# CAP 4453
# Robot Vision

Dr. Gonzalo Vaca-Castaño
gonzalo.vacacastano@ucf.edu

# Administrative details

- Homework 1 doubts

# Questions?

# Robot Vision

## 4. Image Filtering II

# Credits

- Some slides comes directly from:
  - Yogesh S Rawat (UCF)
  - Noah  Snavely (Cornell)
  - Ioannis (Yannis) Gkioulekas (CMU)
  - Mubarak Shah (UCF)
  - S. Seitz
  - James Tompkin
  - Ulas Bagci
  - L. Lazebnik

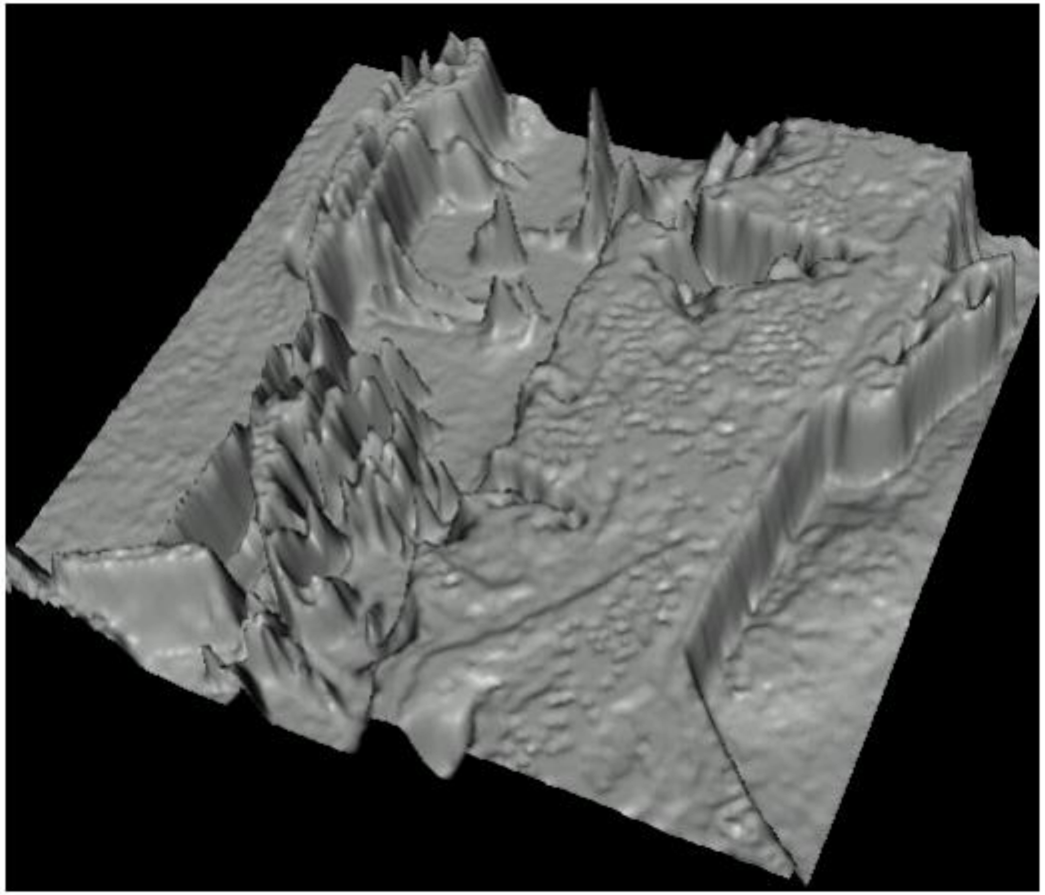
# Outline

- Image as a function
- Extracting useful information from Images
  - ~~Histogram~~
  - ~~Filtering (linear)~~
  - ~~Smoothing/Removing noise~~
  - ~~Convolution/Correlation~~
  - Image Derivatives/Gradient
  - **Edges**

# Edge Detection

- Identify sudden changes in an image
  - Semantic and shape information
  - Marks the border of an object
  - More compact than pixels

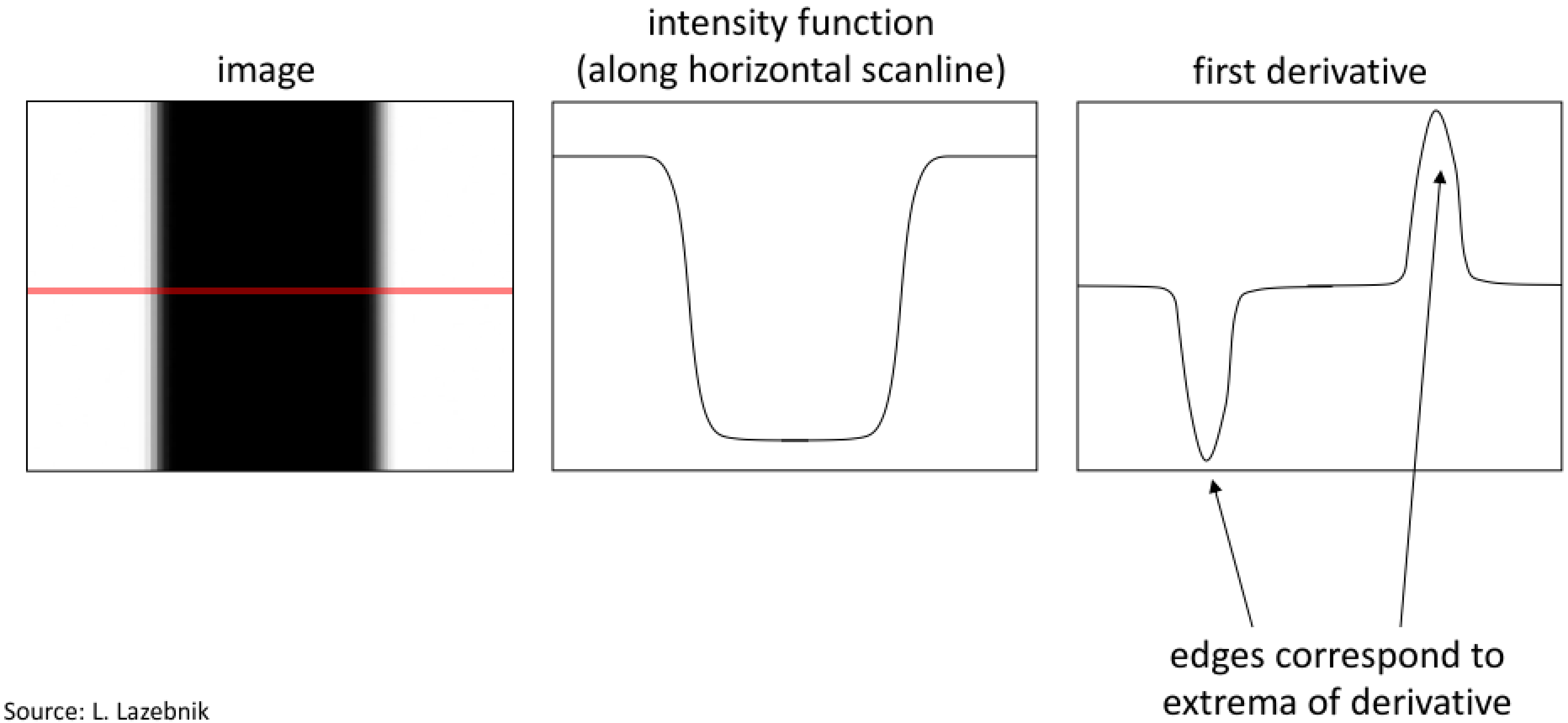
# Images as functions…



- Edges look like steep cliffs

# Characterizing edges

- An edge is a place of *rapid change* in the image intensity function




Source: L. Lazebnik

# Detecting edges

How would you go about detecting edges in an image (i.e., discontinuities in a function)?

