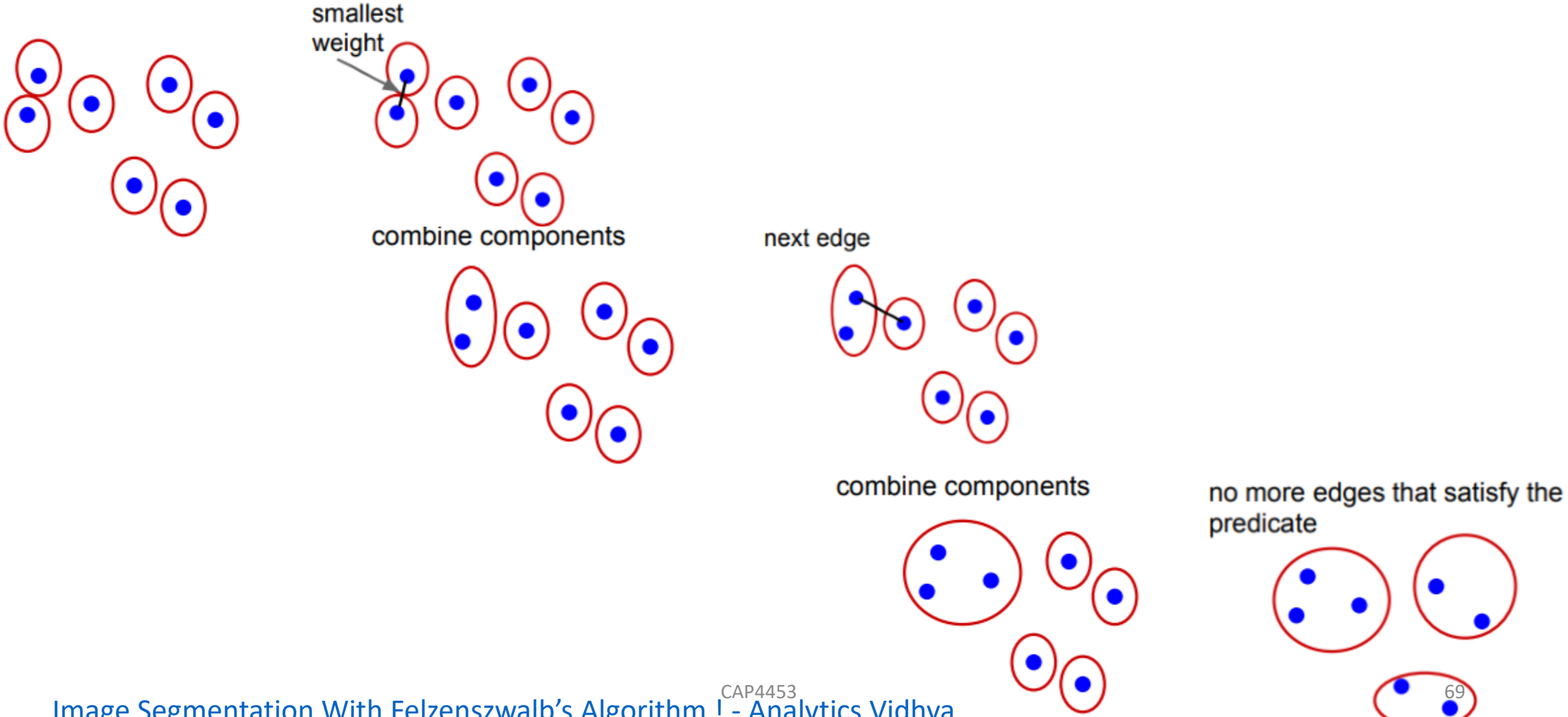

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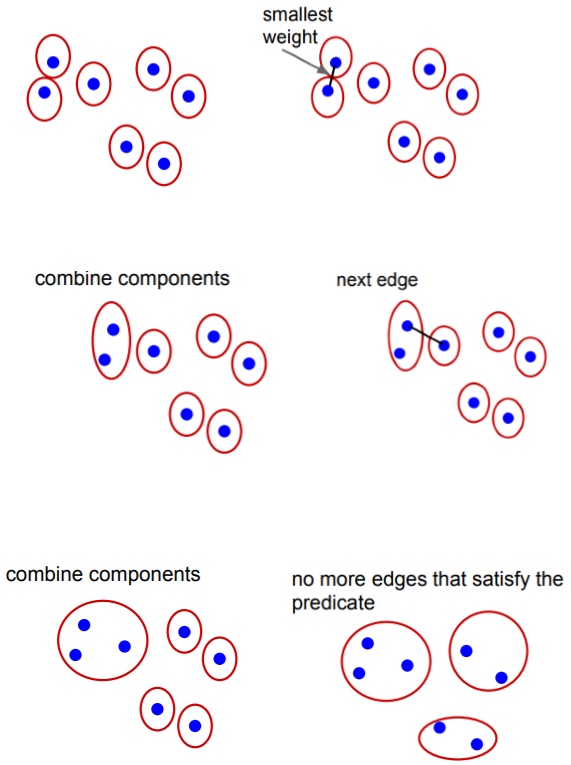

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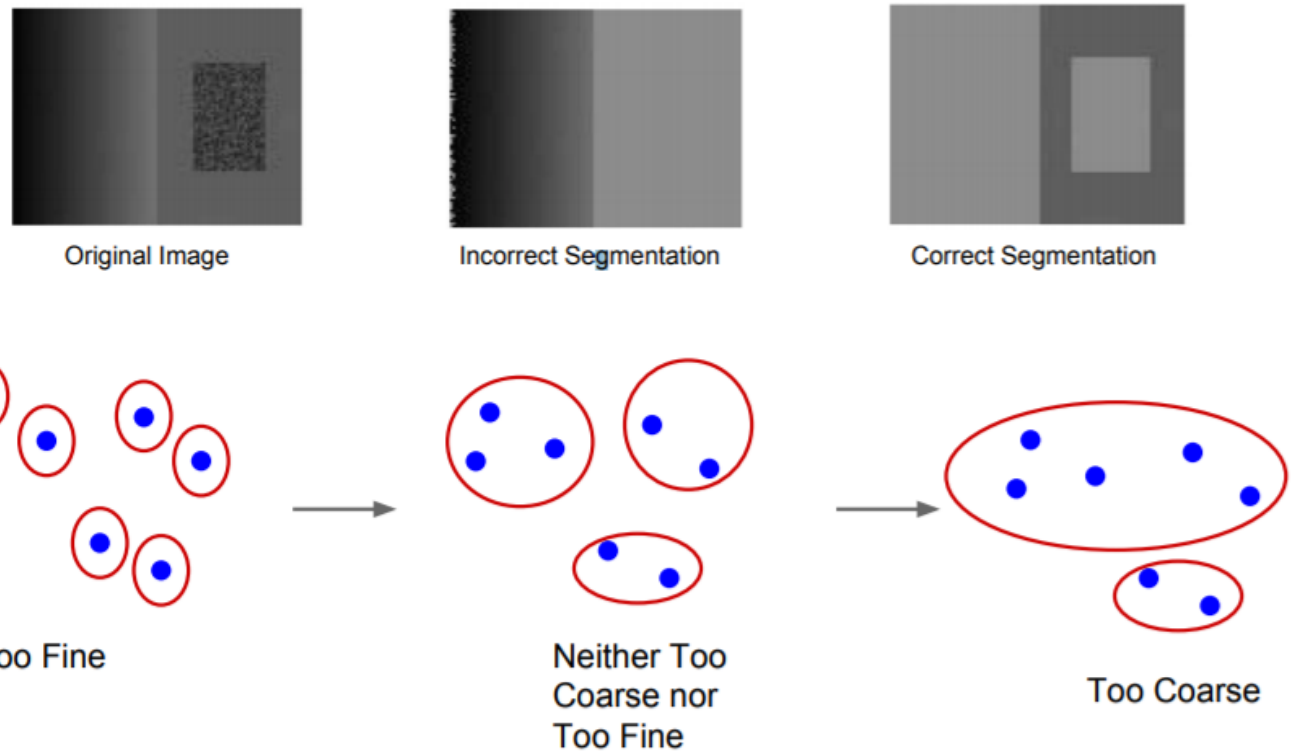

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Internal difference: Int(C) = $\max_{e \in MST(C,E)} \mathbf{w(e)}$, where MST is the minimum spanning tree of the components. A component 'C' can still remain connected even when we have removed all the edges with weights < Int(C). In other words, **Internal Difference is the maximum edge weight that connects two nodes of the same component.**

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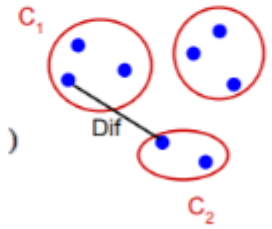
Difference between two components

The difference between the two components is the minimum weight edge that connects a node v_i in component C_1 to node v_j in C_2.



•**Dif (C1,C2)**=$\min_{v_i \in C1, v_j \in C2, (v_i,v_j) \in E}$ **w(vi,vj) and Dif(C1,C2)=∞** if there is no edge in-between. this can also be understood as the difference between two components. The difference between the two components is the minimum weight the edge that connects a node v_i in component C_1 to node v_j in C_2.

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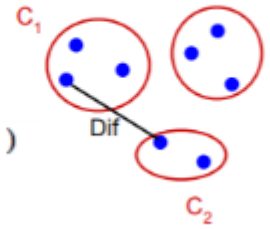
Minimum Internal Difference



MInt(C1,C2) = min(Int(C1) + τ(C1) , Int(C2) + τ(C2))

where **τ(C)=k/|C|**.

$$D(C_1, C_2) = \begin{cases} \text{True} & \text{if } Dif(C_1, C_2) > MInt(C_1, C_2) \\ \text{False} & \text{otherwise} \end{cases}$$

Stop merging criteria

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http://cs.brown.edu/people/pfelzens/papers/seg-ijcv.pdf

Efficient Graph-Based Image Segmentation.

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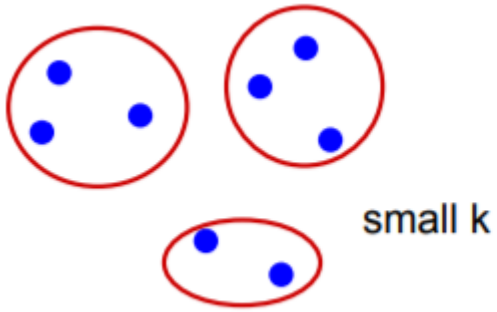
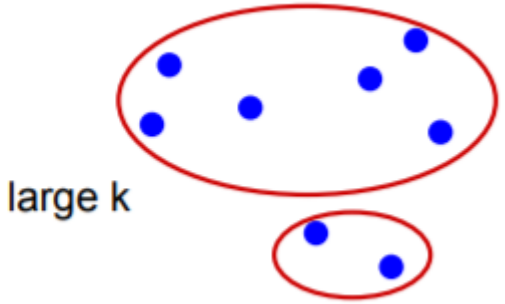
MInt(C1,C2) = min(Int(C1) + τ(C1) , Int(C2) + τ(C2))

where **τ(C)=k/|C|**.

- If k is large, it causes a preference for larger objects.
- k does not set a minimum size for components.





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Stop merging criteria

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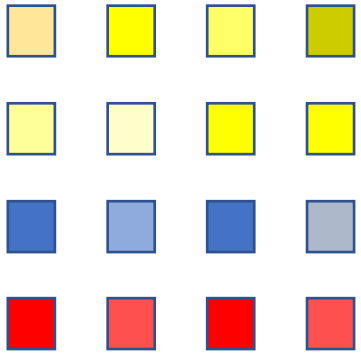
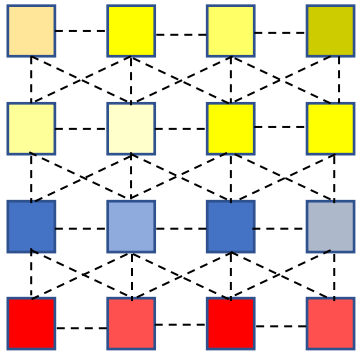
Efficient Graph-Based Image Segmentation. Example (4x4 image)



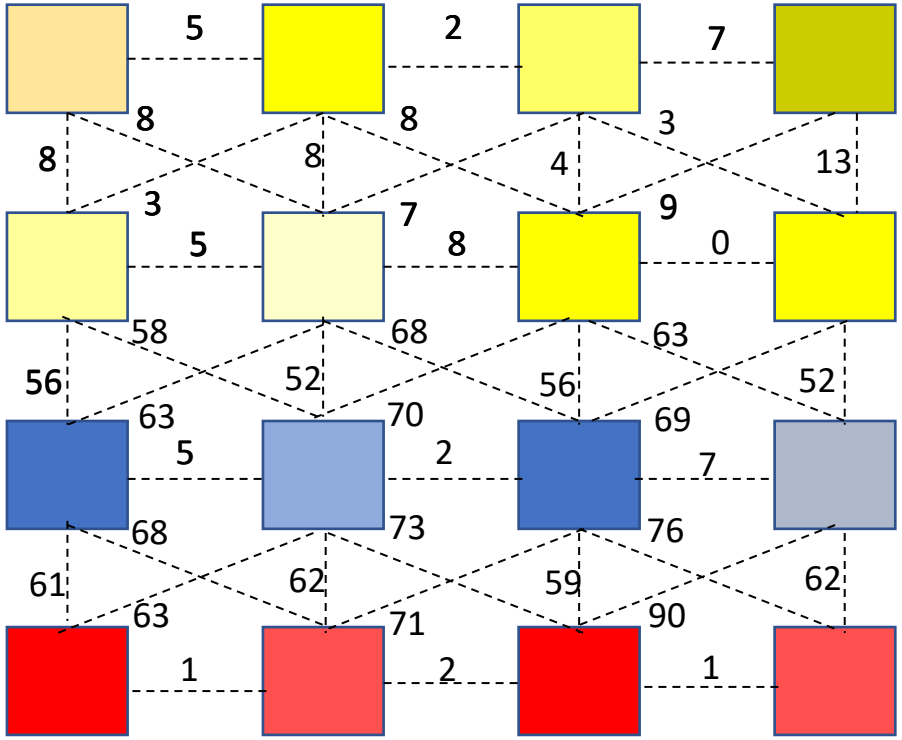
Efficient Graph-Based Image Segmentation. Example (4x4 image)



Build a graph

Efficient Graph-Based Image Segmentation. Example (4x4 image)

