CAP 4453
Robot Vision

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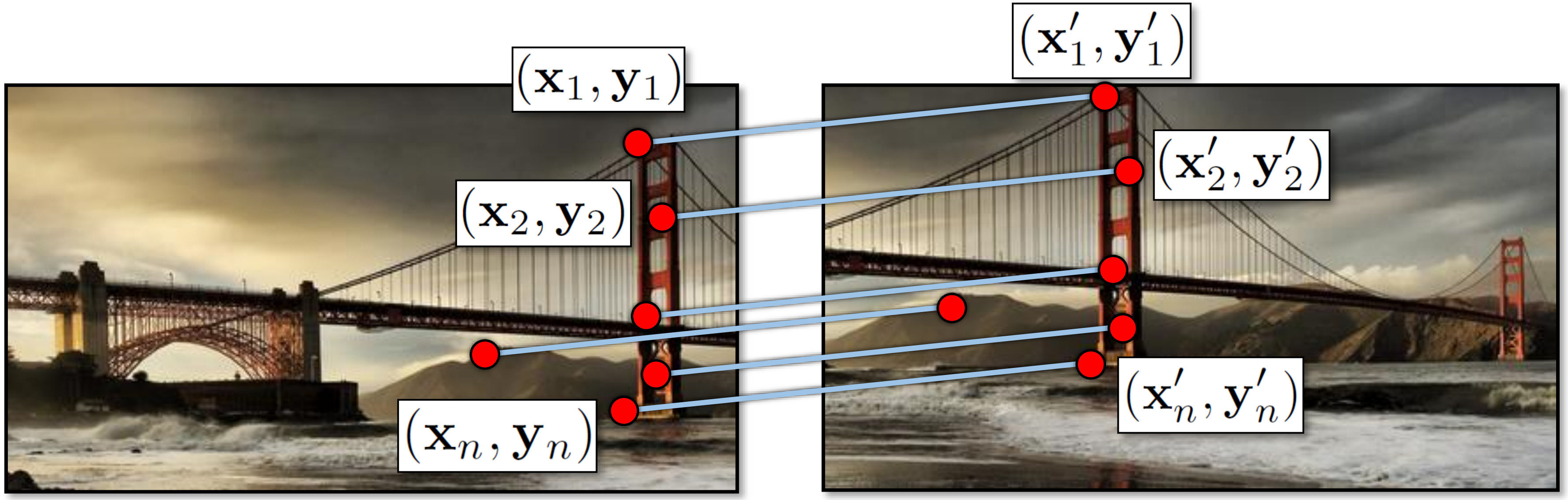
Credits

- Some slides comes directly from these sources:
  - Ioannis (Yannis) Gkioulekas (CMU)
  - Kris Kitani.
  - Fredo Durand (MIT).
  - James Hays (Georgia Tech).
  - Yogesh S Rawat (UCF)
  - Noah Snavely (Cornell)