✓ You take derivatives: derivatives are large at discontinuities.

How do you differentiate a discrete image (or any other discrete signal)?

# Detecting edges

How would you go about detecting edges in an image (i.e., discontinuities in a function)?

✓ You take derivatives: derivatives are large at discontinuities.

How do you differentiate a discrete image (or any other discrete signal)?

✓ You use finite differences.

# Finite differences

High-school reminder: definition of a derivative using forward difference

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

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$$f'(x) = \lim_{h \to 0} \frac{f(x+0.5h) - f(x-0.5h)}{h}$$

For discrete signals: Remove limit and set h = 2

$$f'(x) = \frac{f(x+1) - f(x-1)}{2}$$

What convolution kernel does this correspond to?

# Finite differences

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For discrete signals: Remove limit and set h = 2

$$f'(x) = \frac{f(x+1) - f(x-1)}{2}$$

| | | |
|---|---|---|
| -1 | 0 | 1 |

| | | |
|---|---|---|
| 1 | 0 | -1 |

# Example 1D signal

How do we compute the derivative of a discrete signal?

| | | | | | | |
|---|---|---|---|---|---|---|
| 10 | 20 | 10 | 200 | 210 | 250 | 250 |

$$f'(x) = \frac{f(x+1) - f(x-1)}{2} = \frac{210 - 10}{2} = 100$$

| | | |
|---|---|---|
| -1 | 0 | 1 |

1D derivative filter

# The Sobel filter

$$\begin{array}{|c|c|c|}\hline 1 & 0 & -1 \\ \hline 2 & 0 & -2 \\ \hline 1 & 0 & -1 \\ \hline \end{array} = \begin{array}{|c|}\hline 1 \\ \hline 2 \\ \hline 1 \\ \hline \end{array} * \begin{array}{|c|c|c|}\hline 1 & 0 & -1 \\ \hline \end{array}$$

Sobel filter

What filter is this?

1D derivative filter

# The Sobel filter

$$\begin{array}{|c|c|c|}\hline 1 & 0 & -1 \\ \hline 2 & 0 & -2 \\ \hline 1 & 0 & -1 \\ \hline \end{array} = \begin{array}{|c|}\hline 1 \\ \hline 2 \\ \hline 1 \\ \hline \end{array} * \begin{array}{|c|c|c|}\hline 1 & 0 & -1 \\ \hline \end{array}$$

Sobel filter

Blurring

1D derivative filter

In a 2D image, does this filter responses along horizontal or vertical lines?

# The Sobel filter

$$\begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & -1 \end{bmatrix}$$

Sobel filter

Blurring

1D derivative filter

Does this filter return large responses on vertical or horizontal lines?

# The Sobel filter

Horizontal Sober filter:

$$\begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & -1 \end{bmatrix}$$

What does the vertical Sobel filter look like?

# The Sobel filter

Horizontal Sober filter:

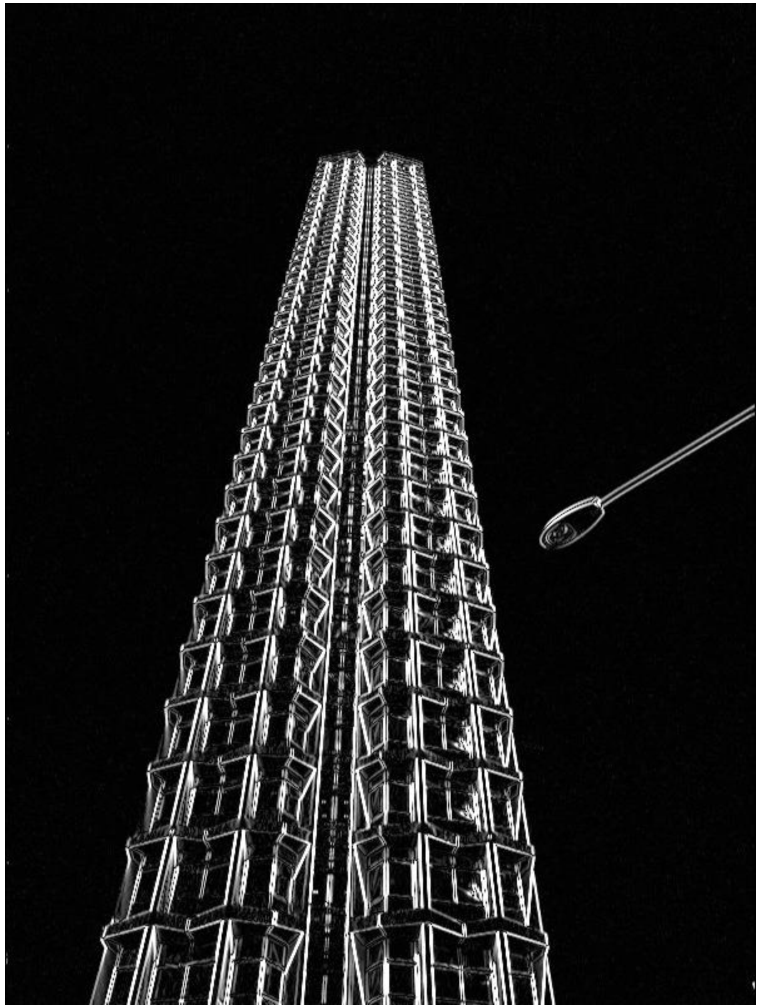
$$\begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & -1 \end{bmatrix}$$

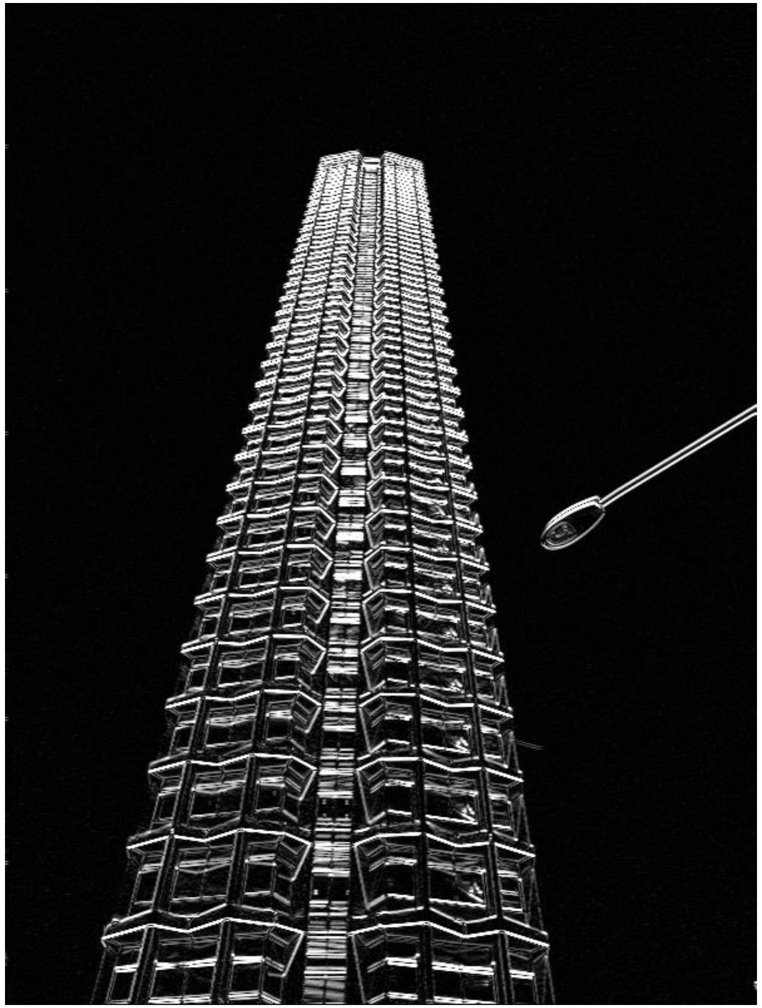
Vertical Sobel filter:

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} * \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$$

# Sobel filter example

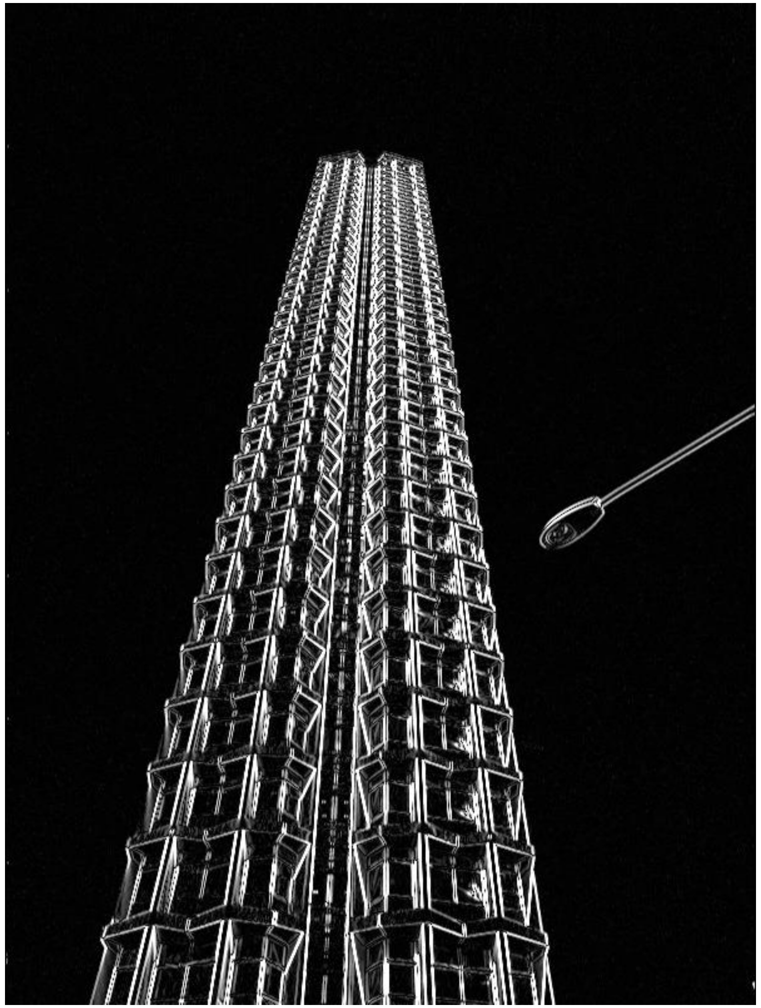

original


which Sobel filter?

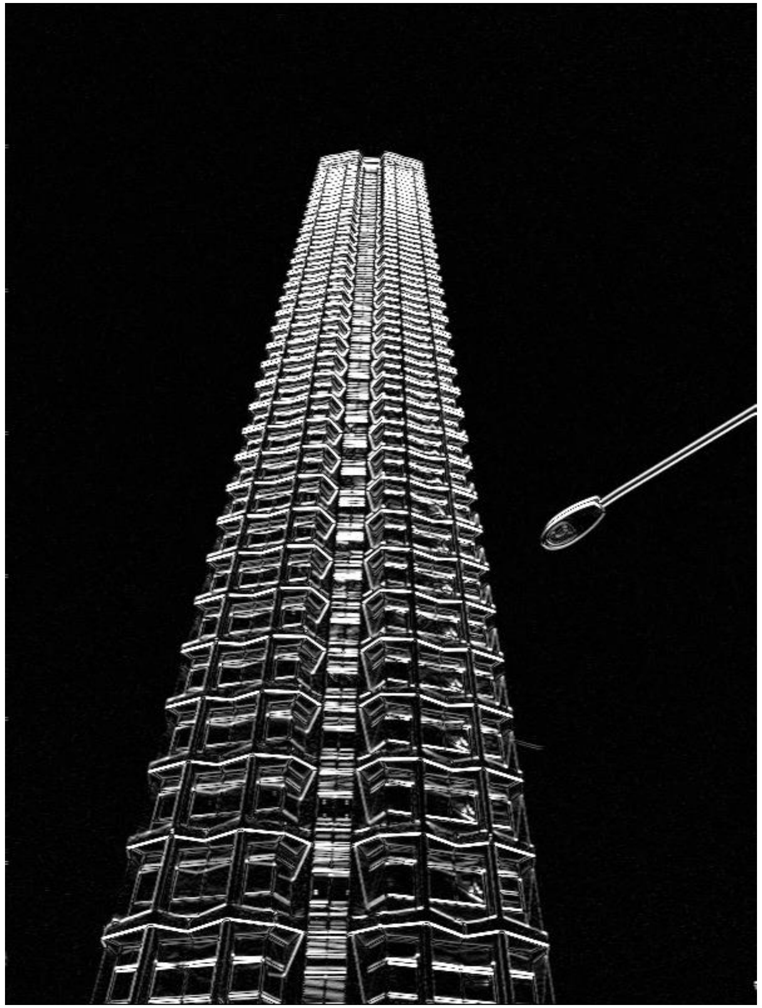

which Sobel filter?