Build a graph



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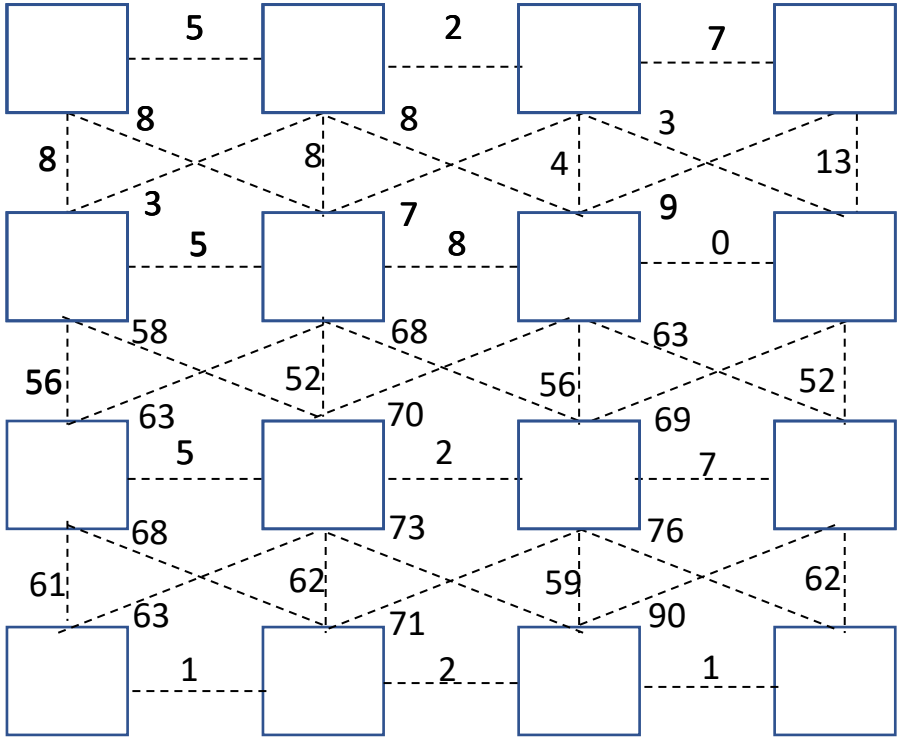

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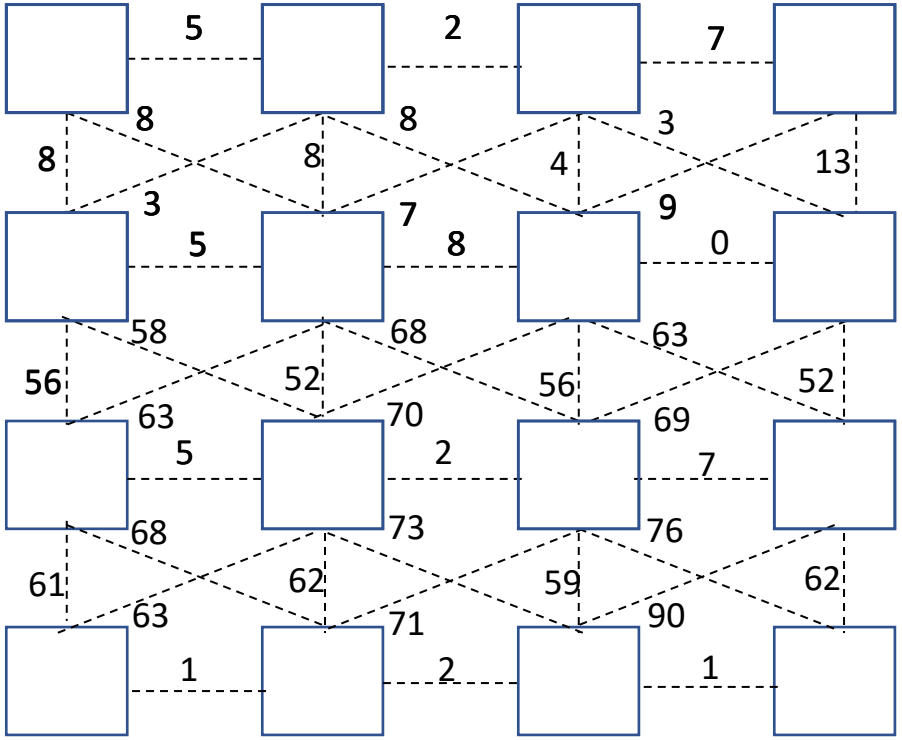

The algorithm follows a bottom-up procedure. Given **G=(V,E)** and **|V|=n, |E|=m.** Where |V| is the number of vertices(pixels) and |E| is the number of edges.

1. Edges are sorted by weight in ascending order, labeled as $e_1,e_2,\ldots,e_m$.
2. Initially, each pixel stays in its own component, so we start with n components.
3. Repeat for k=1,…,m:

- The segmentation snapshot at step k is denoted as Sk.
- We take the k-th edge in the order, ek=(v_i , v_j).
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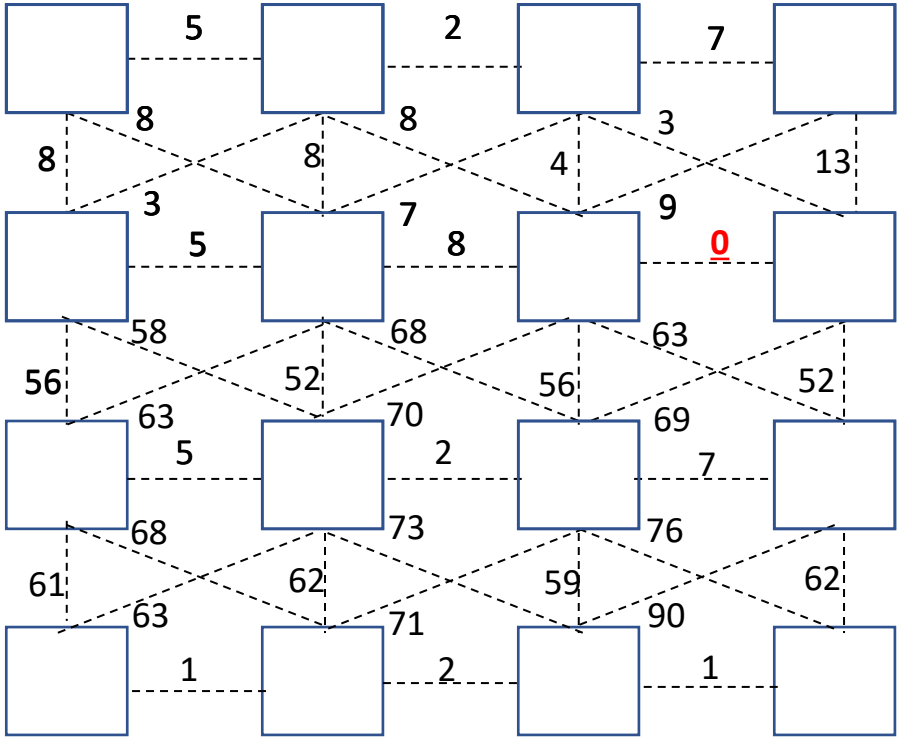

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$$w(v_i, v_j) = 0$$

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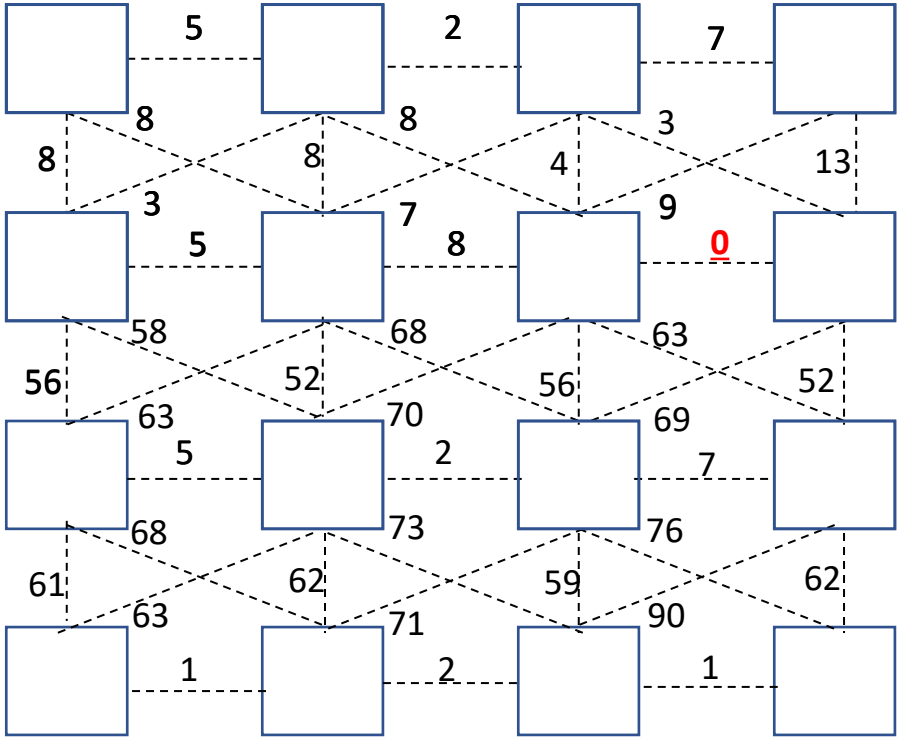

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Let's make k=100

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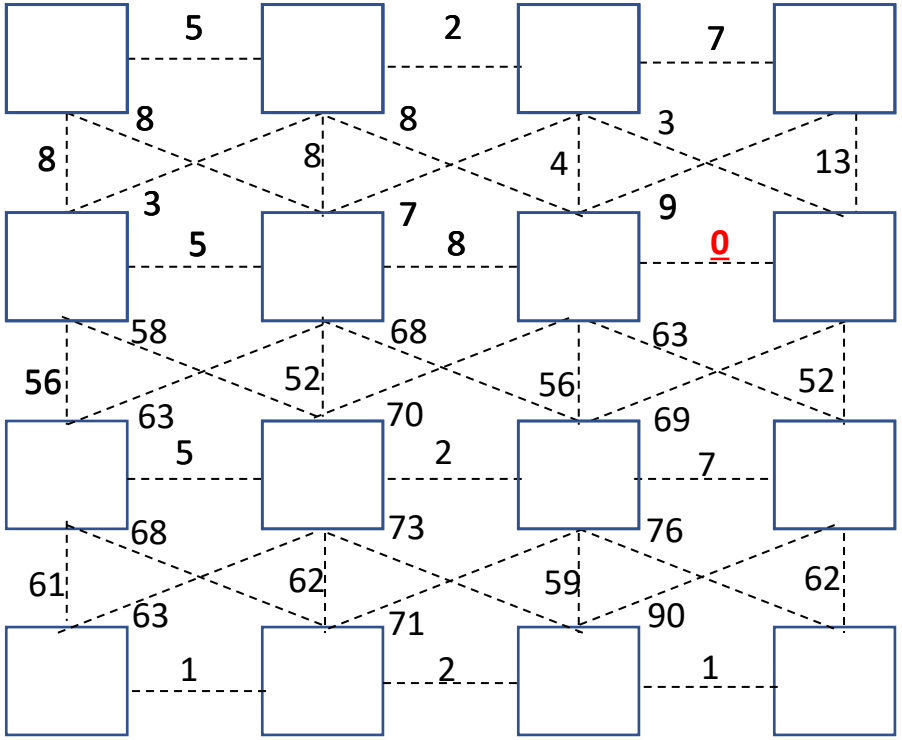

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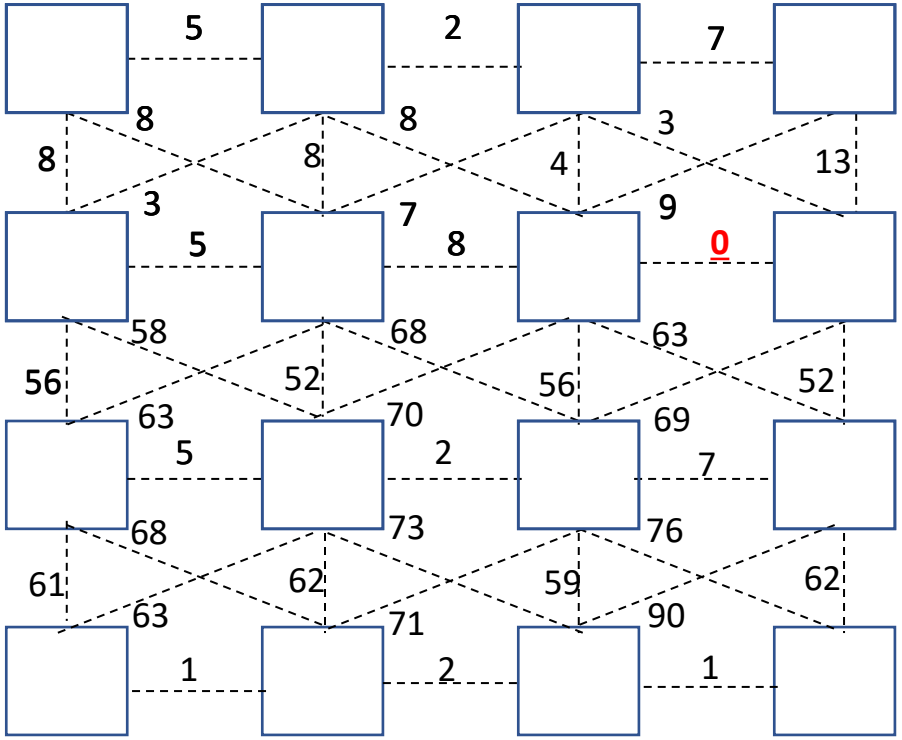
MInt(C1,C2) = min(Int(C1) + τ(C1) , Int(C2) + τ(C2))

Let's make k=100

$$\tau(C_1) = \frac{100}{1}$$

$$\tau(C_2) = \frac{100}{1}$$

Efficient Graph-Based Image Segmentation. Example (4x4 image)



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- If v_i and v_j belong to two different components C_i^{k-1} and C_j^{k-1} as in the segmentation of S^{k-1} we want to merge them into one if $w(v_i,v_j) \le MInt(C_i^{k-1},C_j^{k-1})$; otherwise do nothing.

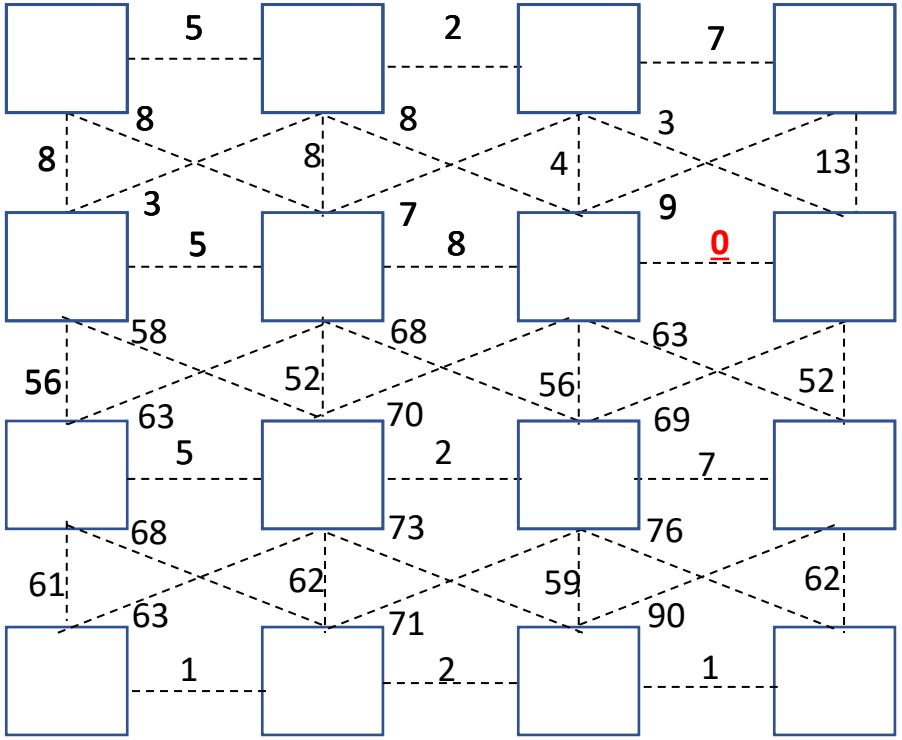
$$w(v_i, v_j) = 0 \quad < \quad \textbf{MInt(C1,C2) = 100}$$

Let's make k=100

$$\tau(C_1) = \frac{100}{1}$$

$$\tau(C_2) = \frac{100}{1}$$

Efficient Graph-Based Image Segmentation. Example (4x4 image)



The algorithm follows a bottom-up procedure. Given **G=(V,E)** and **|V|=n, |E|=m.** Where |V| is the number of vertices(pixels) and |E| is the number of edges.