Short Review from last class

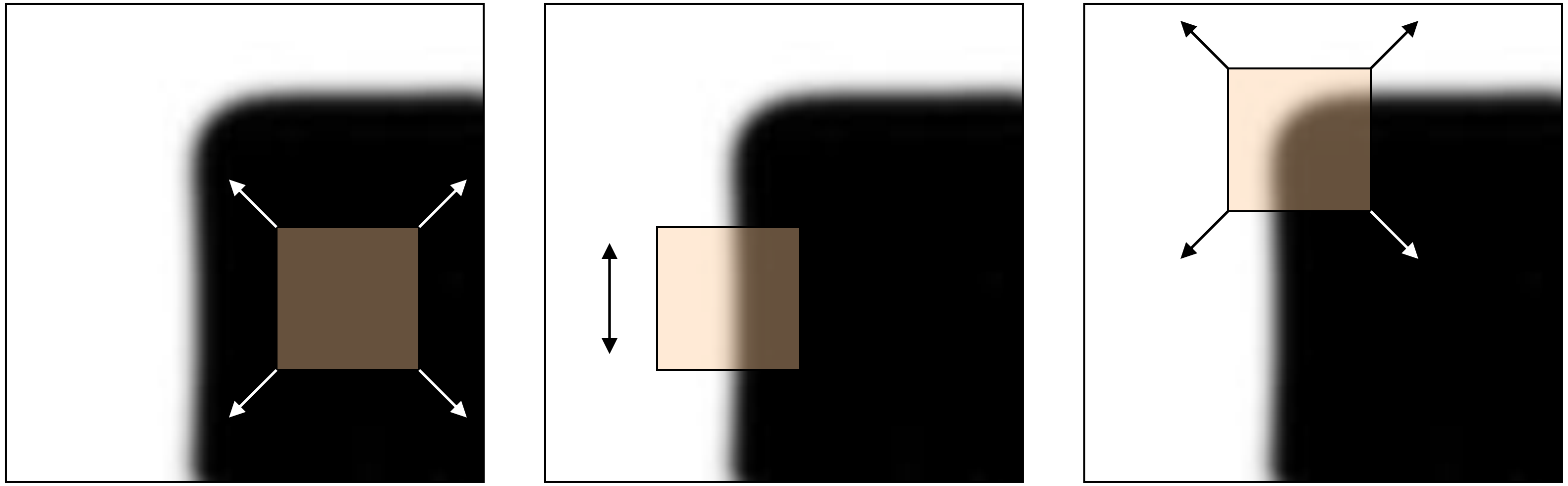
Image warping



How do we find point correspondences automatically?

Corner Detection: Basic Idea

- We should easily recognize the point by looking through a small window
- Shifting a window in *any direction* should give *a large change* in intensity



"flat" region:
no change in all directions

"edge":
no change along the edge direction

"corner":
significant change in all directions

Source: A. Efros

C.Harris and M.Stephens. "A Combined Corner and Edge Detector."1988.

Harris Detector

1. Compute x and y derivatives of image

$$I_x = G_\sigma^x * I \qquad I_y = G_\sigma^y * I$$

2. Compute products of derivatives at every pixel

$$I_{x^2} = I_x \cdot I_x \qquad I_{y^2} = I_y \cdot I_y \quad I_{xy} = I_x \cdot I_y$$

3. Compute the sums of the products of derivatives at each pixel

$$S_{xy} = G_{\sigma'} * I_{xy} \qquad S_{y^2} = G_{\sigma'} * I_{y^2} \qquad S_{x^2} = G_{\sigma'} * I_{x^2}$$