# Sobel filter example

original

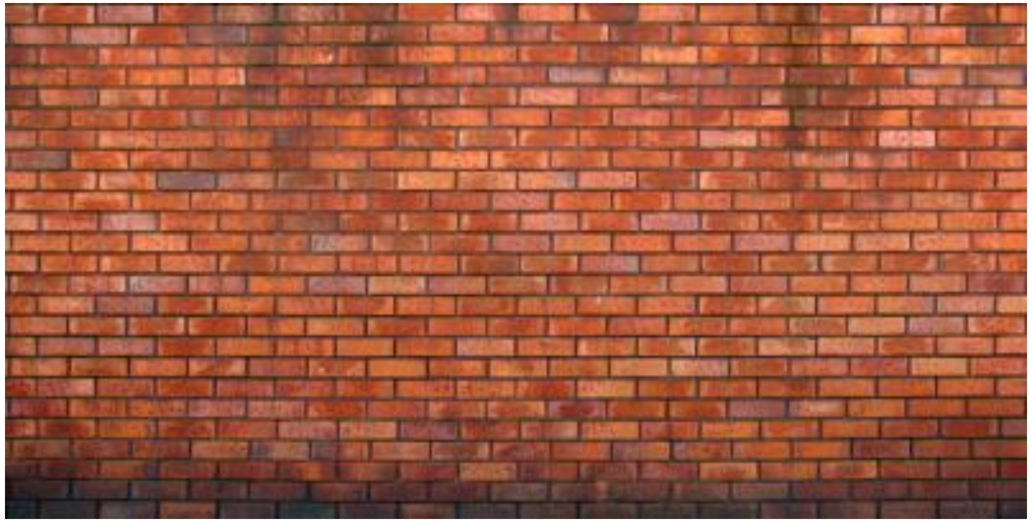


horizontal Sobel filter



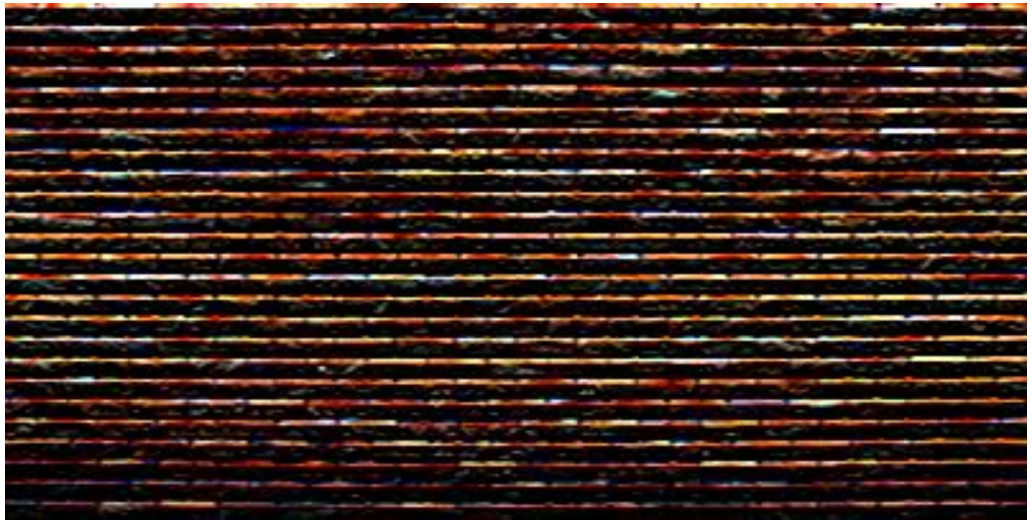
vertical Sobel filter

# Sobel filter example



original

horizontal Sobel filter



vertical Sobel filter

# Several derivative filters

Sobel

| | | |
|---|---|---|
| 1 | 0 | -1 |
| 2 | 0 | -2 |
| 1 | 0 | -1 |

| | | |
|---|---|---|
| 1 | 2 | 1 |
| 0 | 0 | 0 |
| -1 | -2 | -1 |

Scharr

| | | |
|---|---|---|
| 3 | 0 | -3 |
| 10 | 0 | -10 |
| 3 | 0 | -3 |

| | | |
|---|---|---|
| 3 | 10 | 3 |
| 0 | 0 | 0 |
| -3 | -10 | -3 |

Prewitt

| | | |
|---|---|---|
| 1 | 0 | -1 |
| 1 | 0 | -1 |
| 1 | 0 | -1 |

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 0 | 0 |
| -1 | -1 | -1 |

Roberts

| | |
|---|---|
| 0 | 1 |
| -1 | 0 |

| | |
|---|---|
| 1 | 0 |
| 0 | -1 |

- How are the other filters derived and how do they relate to the Sobel filter?
- How would you derive a derivative filter that is larger than 3x3?

# Computing image gradients

1. Select your favorite derivative filters.

$$\boldsymbol{S}_x = \begin{array}{|c|c|c|}\hline 1 & 0 & -1 \\ \hline 2 & 0 & -2 \\ \hline 1 & 0 & -1 \\ \hline \end{array} \qquad \boldsymbol{S}_y = \begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline 0 & 0 & 0 \\ \hline -1 & -2 & -1 \\ \hline \end{array}$$

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2. Convolve with the image to compute derivatives.

$$\frac{\partial \boldsymbol{f}}{\partial x} = \boldsymbol{S}_x \otimes \boldsymbol{f} \qquad \frac{\partial \boldsymbol{f}}{\partial y} = \boldsymbol{S}_y \otimes \boldsymbol{f}$$

# Computing image gradients

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2. Convolve with the image to compute derivatives.

$$\frac{\partial \boldsymbol{f}}{\partial x} = \boldsymbol{S}_x \otimes \boldsymbol{f} \qquad \frac{\partial \boldsymbol{f}}{\partial y} = \boldsymbol{S}_y \otimes \boldsymbol{f}$$

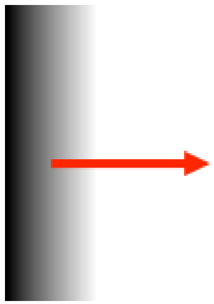
3. Form the image gradient, and compute its direction and amplitude.

$$\nabla \boldsymbol{f} = \left[\frac{\partial \boldsymbol{f}}{\partial x}, \frac{\partial \boldsymbol{f}}{\partial y}\right] \qquad \theta = \tan^{-1}\left(\frac{\partial f}{\partial y} \Big/ \frac{\partial f}{\partial x}\right) \qquad ||\nabla f|| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

gradient direction amplitude

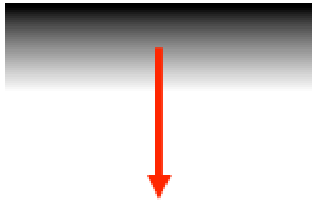
# Image Gradient

Gradient in x only



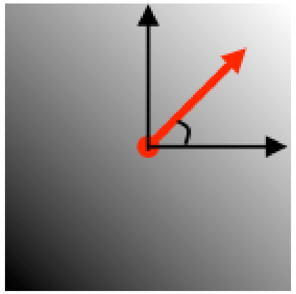
$$\nabla f = \left[\frac{\partial f}{\partial x}, 0\right]$$

Gradient in y only



$$\nabla f = \left[0, \frac{\partial f}{\partial y}\right]$$

Gradient in both x and y



$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

## Gradient direction

$$\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} \Big/ \frac{\partial f}{\partial x}\right)$$