1. Edges are sorted by weight in ascending order, labeled as $e_1,e_2,\ldots,e_m$.
2. Initially, each pixel stays in its own component, so we start with n components.
3. Repeat for k=1,…,m:

- The segmentation snapshot at step k is denoted as Sk.
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$w(v_i, v_j) = 0$ < **MInt(C1,C2) = 100**

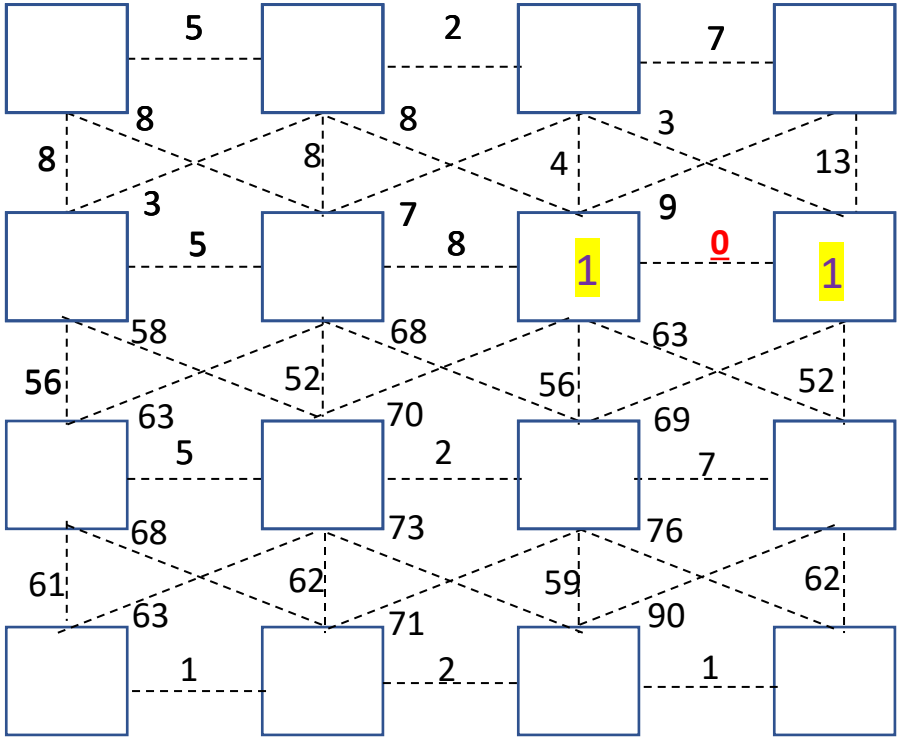
Let's make k=100

$$\tau(C_1) = \frac{100}{1}$$

$$\tau(C_2) = \frac{100}{1}$$

MERGE THEM !

Efficient Graph-Based Image Segmentation. Example (4x4 image)



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Let's make k=100

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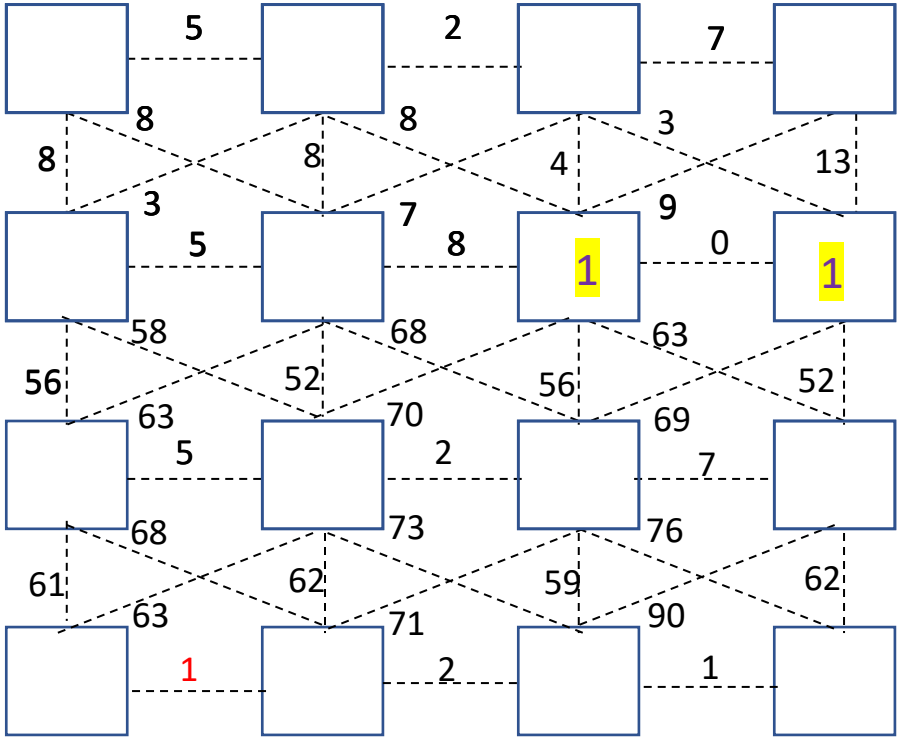
$$\tau(C_2) = \frac{100}{1}$$

MERGE THEM !

Efficient Graph-Based Image Segmentation. Example (4x4 image)



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$$w(v_i, v_j) = 1 \quad < \quad \textbf{MInt(C1,C2) = 100}$$

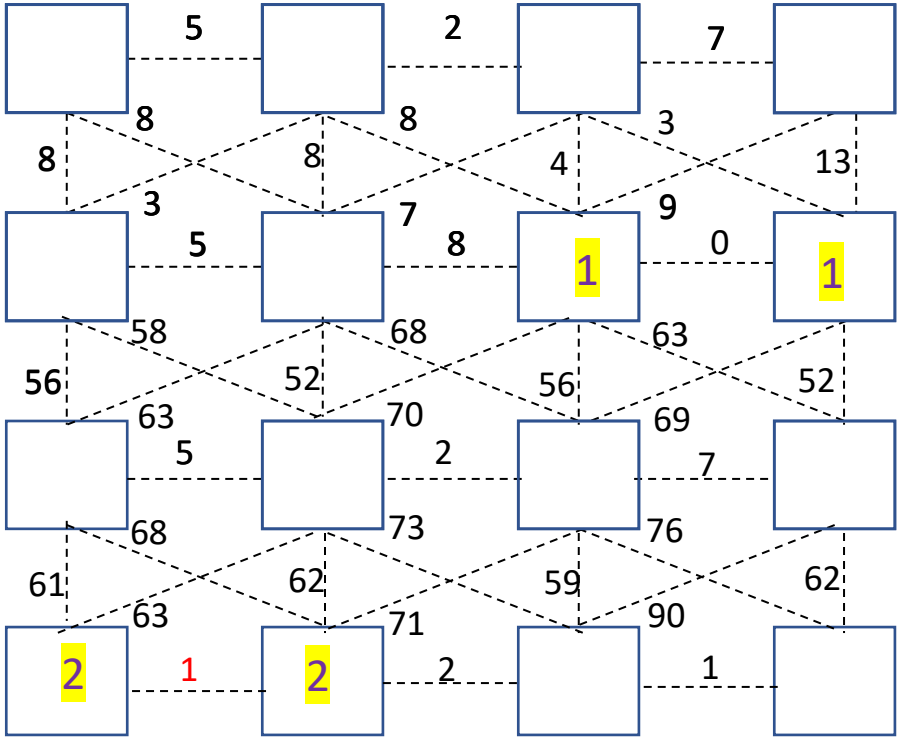
with k=100

$$\tau(C_1) = \frac{100}{1}$$

$$\tau(C_2) = \frac{100}{1}$$

MERGE THEM !

Efficient Graph-Based Image Segmentation. Example (4x4 image)



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with k=100

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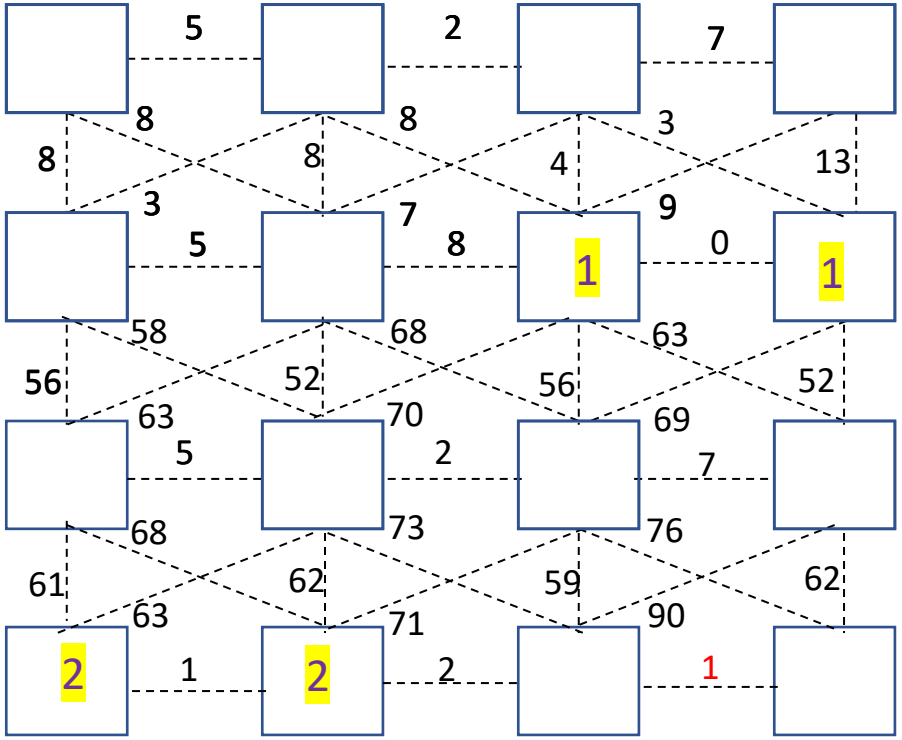
MERGE THEM !

Efficient Graph-Based Image Segmentation. Example (4x4 image)



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with k=100

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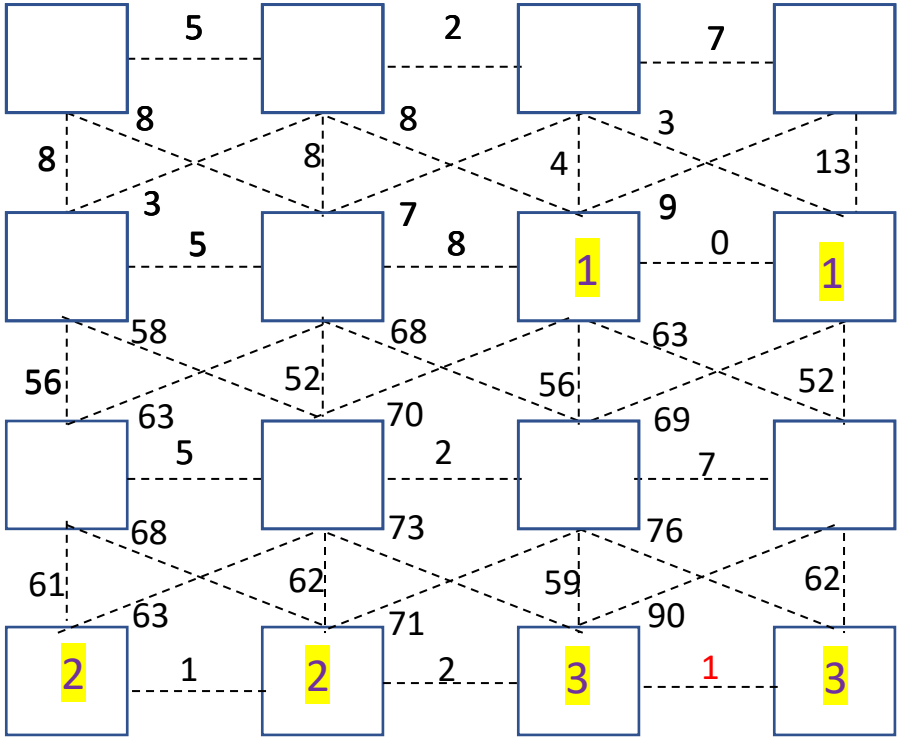
$$\tau(C_2) = \frac{100}{1}$$

MERGE THEM !

Efficient Graph-Based Image Segmentation. Example (4x4 image)



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$w(v_i, v_j) = 1$ < **MInt(C1,C2) = 100**

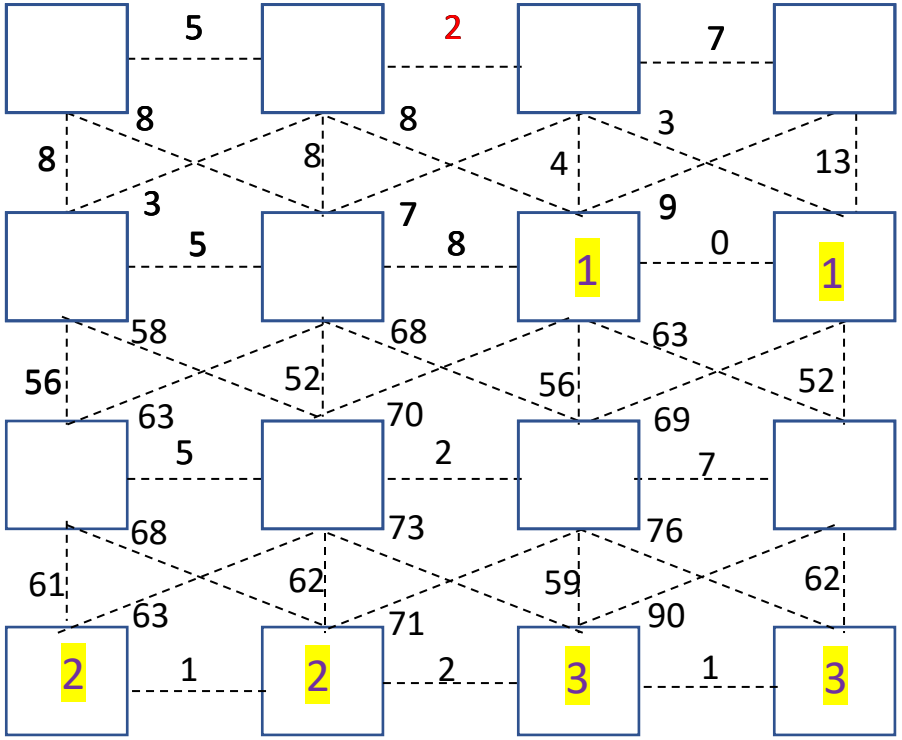
with k=100

$$\tau(C_1) = \frac{100}{1}$$

$$\tau(C_2) = \frac{100}{1}$$

MERGE THEM !