4. Define the matrix at each pixel

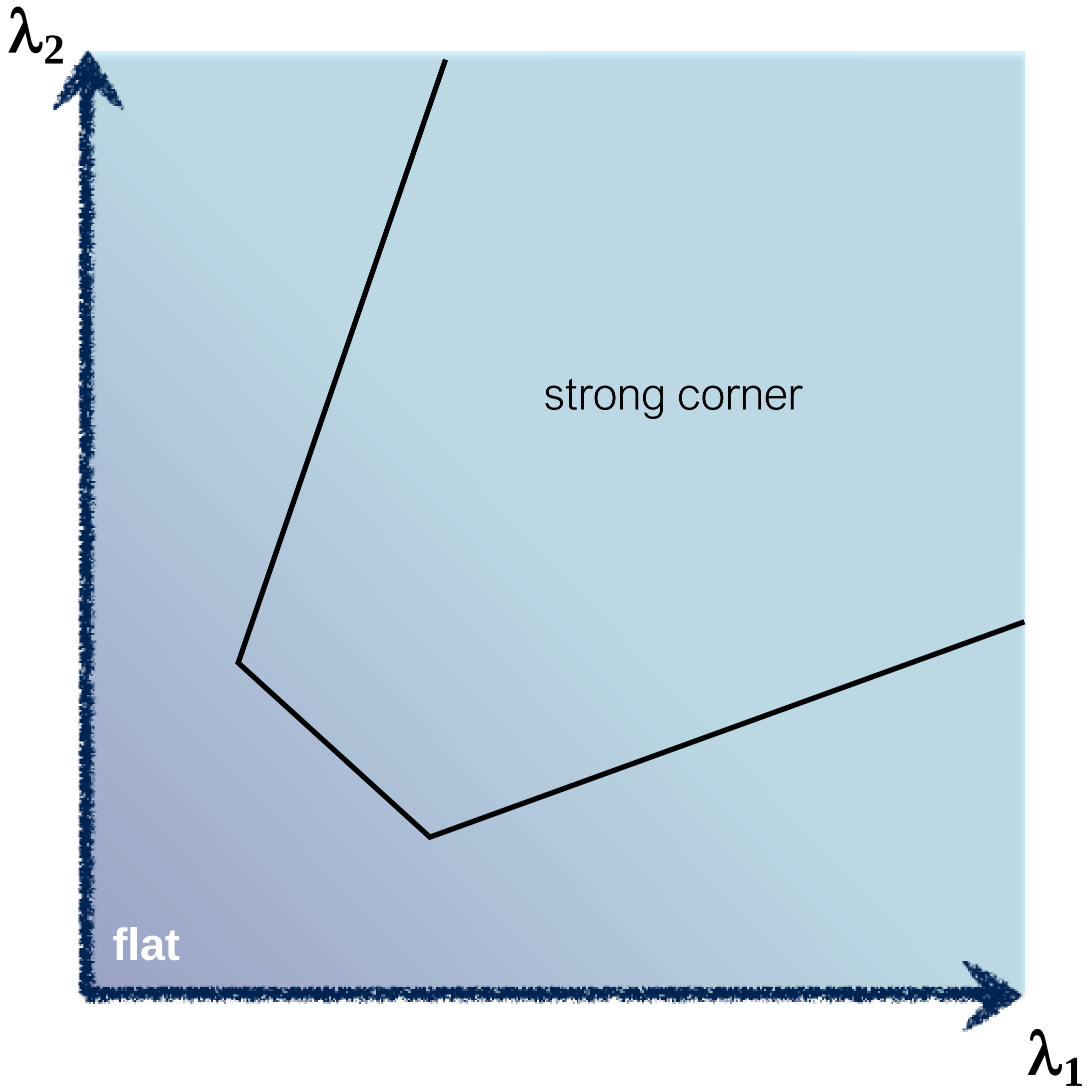
$$M(x,y) = \begin{bmatrix} S_{x^2}(x,y) & S_{xy}(x,y) \\ S_{xy}(x,y) & S_{y^2}(x,y) \end{bmatrix}$$

5. Compute the response of the detector at each pixel

$$R = \det M - k(\mathrm{trace} M)^2$$

6. Threshold on value of R; compute non-max suppression.

Use threshold on eigenvalues to detect corners



Think of a function to score 'cornerness'

Harris & Stephens (1988)

$$R = \det(M) - \kappa \operatorname{trace}^2(M)$$

Kanade & Tomasi (1994)

$$R = \min(\lambda_1, \lambda_2)$$

Nobel (1998)