*How does the gradient direction relate to the edge?*

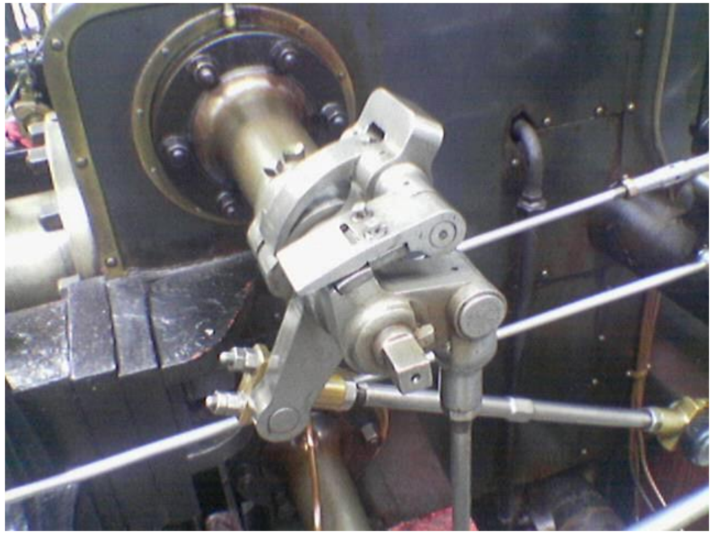
## Gradient magnitude

$$||\nabla f|| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

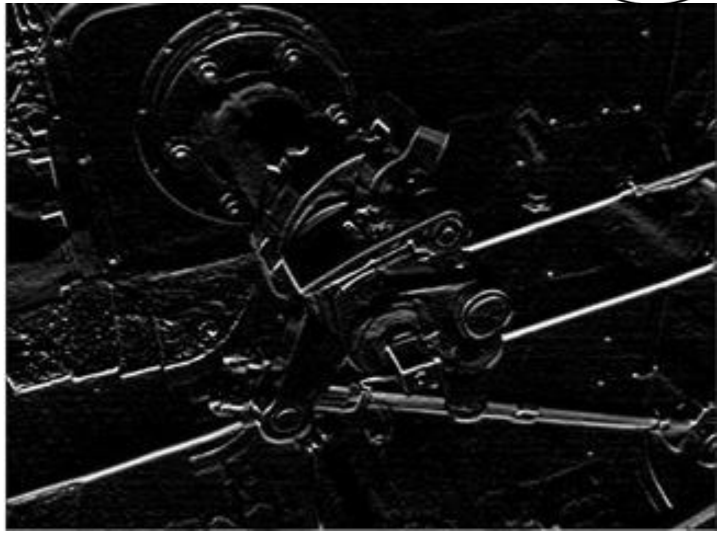
*What does a large magnitude look like in the image?*

# Image gradient example

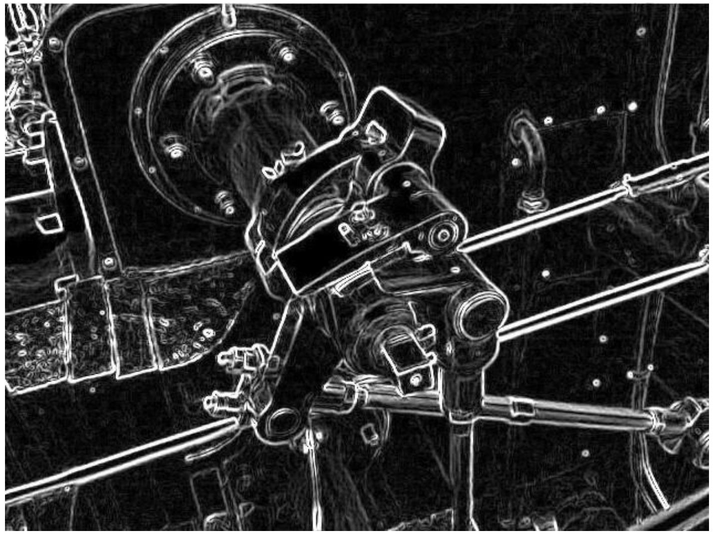
original



vertical
derivative



gradient
amplitude

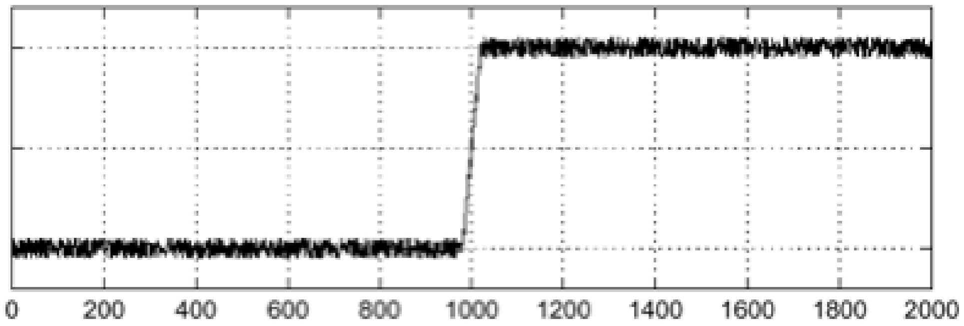


horizontal
derivative

How does the gradient direction relate to these edges?

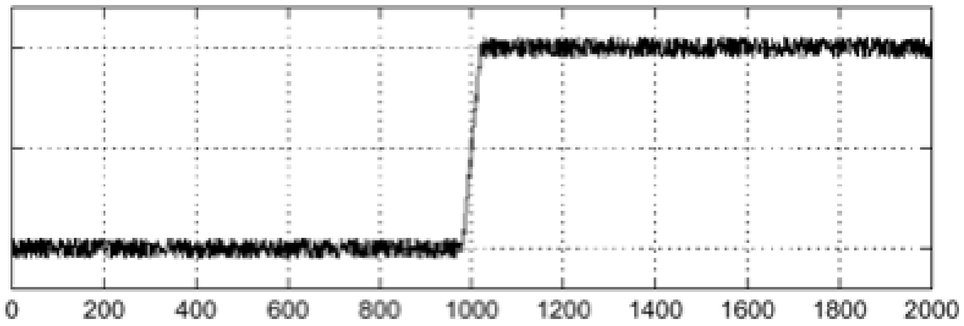
# How do you find the edge of this signal?

intensity plot



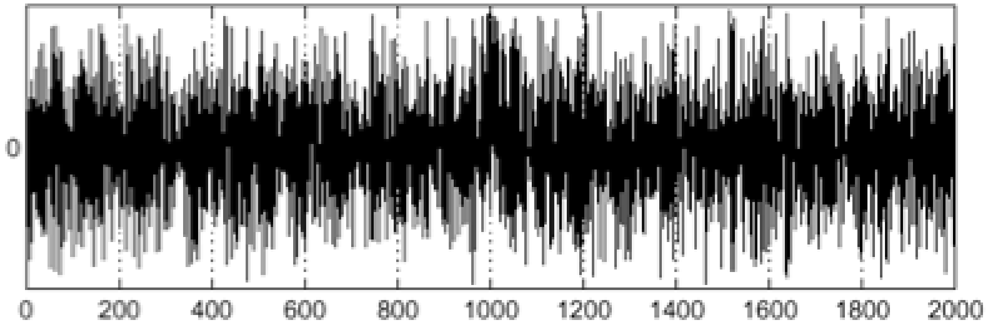
# How do you find the edge of this signal?

intensity plot




Using a derivative filter:

derivative plot




What's the problem here?