Efficient Graph-Based Image Segmentation. Example (4x4 image)



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$$w(v_i, v_j) = 2 \quad < \quad \textbf{MInt(C1,C2) = 100}$$

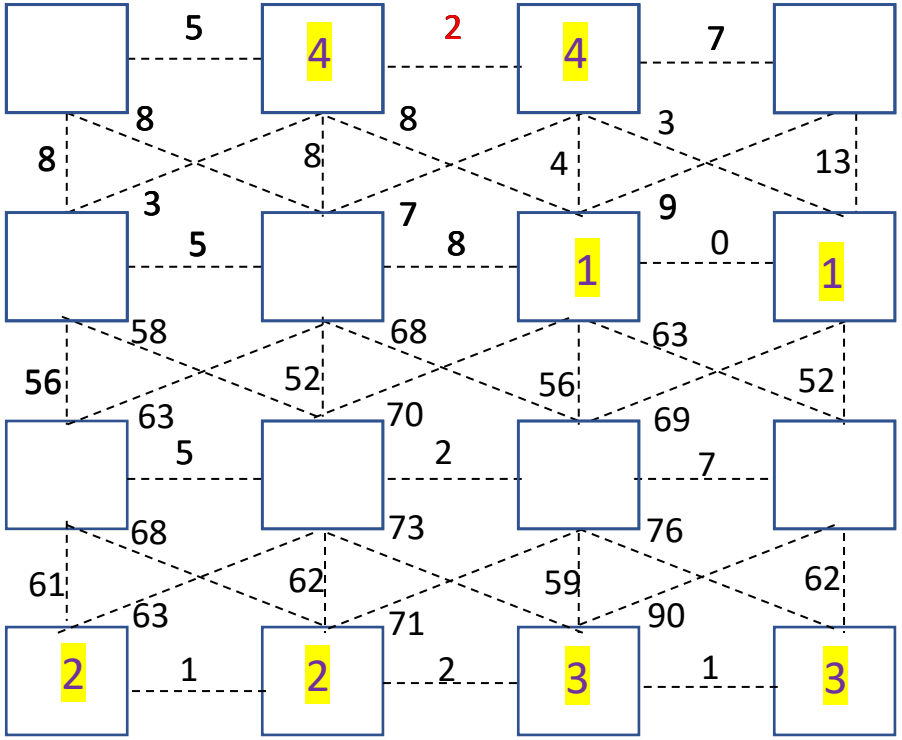
with k=100

$$\tau(C_1) = \frac{100}{1}$$

$$\tau(C_2) = \frac{100}{1}$$

MERGE THEM !

Efficient Graph-Based Image Segmentation. Example (4x4 image)



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$w(v_i, v_j) = 2$ < **MInt(C1,C2) = 100**

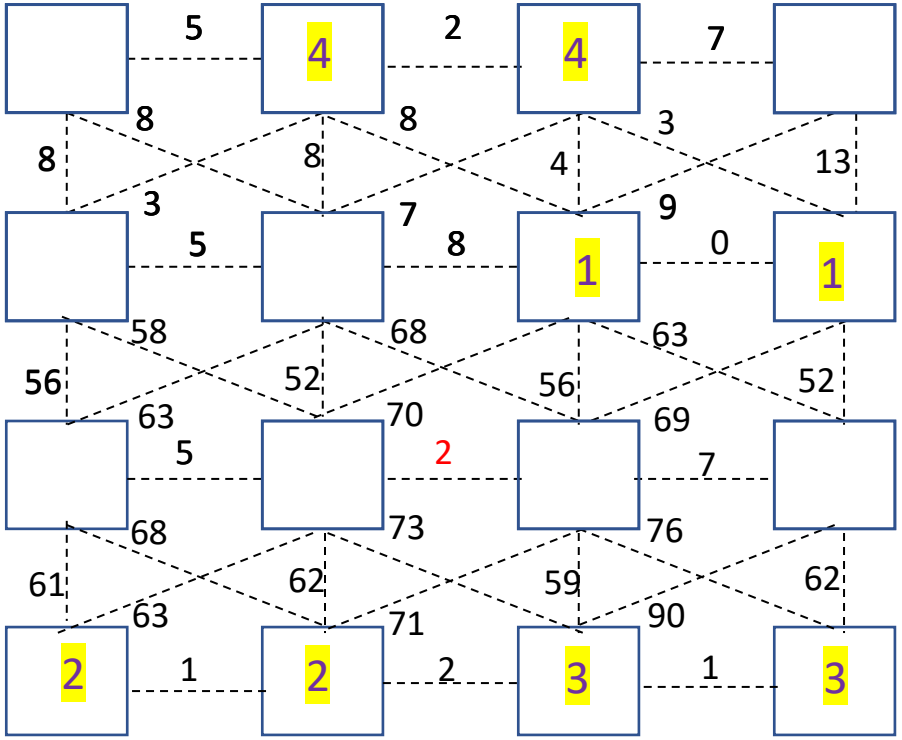
with k=100

$$\tau(C_1) = \frac{100}{1}$$

$$\tau(C_2) = \frac{100}{1}$$

MERGE THEM !

Efficient Graph-Based Image Segmentation. Example (4x4 image)



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$$w(v_i, v_j) = 2 \quad < \quad \textbf{MInt(C1,C2) = 100}$$

with k=100

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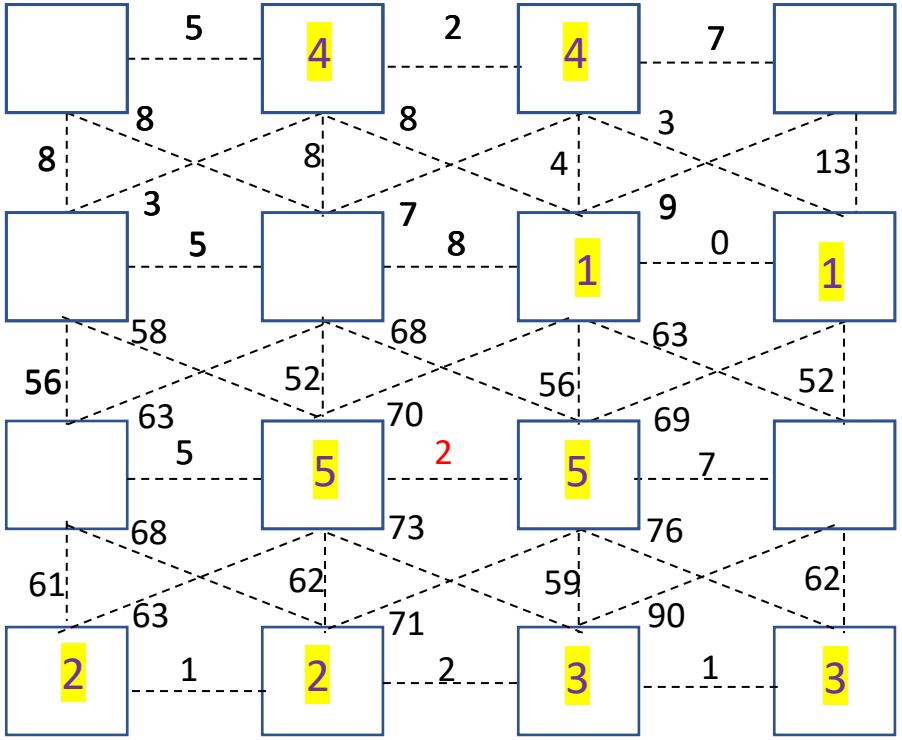
$$\tau(C_2) = \frac{100}{1}$$

MERGE THEM !

Efficient Graph-Based Image Segmentation. Example (4x4 image)



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$$w(v_i, v_j) = 2 \quad < \quad \textbf{MInt(C1,C2) = 100}$$

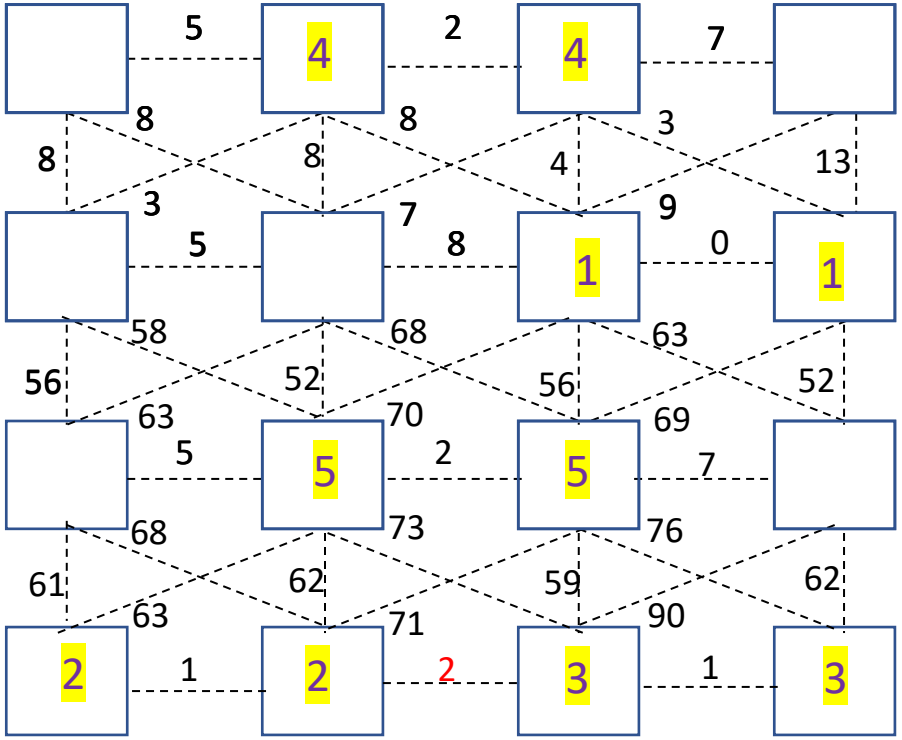
with k=100

$$\tau(C_1) = \frac{100}{1}$$

$$\tau(C_2) = \frac{100}{1}$$

MERGE THEM !

Efficient Graph-Based Image Segmentation. Example (4x4 image)



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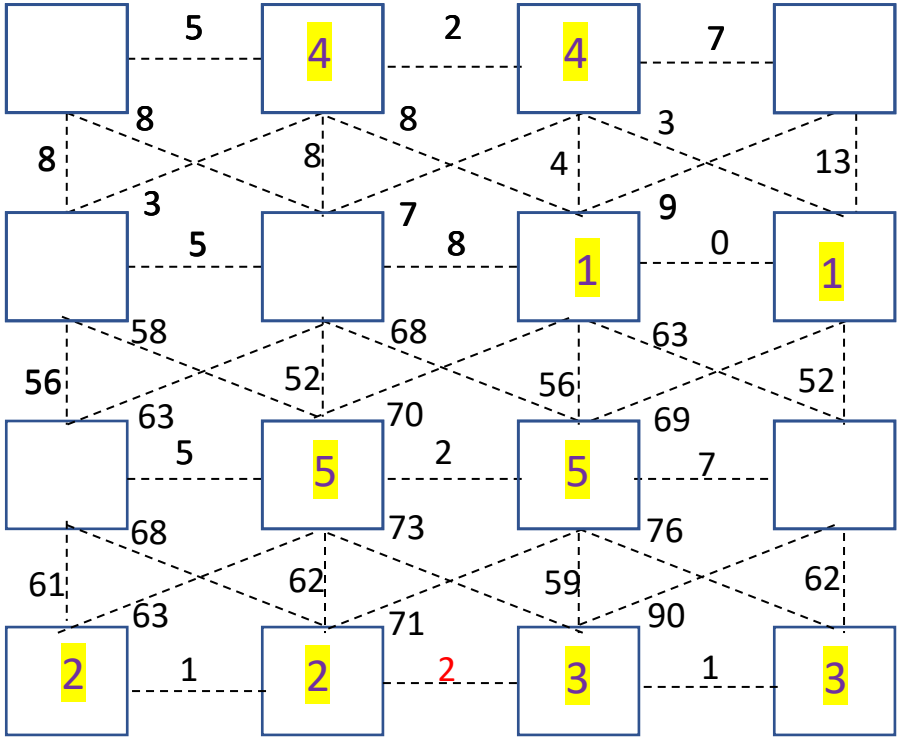
$w(v_i, v_j) = 2$ < **MInt(C2,C3) = min(Int(C2) + τ(C2) , Int(C3) + τ(C3)) = 51**

with k=100

$$\tau(C_2) = \frac{100}{2} = 50 \quad Int(C_2) = 1$$

$$\tau(C_2) = \frac{100}{2} = 50 \quad Int(C_3) = 1$$

Efficient Graph-Based Image Segmentation. Example (4x4 image)



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$w(v_i, v_j) = 2$ < **MInt(C2,C3) = min(Int(C2) + τ(C2) , Int(C3) + τ(C3)) = 51**

MERGE THEM !!

with k=100

$$\tau(C_2) = \frac{100}{2} = 50 \quad Int(C_2) = 1$$

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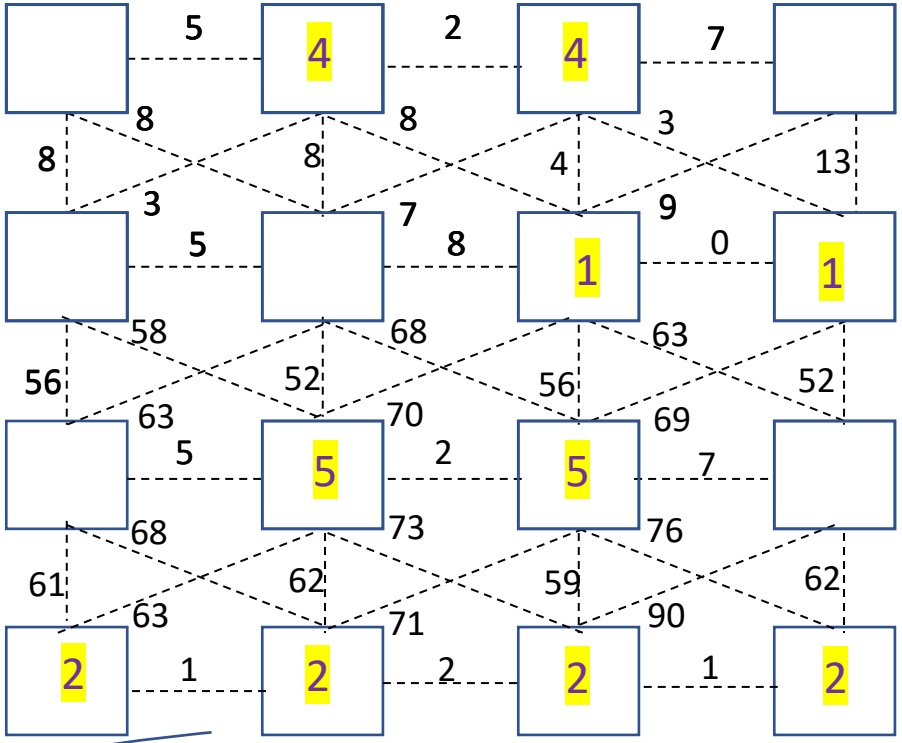
Efficient Graph-Based Image Segmentation. Example (4x4 image)