$$R = \frac{\det(M)}{\operatorname{trace}(M) + \epsilon}$$

$$\det M = \lambda_1 \lambda_2$$

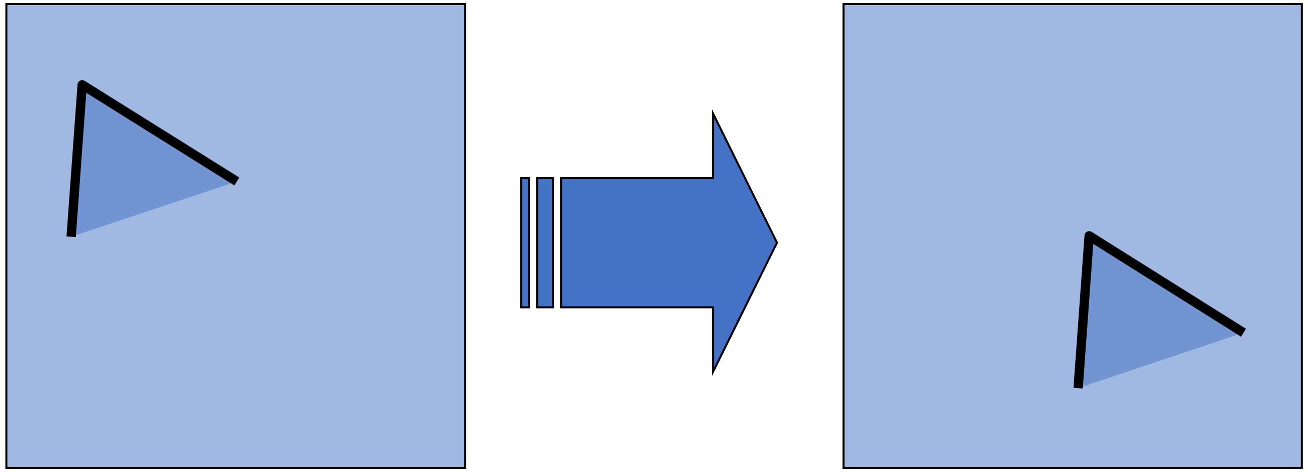
$$\operatorname{trace} M = \lambda_1 + \lambda_2$$

$$det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = ad - bc$$

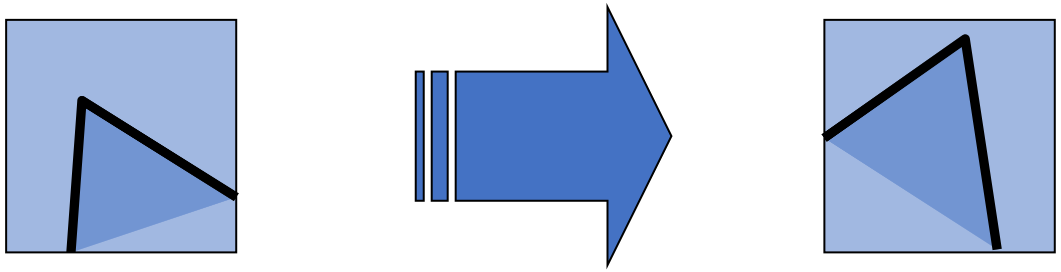
$$trace\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = a + d$$

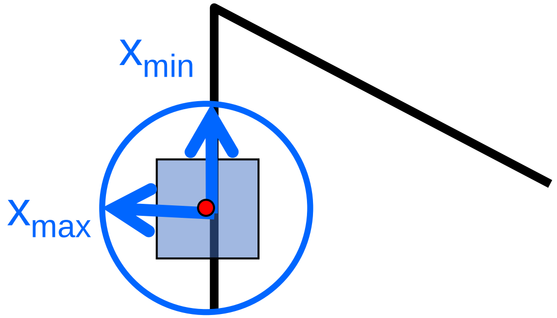
Harris corner detection and translation

- What happens if image is translated?
- Derivatives, second moment matrix obtained through convolution, which is *translation equivariant*
- Eigenvalues based only on derivatives so cornerness is *invariant*
- Thus Harris corner detection location is *equivariant to translation,* and response is *invariant to translation*



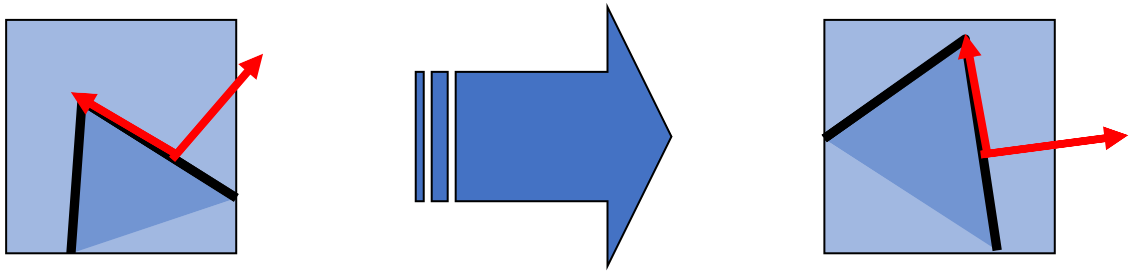
What about rotation?