# Differentiation is very sensitive to noise

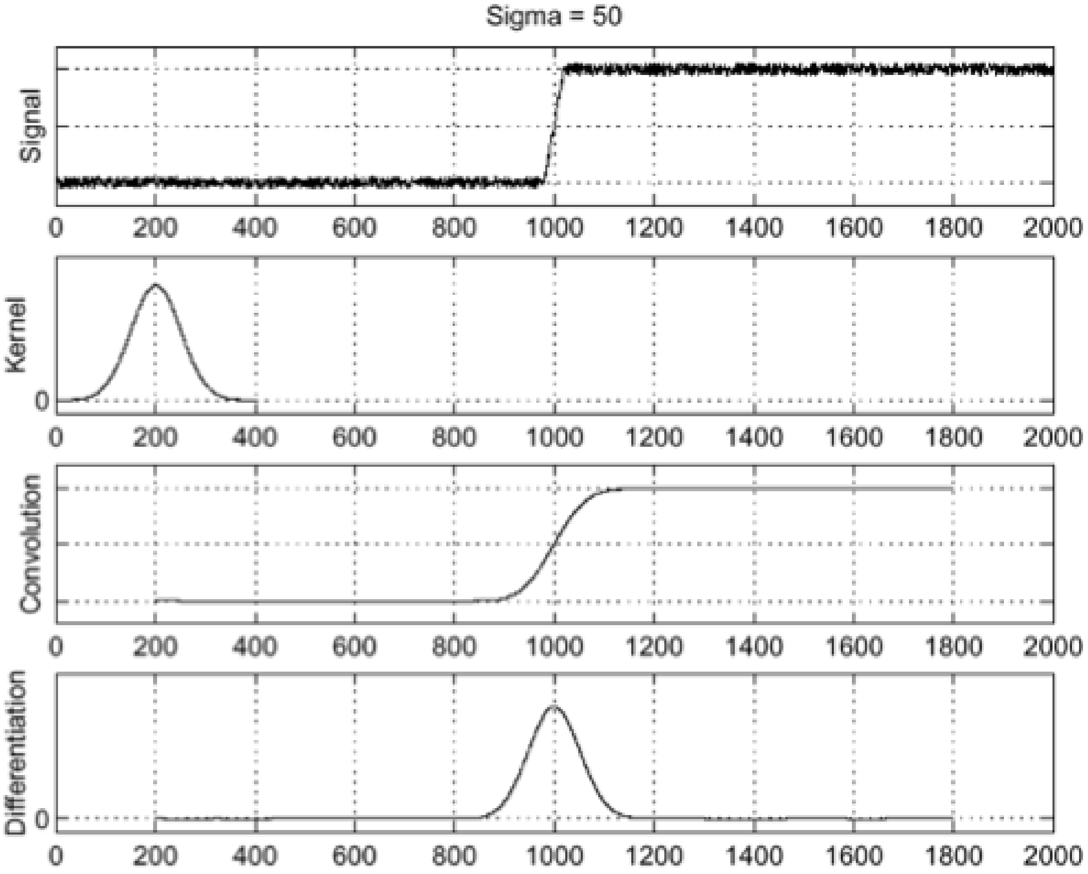
When using derivative filters, it is critical to blur first!

input

Gaussian

blurred

derivative of blurred




How much should we blur?

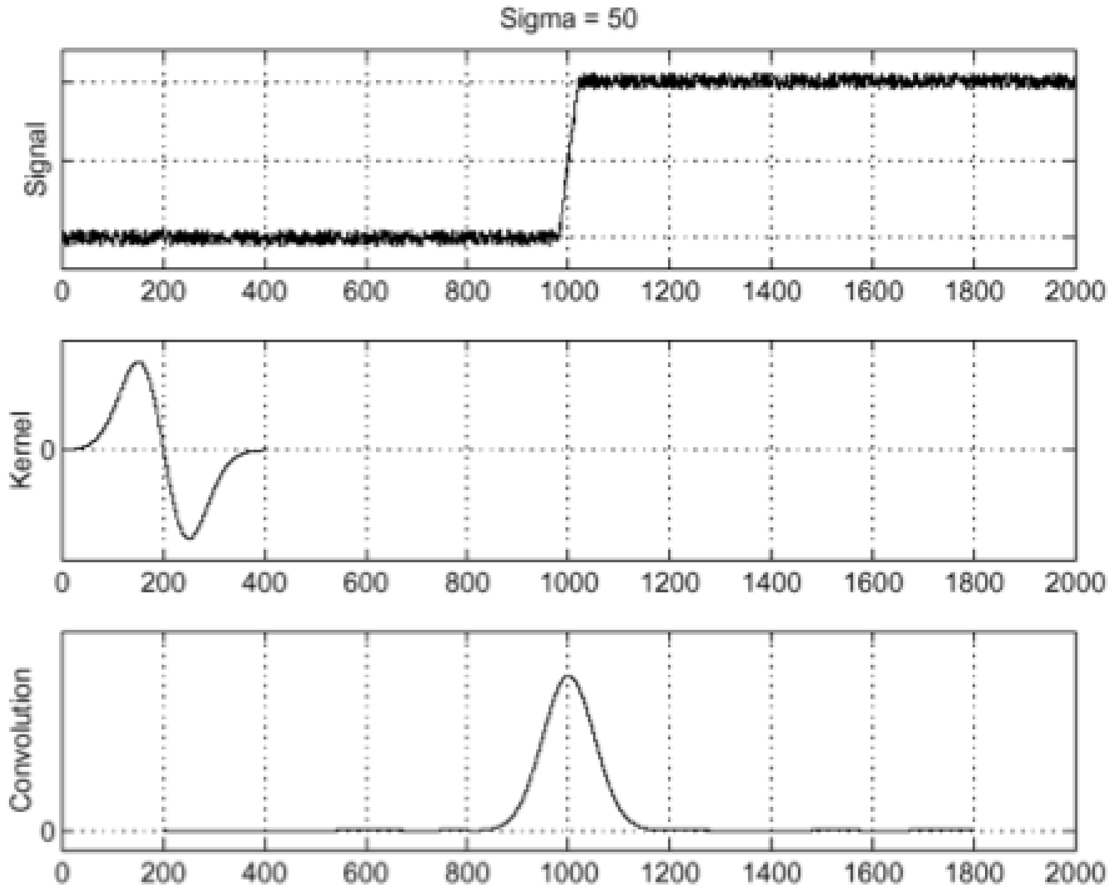
# Derivative of Gaussian (DoG) filter

Derivative theorem of convolution: $\frac{\partial}{\partial x}(h \star f) = (\frac{\partial}{\partial x}h) \star f$

input

derivative of Gaussian

output (same as before)




- How many operations did we save?
- Any other advantages beyond efficiency?

# Laplace filter

Basically a second derivative filter.

- We can use finite differences to derive it, as with first derivative filter.

first-order finite difference

$$f'(x) = \lim_{h \to 0} \frac{f(x + 0.5h) - f(x - 0.5h)}{h} \longrightarrow$$

1D derivative filter

| | | |
|---|---|---|
| 1 | 0 | -1 |

second-order finite difference

$$f''(x) = \lim_{h \to 0} \frac{f(x + h) - 2f(x) + f(x - h)}{h^2} \longrightarrow$$

Laplace filter

?

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$$f'(x) = \lim_{h \to 0} \frac{f(x + 0.5h) - f(x - 0.5h)}{h} \longrightarrow$$

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second-order finite difference

$$f''(x) = \lim_{h \to 0} \frac{f(x + h) - 2f(x) + f(x - h)}{h^2} \longrightarrow$$

Laplace filter

| | | |
|---|---|---|
| 1 | -2 | 1 |

# Laplacian of a Gaussian

The Laplace of Gaussian (LoG) of image $f$ can be written as

$$\nabla^2(f * g) = f * \nabla^2 g$$

with $g$ the Gaussian kernel and $*$ the convolution. That is, the Laplace of the image smoothed by a Gaussian kernel is identical to the image convolved with the Laplace of the Gaussian kernel. This convolution can be further expanded, in the 2D case, as

$$f * \nabla^2 g = f * \left( \frac{\partial^2}{\partial x^2} g + \frac{\partial^2}{\partial y^2} g \right) = f * \frac{\partial^2}{\partial x^2} g + f * \frac{\partial^2}{\partial y^2} g$$

Thus, it is possible to compute it as the addition of two convolutions of the input image with second derivatives of the Gaussian kernel (in 3D this is 3 convolutions, etc.). This is interesting because the Gaussian kernel is separable, as are its derivatives. That is,

$$f(x, y) * g(x, y) = f(x, y) * (g(x) * g(y)) = (f(x, y) * g(x)) * g(y)$$

meaning that instead of a 2D convolution, we can compute the same thing using two 1D convolutions. This saves a lot of computations. For the smallest thinkable Gaussian kernel you'd have 5 samples along each dimension. A 2D convolution requires 25 multiplications and additions, two 1D convolutions require 10. The larger the kernel, or the more dimensions in the image, the more significant these computational savings are.