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MERGE THEM !!

with k=100

$$\tau(C_2) = \frac{100}{2} = 50 \quad Int(C_2) = 1$$

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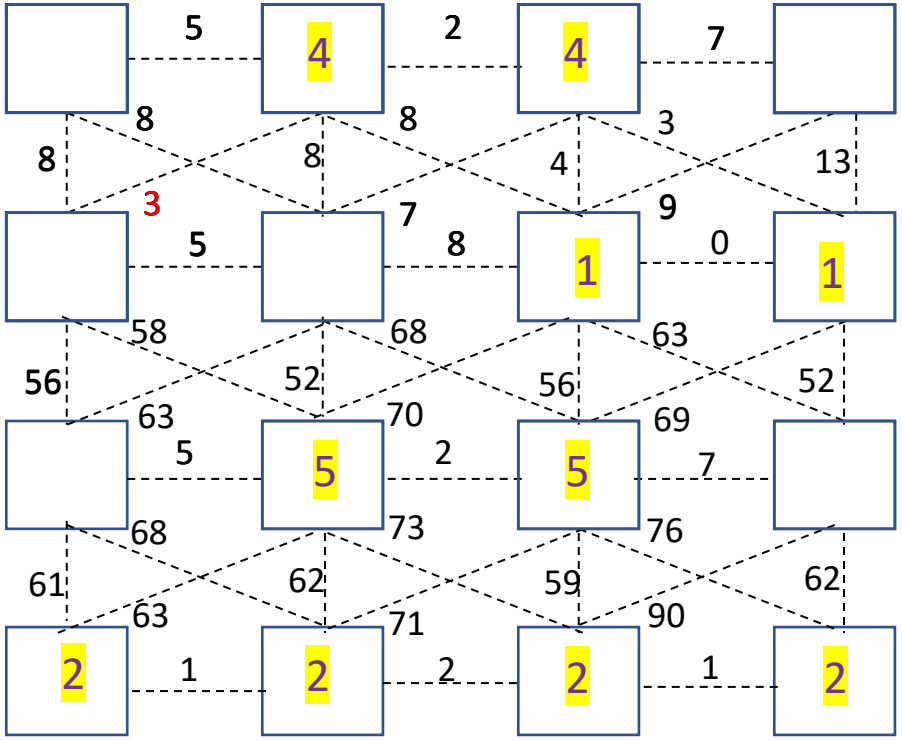
Efficient Graph-Based Image Segmentation. Example (4x4 image)



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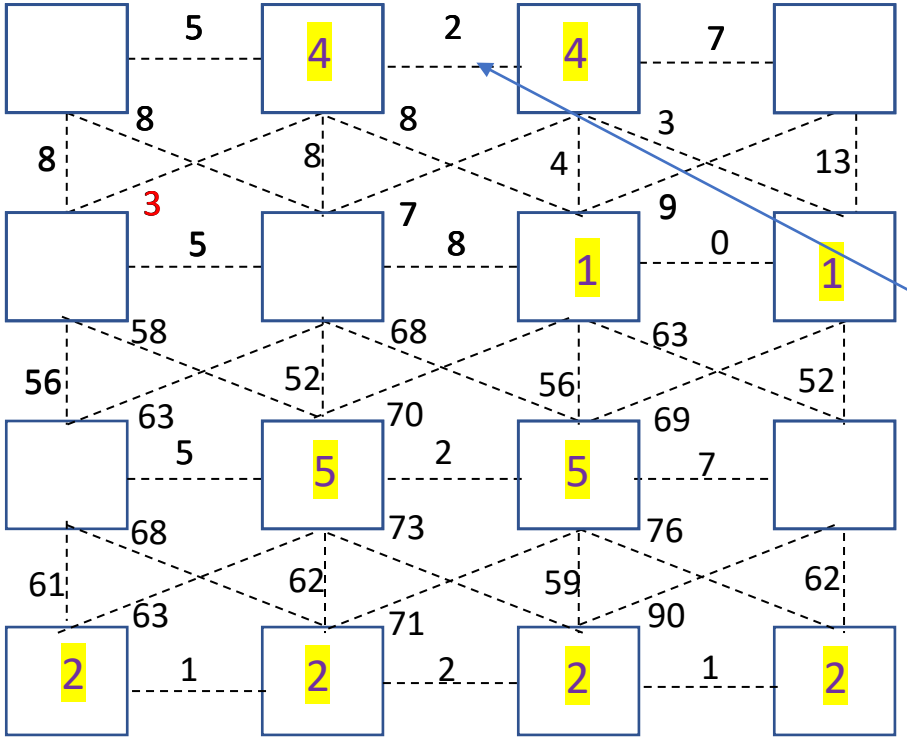
$w(v_i, v_j) = 3$ **< MInt(Ci,C4) = min(Int(Ci) + τ(Ci) , Int(C4) + τ(C4)) = 52**

with k=100

$$\tau(C_i) = \frac{100}{1} = 100 \quad Int(C_i) = 0$$

$$\tau(C_4) = \frac{100}{2} = 50 \quad Int(C_4) = 2$$

Efficient Graph-Based Image Segmentation. Example (4x4 image)



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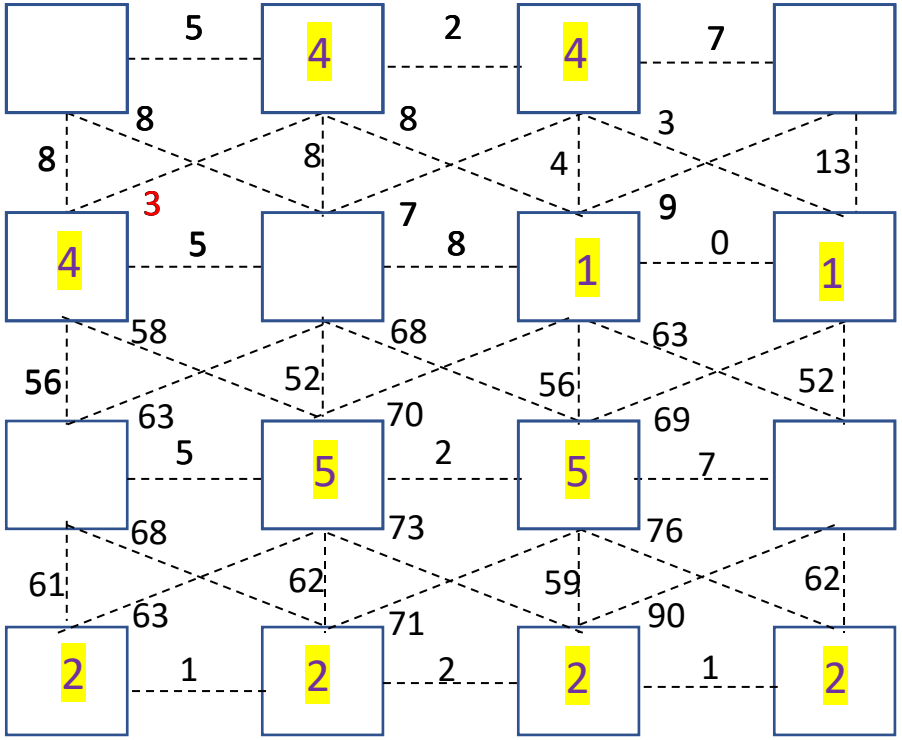
$w(v_i, v_j) = 3$ < **MInt(Ci,C4) = min(Int(Ci) + τ(Ci) , Int(C4) + τ(C4)) = 52**

with k=100

$$\tau(C_i) = \frac{100}{1} = 100 \quad Int(C_i) = 0$$

$$\tau(C_4) = \frac{100}{2} = 50 \quad Int(C_4) = 2$$

Efficient Graph-Based Image Segmentation. Example (4x4 image)



The algorithm follows a bottom-up procedure. Given **G=(V,E)** and **|V|=n, |E|=m.** Where |V| is the number of vertices(pixels) and |E| is the number of edges.

1. Edges are sorted by weight in ascending order, labeled as $e_1, e_2, \ldots, e_m$.
2. Initially, each pixel stays in its own component, so we start with n components.
3. Repeat for k=1,...,m:

- The segmentation snapshot at step k is denoted as Sk.
- We take the k-th edge in the order, ek=(v_i , v_j).
- If v_i and v_j belong to the same component, do nothing and thus $S^k=S^{k-1}$.
- If v_i and v_j belong to two different components C_i^{k-1} and C_j^{k-1} as in the segmentation of S^{k-1} we want to merge them into one if $w(v_i,v_j) \leq MInt(C_i^{k-1},C_j^{k-1})$; otherwise do nothing.

$w(v_i, v_j) = 3$ **< MInt(Ci,C4) = min(Int(Ci) + τ(Ci) , Int(C4) + τ(C4)) = 52**

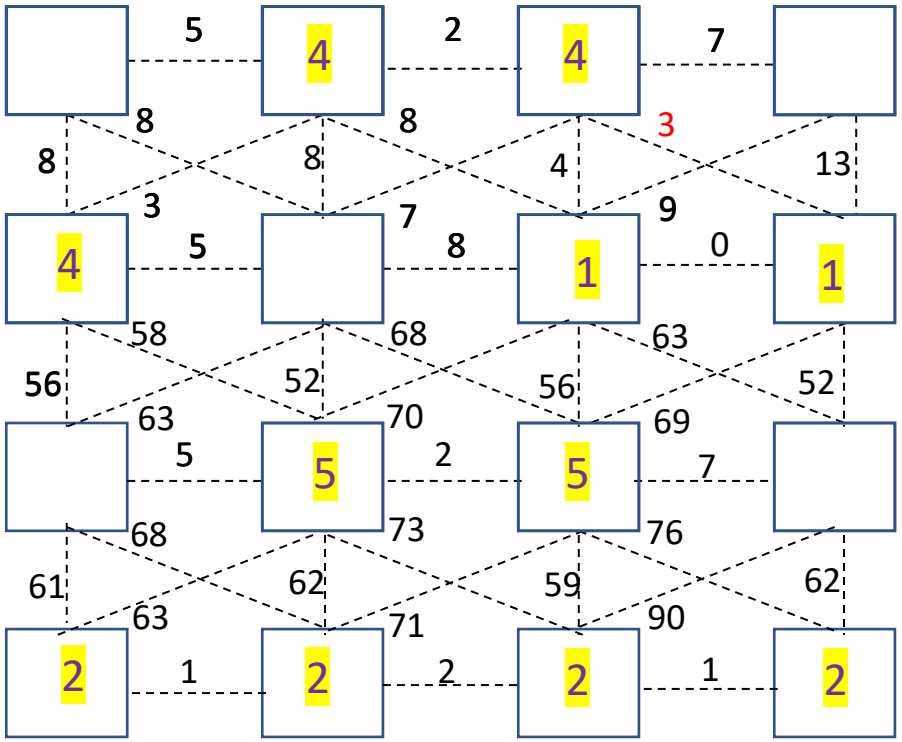
MERGE THEM !!

with k=100

$$\tau(C_i) = \frac{100}{1} = 100 \quad Int(C_i) = 0$$

$$\tau(C_4) = \frac{100}{2} = 50 \quad Int(C_4) = 2$$

Efficient Graph-Based Image Segmentation. Example (4x4 image)



The algorithm follows a bottom-up procedure. Given **G=(V,E)** and **|V|=n, |E|=m.** Where |V| is the number of vertices(pixels) and |E| is the number of edges.

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$w(v_i, v_j) = 3$ < **MInt(C1,C4) = min(Int(C1) + τ(C1) , Int(C4) + τ(C4)) = 36.3**

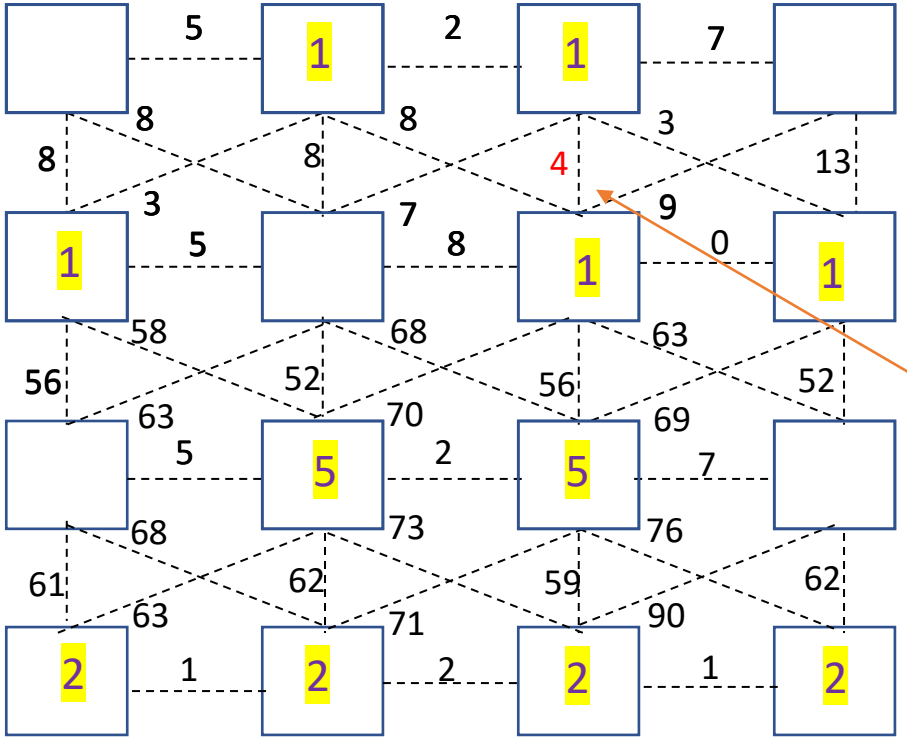
MERGE THEM !!

with k=100

$$\tau(C_1) = \frac{100}{2} = 50 \quad Int(C_i) = 0$$

$$\tau(C_4) = \frac{100}{3} = 33.3 \quad Int(C_4) = 3$$

Efficient Graph-Based Image Segmentation. Example (4x4 image)



The algorithm follows a bottom-up procedure. Given **G=(V,E)** and **|V|=n, |E|=m.** Where |V| is the number of vertices(pixels) and |E| is the number of edges.