- Now every patch is rotated, so problem?
- Recall properties of second moment matrix
- Eigenvalues and eigenvectors of M
  - Define shift directions with the smallest and largest change in error
  - x_{max}= direction of largest increase in E (across the edge)
  - λ_{max} = amount of increase in direction x_{max}
  - x_{min}= direction of smallest increase in E (along the edge)
  - λ_{min} = amount of increase in direction x_{min}

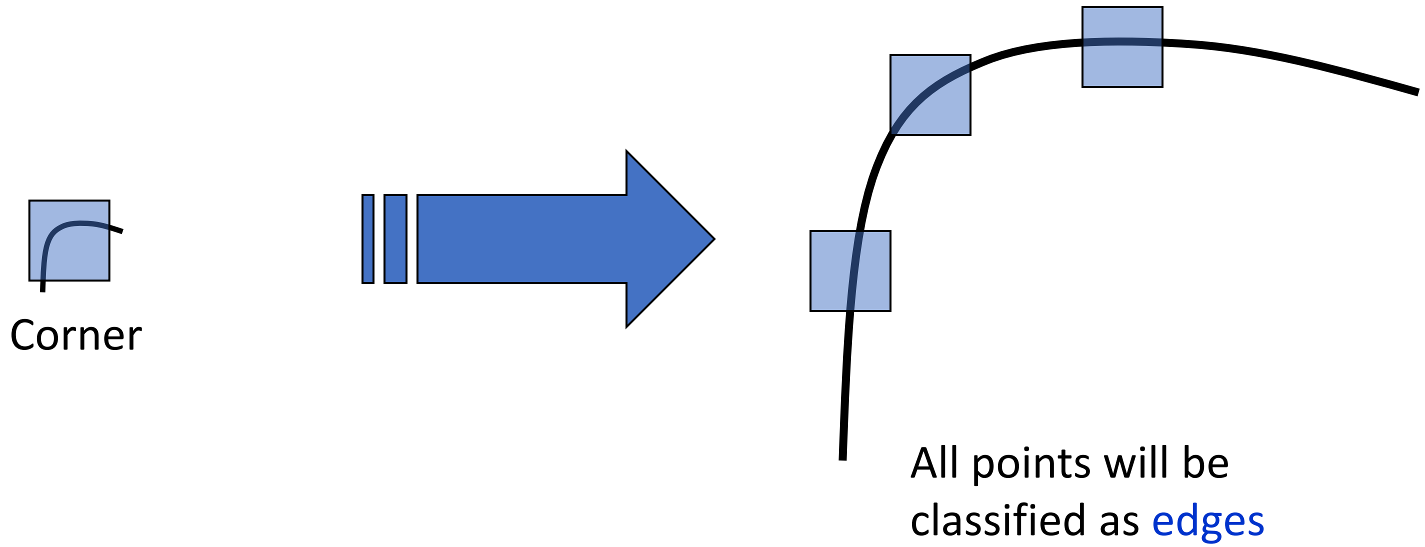
What about rotation?



- What happens to eigenvalues and eigenvectors when a patch rotates?
- Eigenvectors represent the *direction* of maximum / minimum change in appearance, so they rotate *with the patch*
- Eigenvalues represent the corresponding *magnitude* of maximum/minimum change so they *stay constant*
- Corner response is only dependent on the eigenvalues so is *invariant to* rotation
- Corner location is as before equivariant to rotation.

What about scaling?