Thus, the LoG can be computed using four 1D convolutions. The LoG kernel itself, though, is not separable.

There is an approximation where the image is first convolved with a Gaussian kernel and then $\nabla^2$ is implemented using finite differences, leading to the 3x3 kernel with -4 in the middle and 1 in its four edge neighbors.

The Ricker wavelet or Mexican hat operator are identical to the LoG, up to scaling and normalization.

image processing - What Is the Difference between Difference of Gaussian, Laplace of Gaussian, and Mexican Hat Wavelet? - Signal Processing Stack Exchange

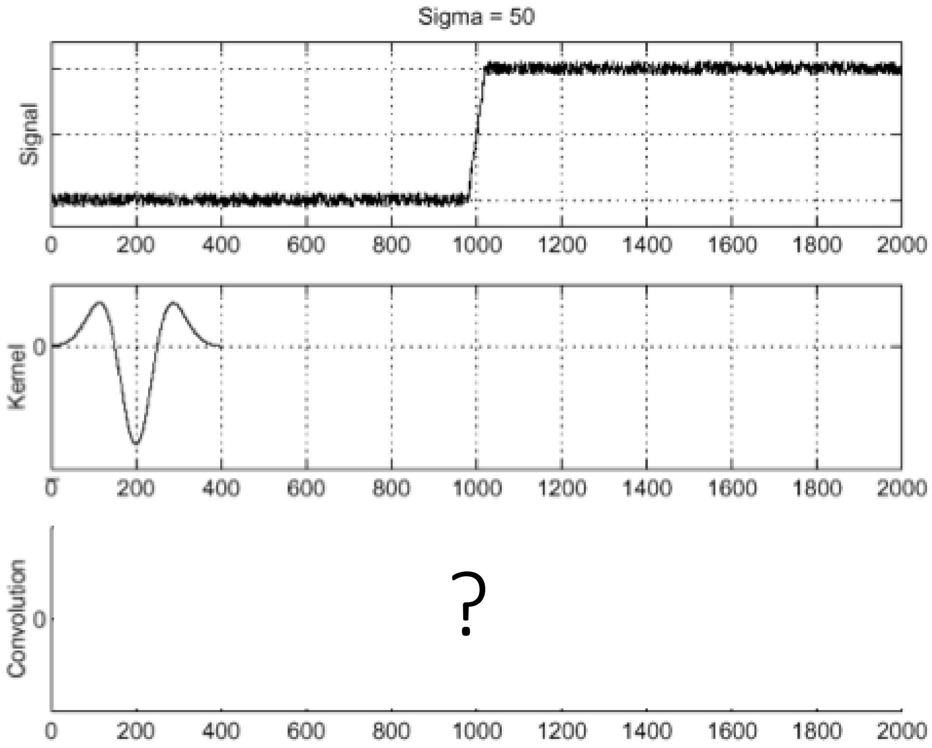
# Laplacian of Gaussian (LoG) filter

As with derivative, we can combine Laplace filtering with Gaussian filtering

input

Laplacian of Gaussian

output



# Laplacian of Gaussian (LoG) filter

As with derivative, we can combine Laplace filtering with Gaussian filtering

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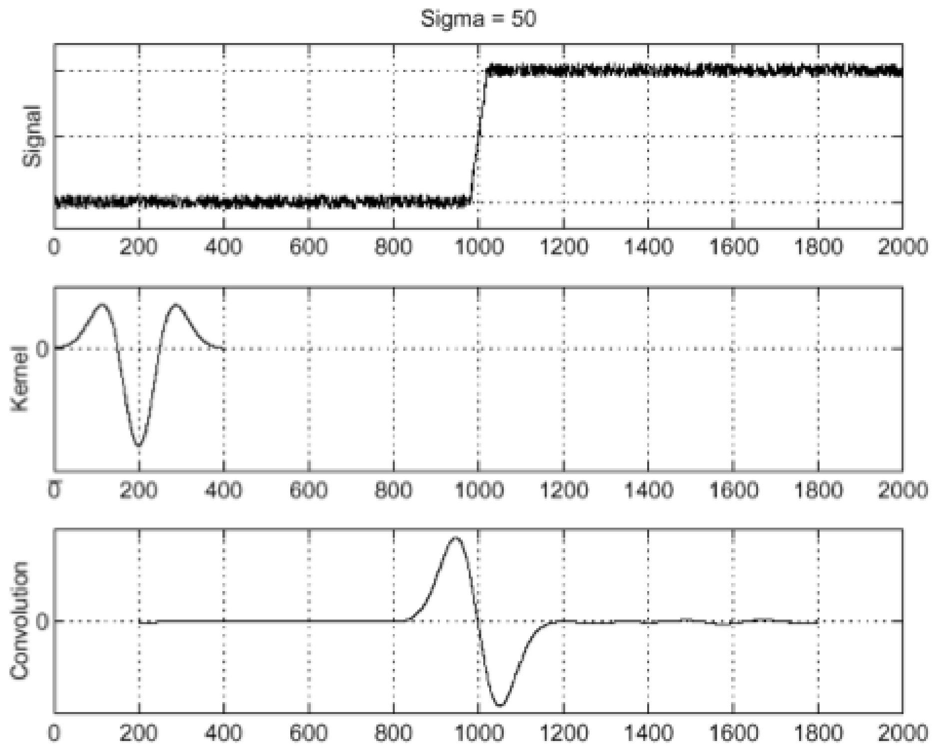
Laplacian of Gaussian