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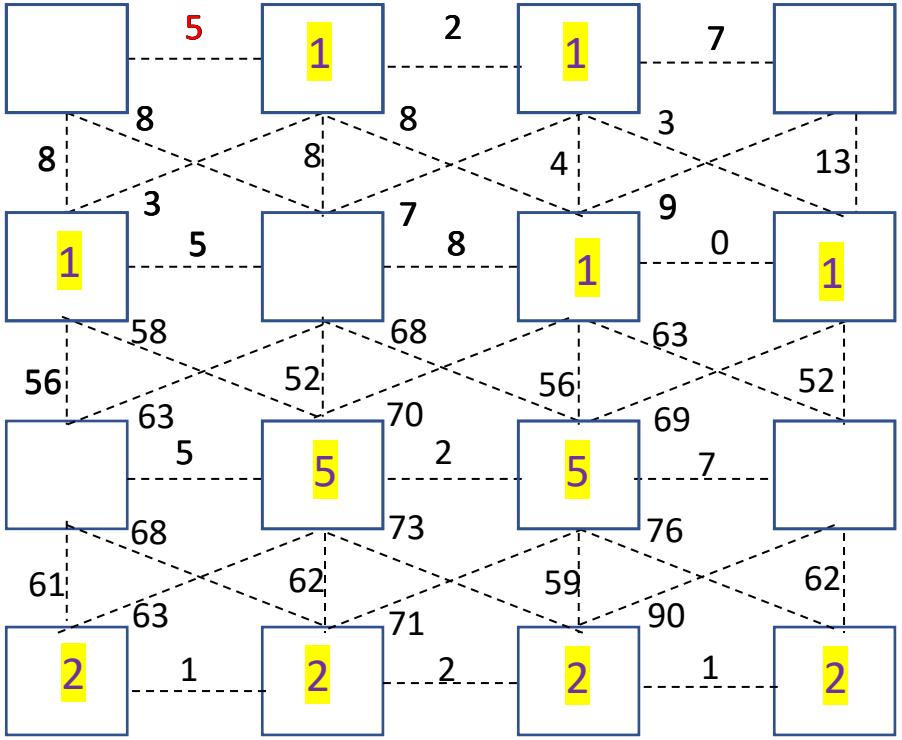
DO NOTHING !!

Efficient Graph-Based Image Segmentation. Example (4x4 image)



The algorithm follows a bottom-up procedure. Given **G=(V,E)** and **|V|=n, |E|=m.** Where |V| is the number of vertices(pixels) and |E| is the number of edges.

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$w(v_i, v_j) = 5$ < **MInt(Ci,C1) = min(Int(Ci) + τ(Ci) , Int(C1) + τ(C1)) = 24**

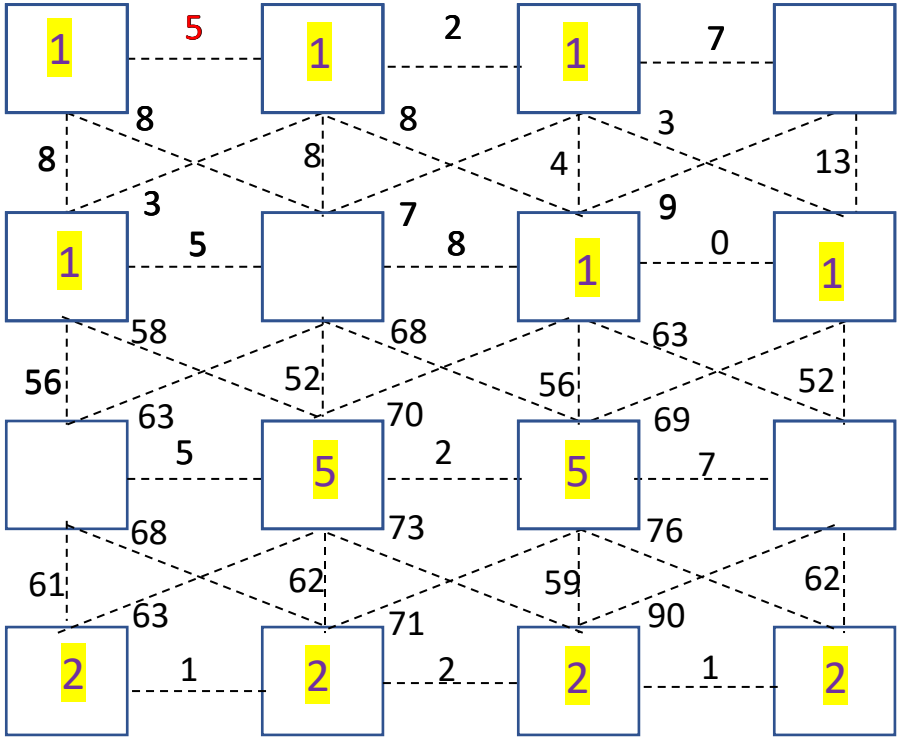
MERGE THEM !!

with k=100

$$\tau(C_i) = \frac{100}{1} = 100 \quad Int(C_i) = 0$$

$$\tau(C_1) = \frac{100}{5} = 20 \quad Int(C_1) = 4$$

Efficient Graph-Based Image Segmentation. Example (4x4 image)



The algorithm follows a bottom-up procedure. Given **G=(V,E)** and **|V|=n, |E|=m.** Where |V| is the number of vertices(pixels) and |E| is the number of edges.

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$w(v_i, v_j) = 5$ < **MInt(Ci,C1) = min(Int(Ci) + τ(Ci) , Int(C1) + τ(C1)) = 24**

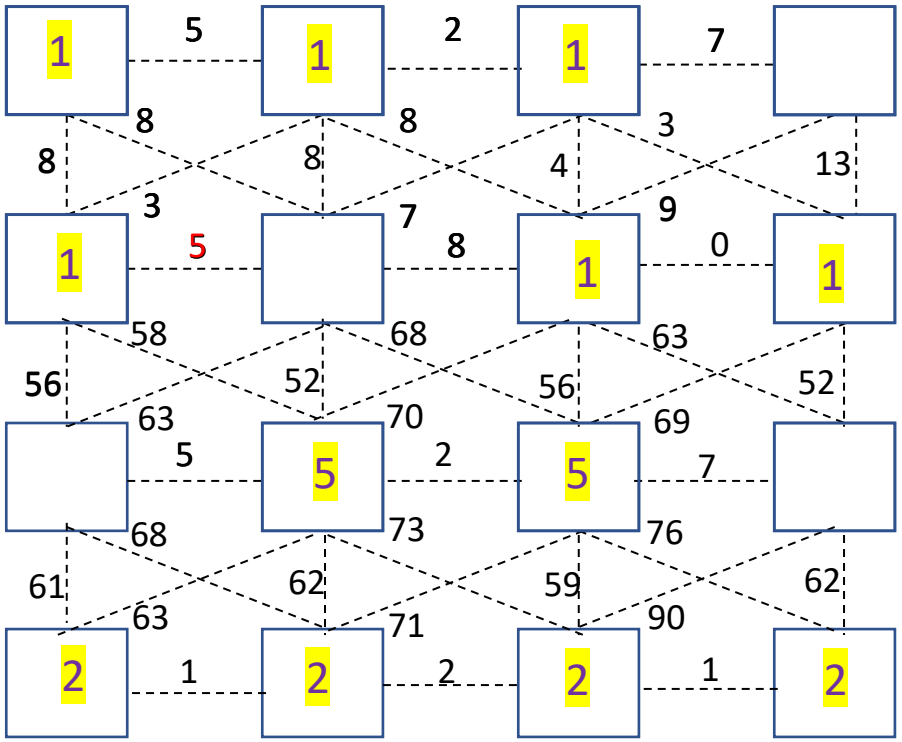
MERGE THEM !!

with k=100

$$\tau(C_i) = \frac{100}{1} = 100 \quad Int(C_i) = 0$$

$$\tau(C_1) = \frac{100}{5} = 20 \quad Int(C_1) = 4$$

Efficient Graph-Based Image Segmentation. Example (4x4 image)



The algorithm follows a bottom-up procedure. Given **G=(V,E)** and **|V|=n, |E|=m.** Where |V| is the number of vertices(pixels) and |E| is the number of edges.

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$w(v_i, v_j) = 5$ < **MInt(Ci,C1) = min(Int(Ci) + τ(Ci) , Int(C1) + τ(C1)) = 24.6**

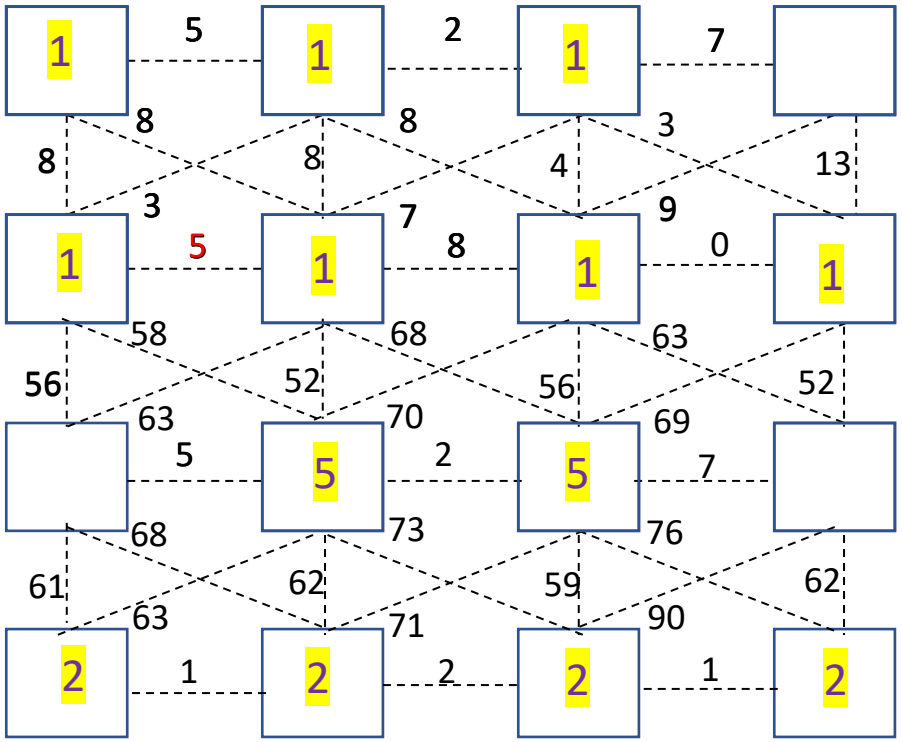
MERGE THEM !!

with k=100

$$\tau(C_i) = \frac{100}{1} = 100 \quad Int(C_i) = 0$$

$$\tau(C_1) = \frac{100}{6} = 16.6 \quad Int(C_1) = 8$$

Efficient Graph-Based Image Segmentation. Example (4x4 image)



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- If v_i and v_j belong to two different components C_i^{k-1} and C_j^{k-1} as in the segmentation of S^{k-1} we want to merge them into one if $w(v_i,v_j) \le MInt(C_i^{k-1},C_j^{k-1})$; otherwise do nothing.

$w(v_i, v_j) = 5$ < **MInt(Ci,C1) = min(Int(Ci) + τ(Ci) , Int(C1) + τ(C1)) = 24.6**

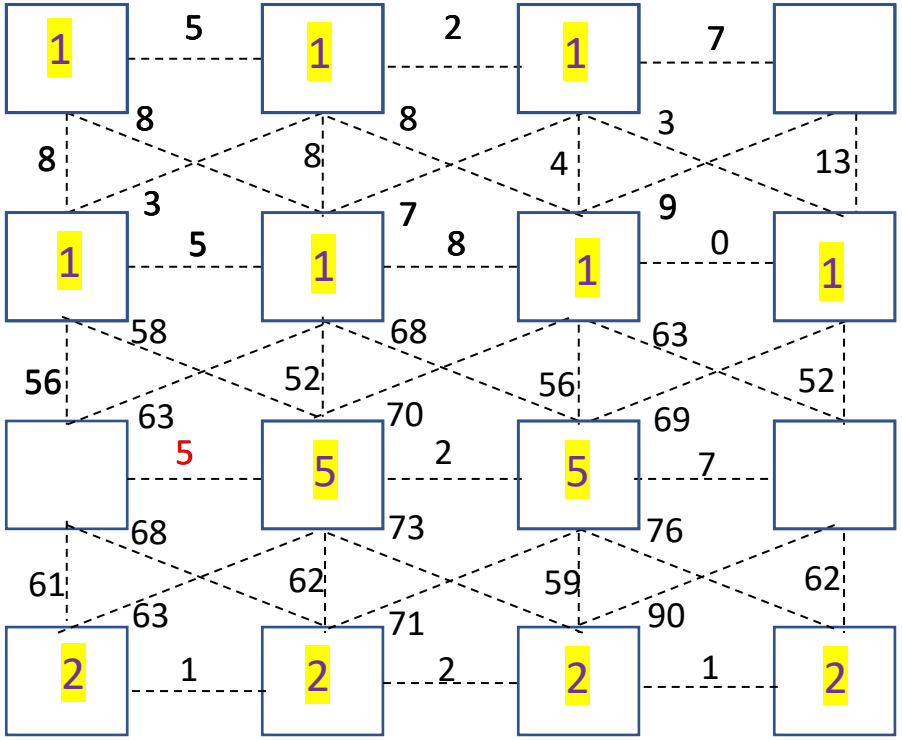
MERGE THEM !!

with k=100

$$\tau(C_i) = \frac{100}{1} = 100 \quad Int(C_i) = 0$$

$$\tau(C_1) = \frac{100}{6} = 16.6 \quad Int(C_1) = 8$$

Efficient Graph-Based Image Segmentation. Example (4x4 image)



The algorithm follows a bottom-up procedure. Given **G=(V,E)** and **|V|=n, |E|=m.** Where |V| is the number of vertices(pixels) and |E| is the number of edges.

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$w(v_i, v_j) = 5$ < **MInt(Ci,C5) = min(Int(Ci) + τ(Ci) , Int(C5) + τ(C5)) = 52**

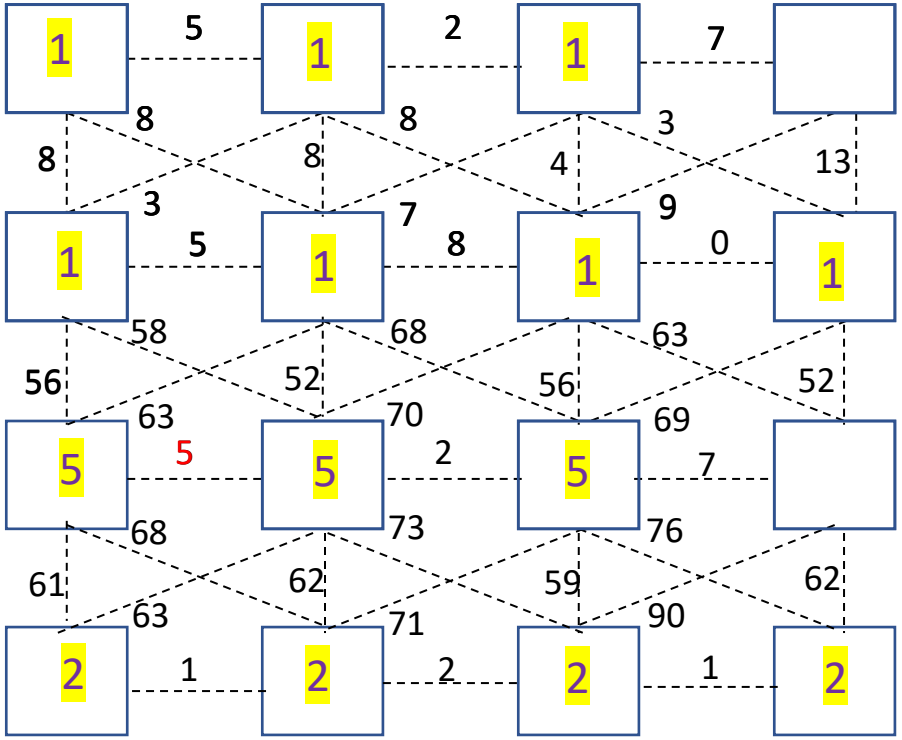
MERGE THEM !!

with k=100

$$\tau(C_i) = \frac{100}{1} = 100 \quad Int(C_i) = 0$$

$$\tau(C_5) = \frac{100}{2} = 50 \quad Int(C_5) = 2$$

Efficient Graph-Based Image Segmentation. Example (4x4 image)



The algorithm follows a bottom-up procedure. Given **G=(V,E)** and **|V|=n, |E|=m.** Where |V| is the number of vertices(pixels) and |E| is the number of edges.

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$w(v_i, v_j) = 5$ < **MInt(Ci,C5) = min(Int(Ci) + τ(Ci) , Int(C5) + τ(C5)) = 52**

MERGE THEM !!

with k=100

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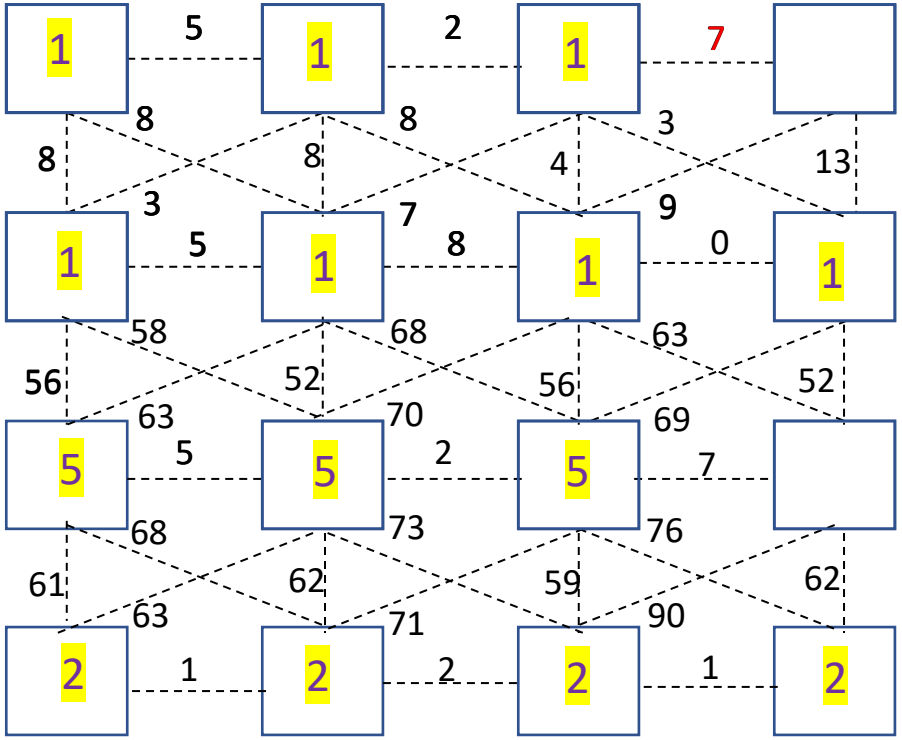
Efficient Graph-Based Image Segmentation. Example (4x4 image)



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$w(v_i, v_j) = 7$ < **MInt(Ci,C1) = min(Int(Ci) + τ(Ci) , Int(C1) + τ(C1)) = 22.3**

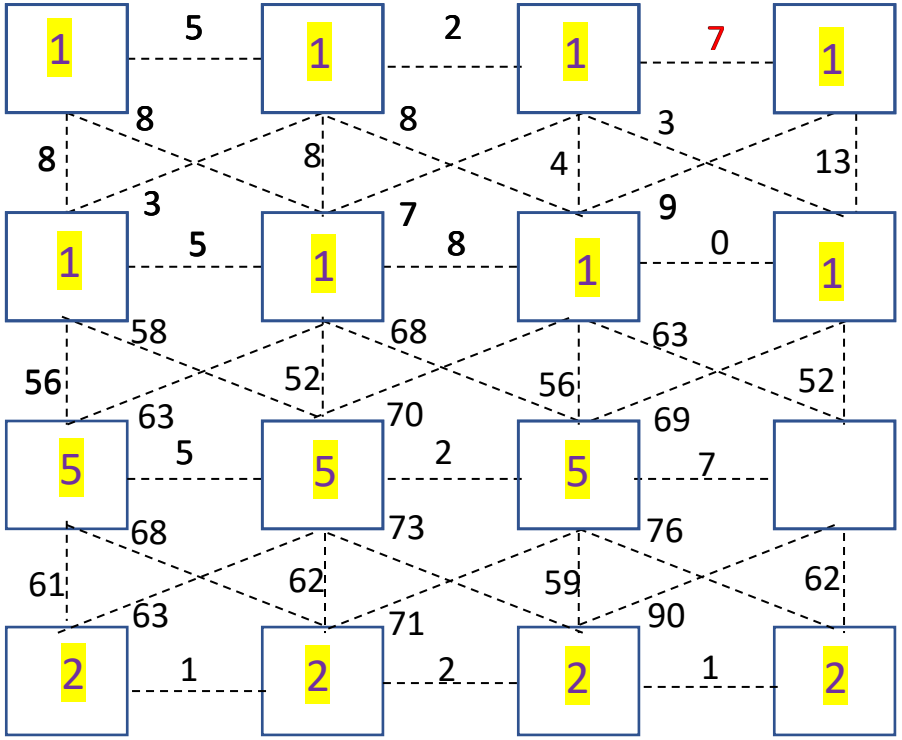
MERGE THEM !!

with k=100

$$\tau(C_i) = \frac{100}{1} = 100 \quad Int(C_i) = 0$$

$$\tau(C_1) = \frac{100}{7} = 14.3 \quad Int(C_1) = 8$$

Efficient Graph-Based Image Segmentation. Example (4x4 image)



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$w(v_i, v_j) = 7$ < **MInt(Ci,C1) = min(Int(Ci) + τ(Ci) , Int(C1) + τ(C1)) = 22.3**

MERGE THEM !!

with k=100

$$\tau(C_i) = \frac{100}{1} = 100 \quad Int(C_i) = 0$$

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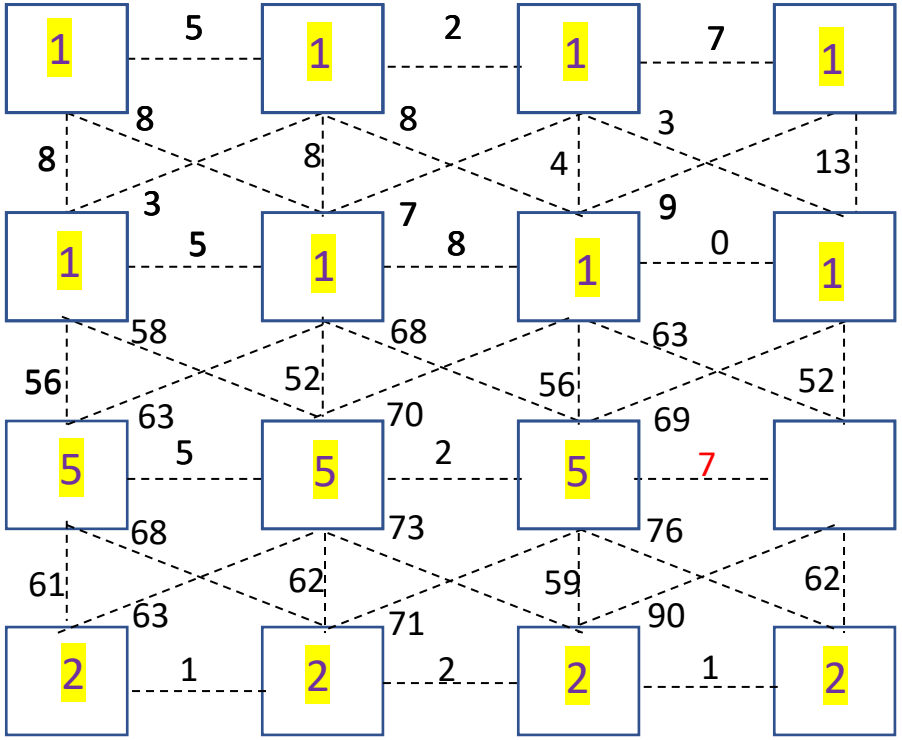
Efficient Graph-Based Image Segmentation. Example (4x4 image)



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$w(v_i, v_j) = 7$ < **MInt(Ci,C5) = min(Int(Ci) + τ(Ci) , Int(C5) + τ(C5)) = 38.3**

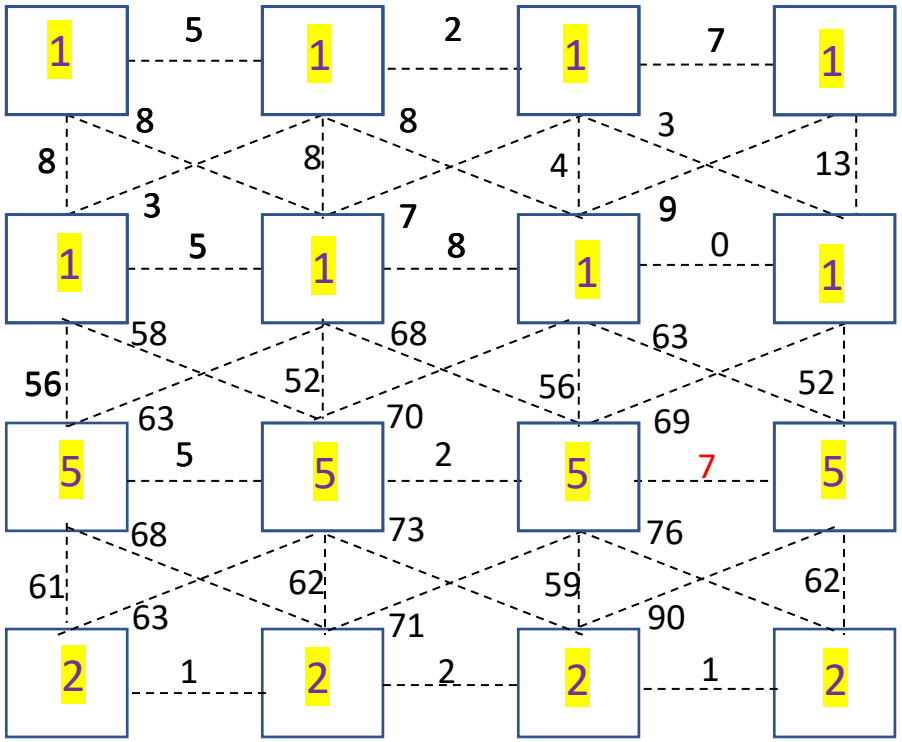
MERGE THEM !!

with k=100

$$\tau(C_i) = \frac{100}{1} = 100 \quad Int(C_i) = 0$$

$$\tau(C_5) = \frac{100}{3} = 33.3 \quad Int(C_5) = 5$$

Efficient Graph-Based Image Segmentation. Example (4x4 image)



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$w(v_i, v_j) = 7$ < **MInt(Ci,C5) = min(Int(Ci) + τ(Ci) , Int(C5) + τ(C5)) = 38.3**

MERGE THEM !!

with k=100

$$\tau(C_i) = \frac{100}{1} = 100 \quad Int(C_i) = 0$$

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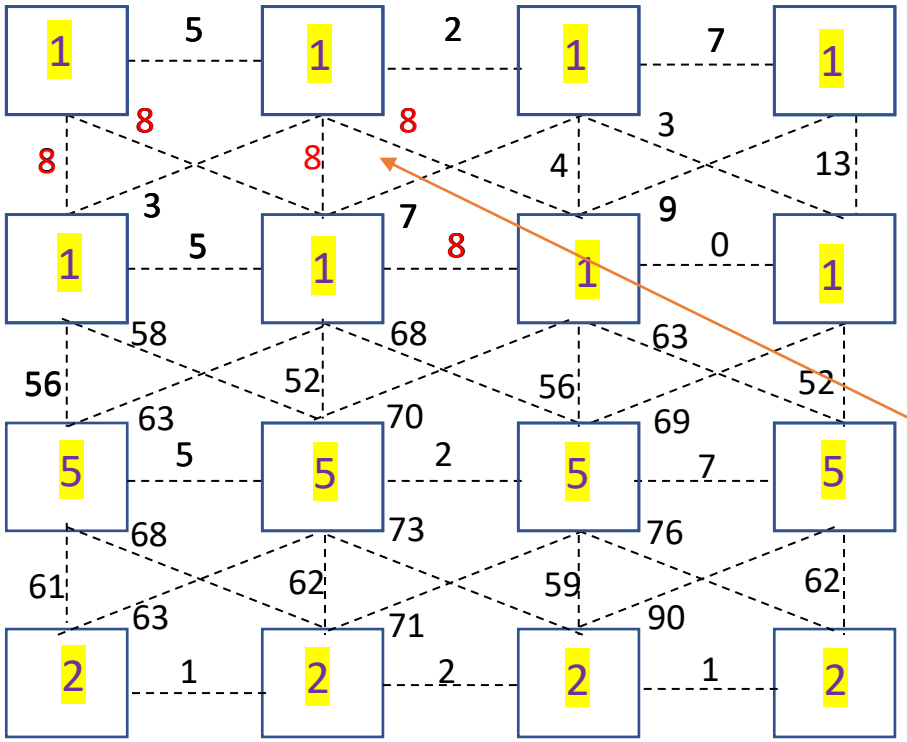
Efficient Graph-Based Image Segmentation. Example (4x4 image)



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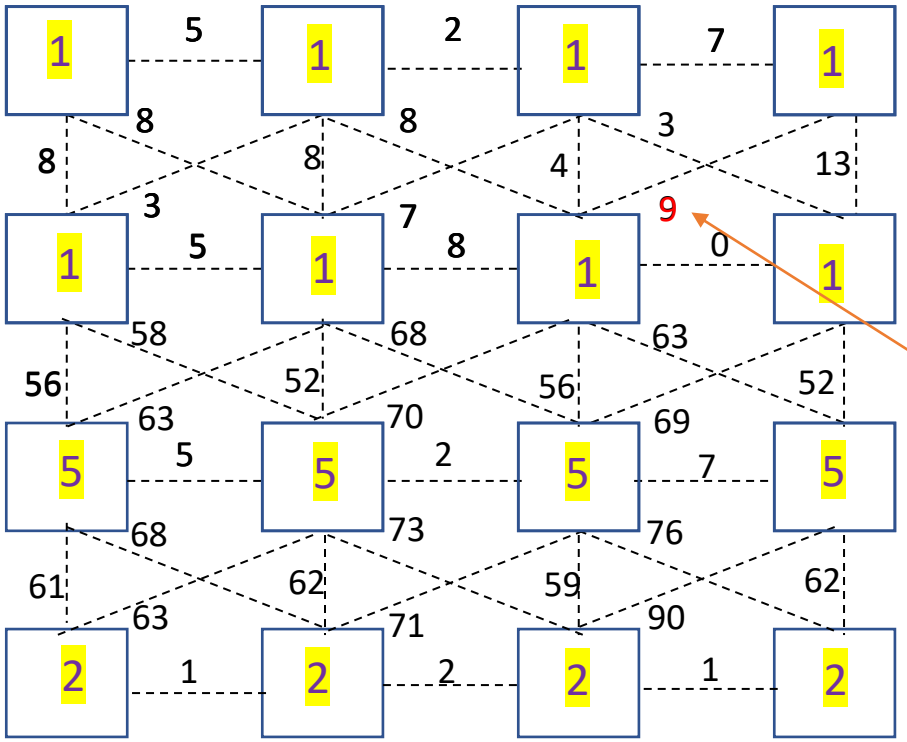
DO NOTHING !!