output




“zero crossings” at edges

# Laplace and LoG filtering examples

Laplacian of Gaussian filtering

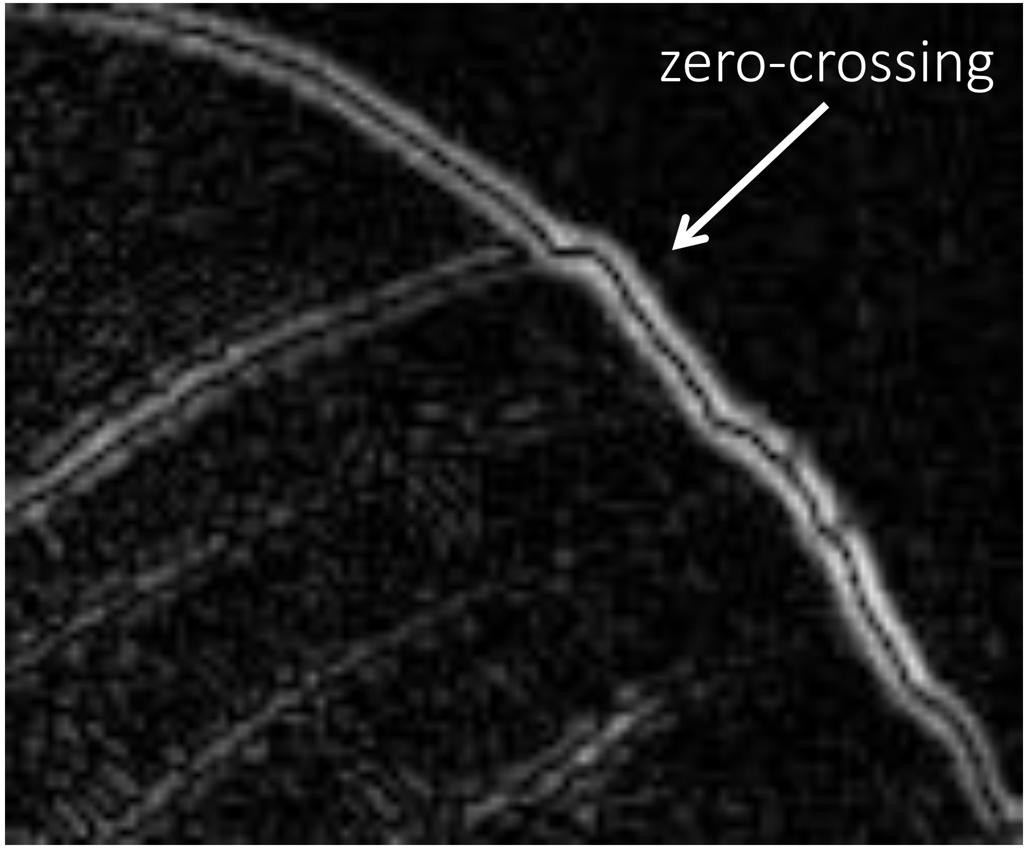
Laplace filtering

# Laplacian of Gaussian vs Derivative of Gaussian

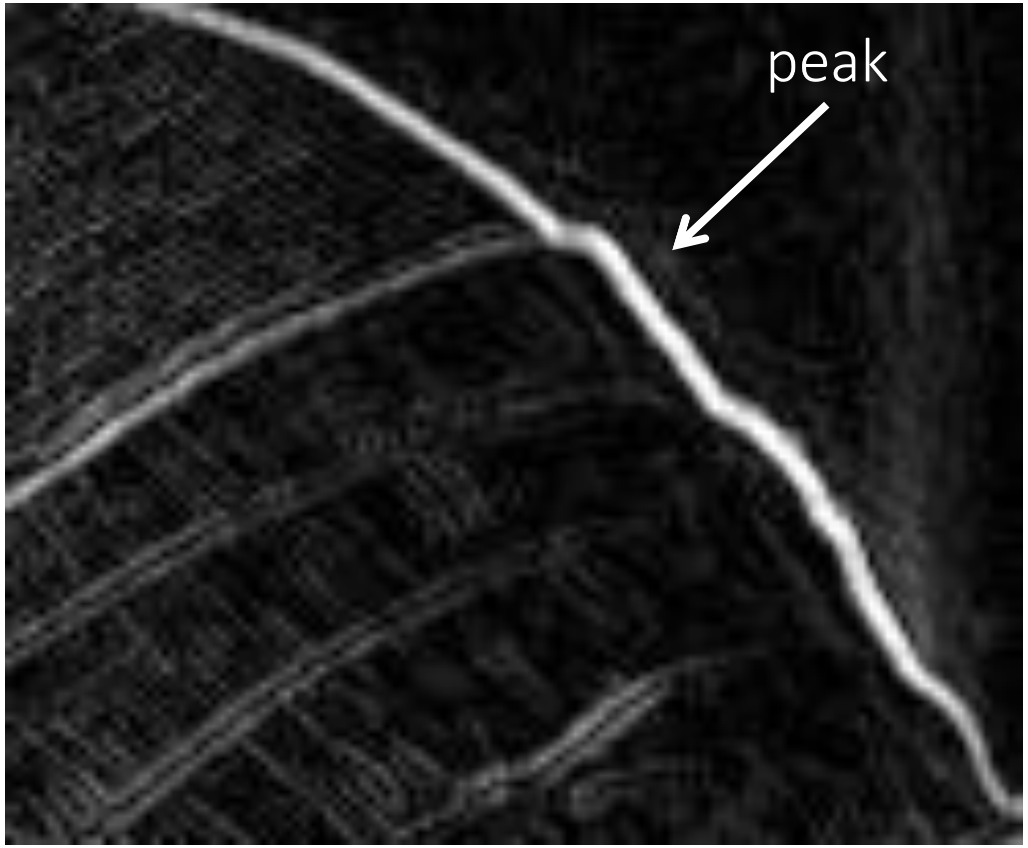
Laplacian of Gaussian filtering

Derivative of Gaussian filtering

# Laplacian of Gaussian vs Derivative of Gaussian



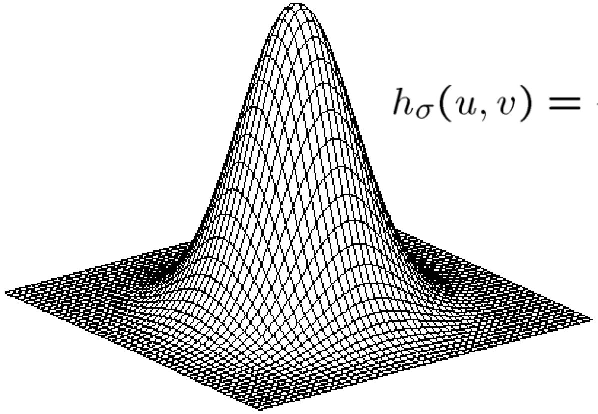

Laplacian of Gaussian filtering




Derivative of Gaussian filtering

Zero crossings are more accurate at localizing edges (but not very convenient).

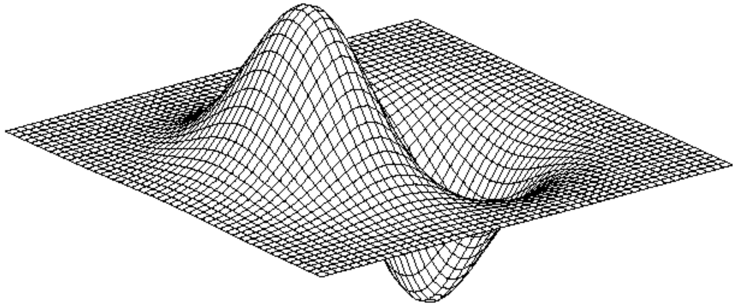
# 2D Gaussian filters



$$h_\sigma(u,v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$$

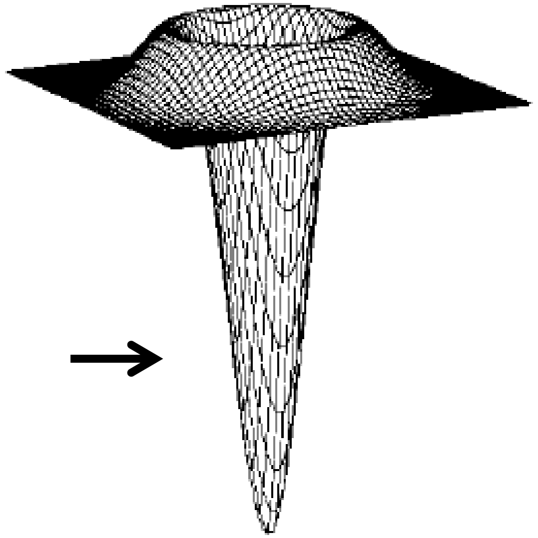
Gaussian

$$\frac{\partial}{\partial x} h_\sigma(u,v)$$



Derivative of Gaussian

$$\nabla^2 h_\sigma(u,v)$$



Mexican hat

Laplacian of Gaussian

# References

Basic reading:

- Szeliski textbook, Section 3.2

# Questions?