Efficient Graph-Based Image Segmentation. Example (4x4 image)



The algorithm follows a bottom-up procedure. Given **G=(V,E)** and **|V|=n, |E|=m.** Where |V| is the number of vertices(pixels) and |E| is the number of edges.

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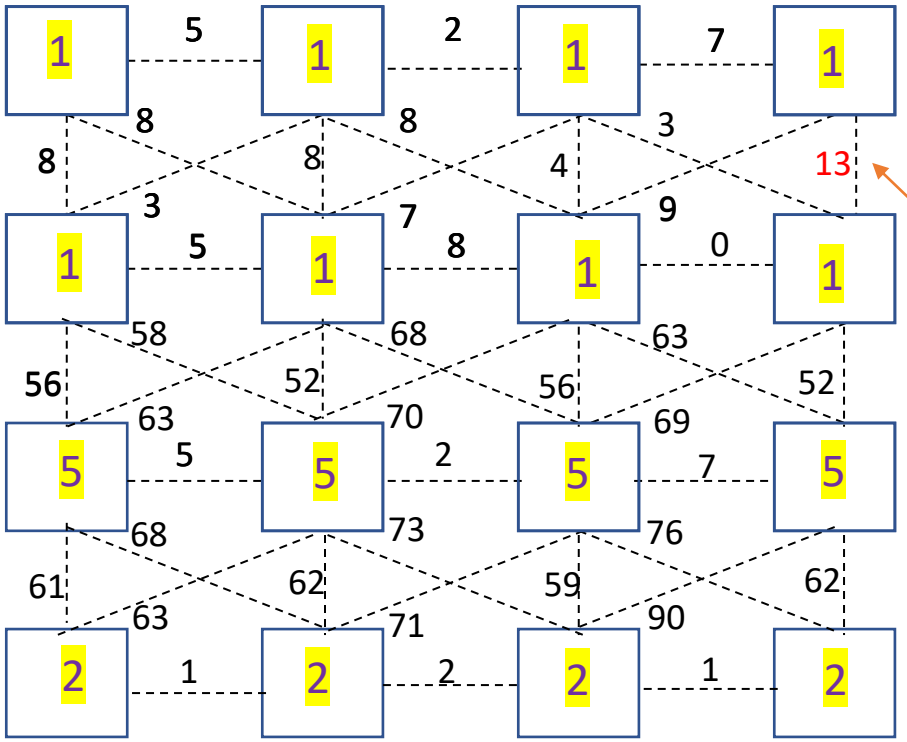
DO NOTHING !!

Efficient Graph-Based Image Segmentation. Example (4x4 image)



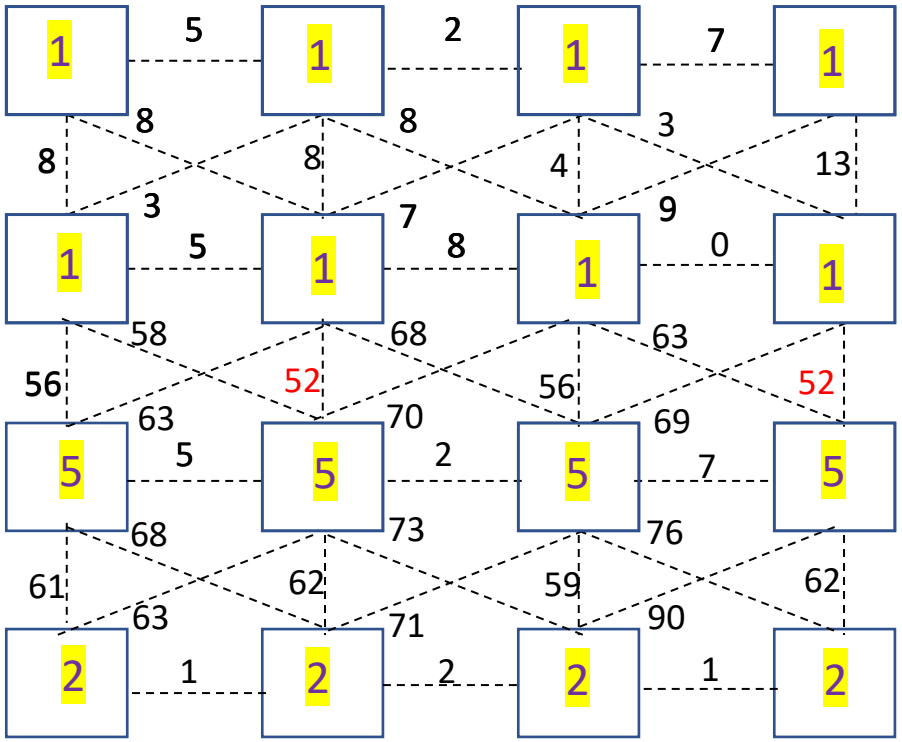
The algorithm follows a bottom-up procedure. Given **G=(V,E)** and **|V|=n, |E|=m.** Where |V| is the number of vertices(pixels) and |E| is the number of edges.

1. Edges are sorted by weight in ascending order, labeled as $e_1, e_2, \ldots, e_m$.
2. Initially, each pixel stays in its own component, so we start with n components.
3. Repeat for k=1,...,m:

- The segmentation snapshot at step k is denoted as Sk.
- We take the k-th edge in the order, ek=(v_i , v_j).
- If v_i and v_j belong to the same component, do nothing and thus $S^k=S^{k-1}$.
- If v_i and v_j belong to two different components C_i^{k-1} and C_j^{k-1} as in the segmentation of S^{k-1} we want to merge them into one if $w(v_i,v_j) \leq MInt(C_i^{k-1},C_j^{k-1})$; otherwise do nothing.

DO NOTHING !!

Efficient Graph-Based Image Segmentation. Example (4x4 image)



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$w(v_i, v_j) = 52$ > **MInt(C5,C1) = min(Int(C5) + τ(C5) , Int(C1) + τ(C1)) =25.5**

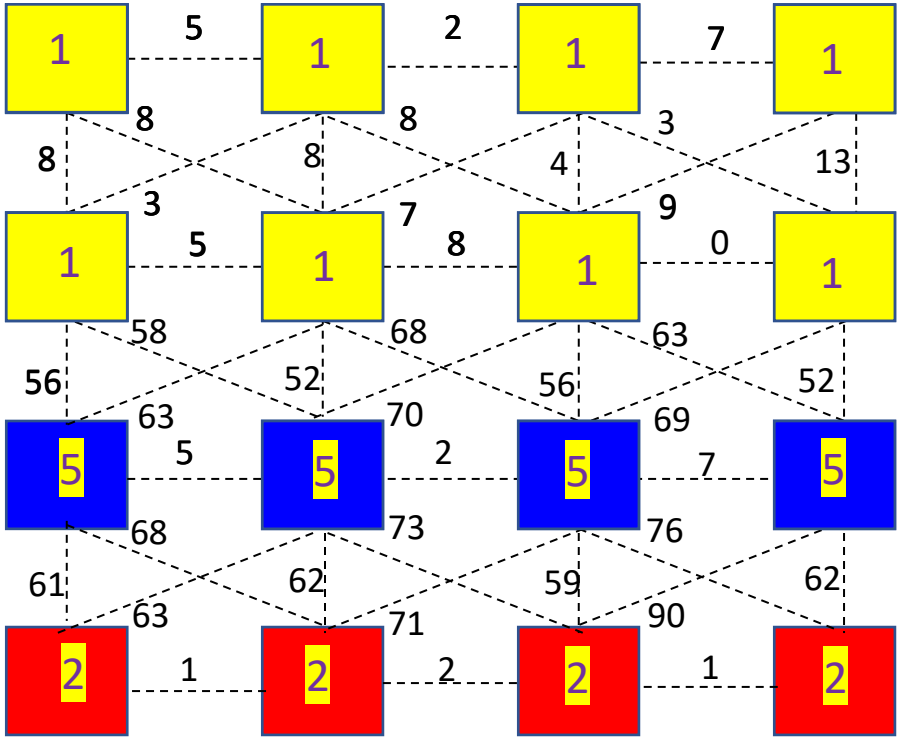
STOP MERGING!!

with k=100

$$\tau(C_5) = \frac{100}{4} = 25 \quad Int(C_5) = 7$$

$$\tau(C_1) = \frac{100}{8} = 12.5 \quad Int(C_1) = 13$$

Efficient Graph-Based Image Segmentation. Example (4x4 image)



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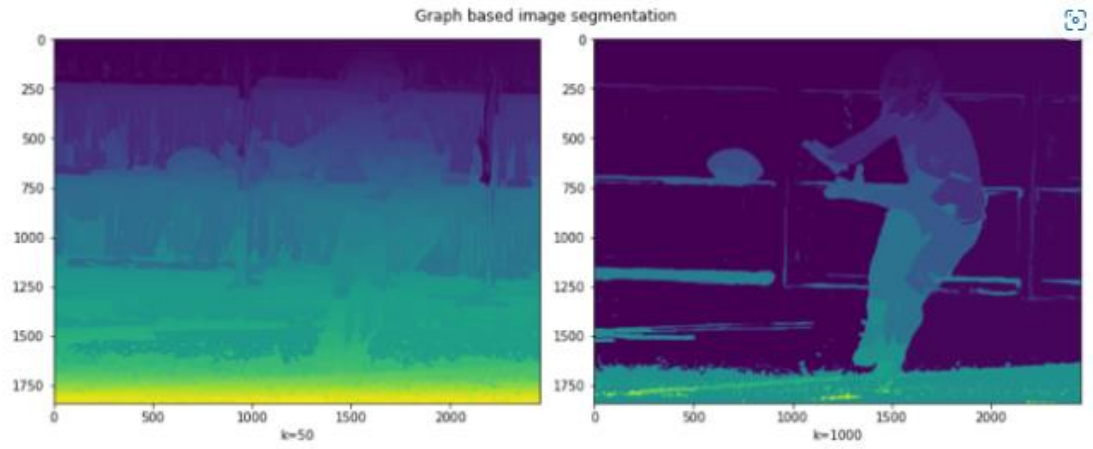
Efficient Graph-Based Image Segmentation.

Pedro F. Felzenszwalb, Daniel P. Huttenlocher

Result



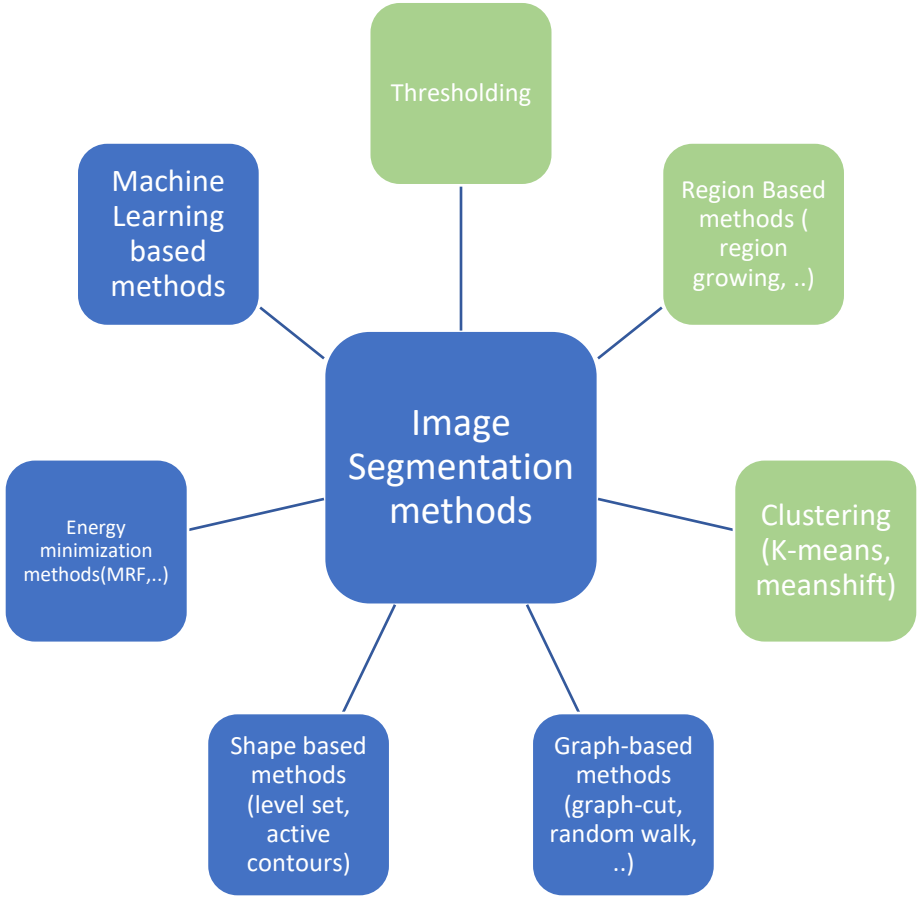
CAP4

[Image Segmentation With Felzenszwalb's Algorithm ! - A]

Region splitting and Merging Segmentation

- Region merging:
  - Region merging is the opposite of splitting, and works as a way of avoiding over-segmentation
  - Start with small regions (2x2 or 4x4 regions) and merge the regions that have similar characteristics (such as gray level, variance).
- **Region splitting:**
  - **Unlike region growing, which starts from a set of seed points, region splitting starts with the whole image as a single region and subdivides it into subsidiary regions recursively while a condition of homogeneity is not satisfied.**

Image segmentation methods



Questions?