- What was one patch earlier is now many



Not invariant to scaling

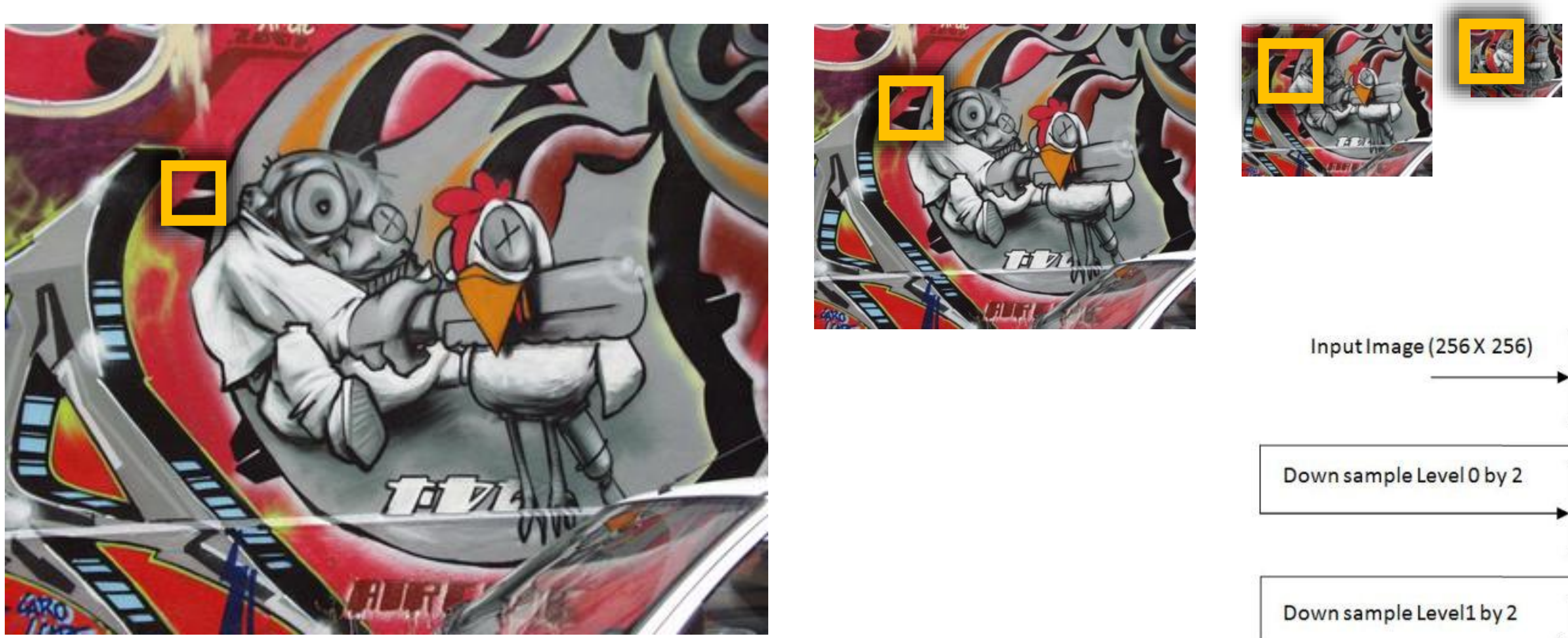
implementation

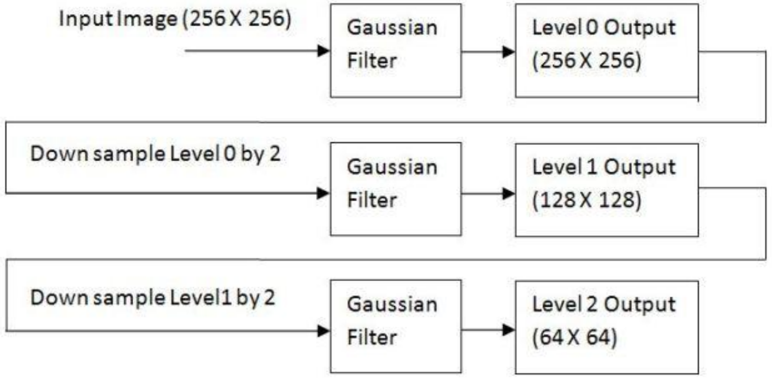
```
For each level of the Gaussian pyramid

    compute feature response (e.g. Harris, Laplacian)


For each level of the Gaussian pyramid

    if local maximum and cross-scale

    save scale and location of feature
```

(x, y, s)

Implementation

- Instead of computing *f* for larger and larger windows, we can implement using a fixed window size with a Gaussian pyramid

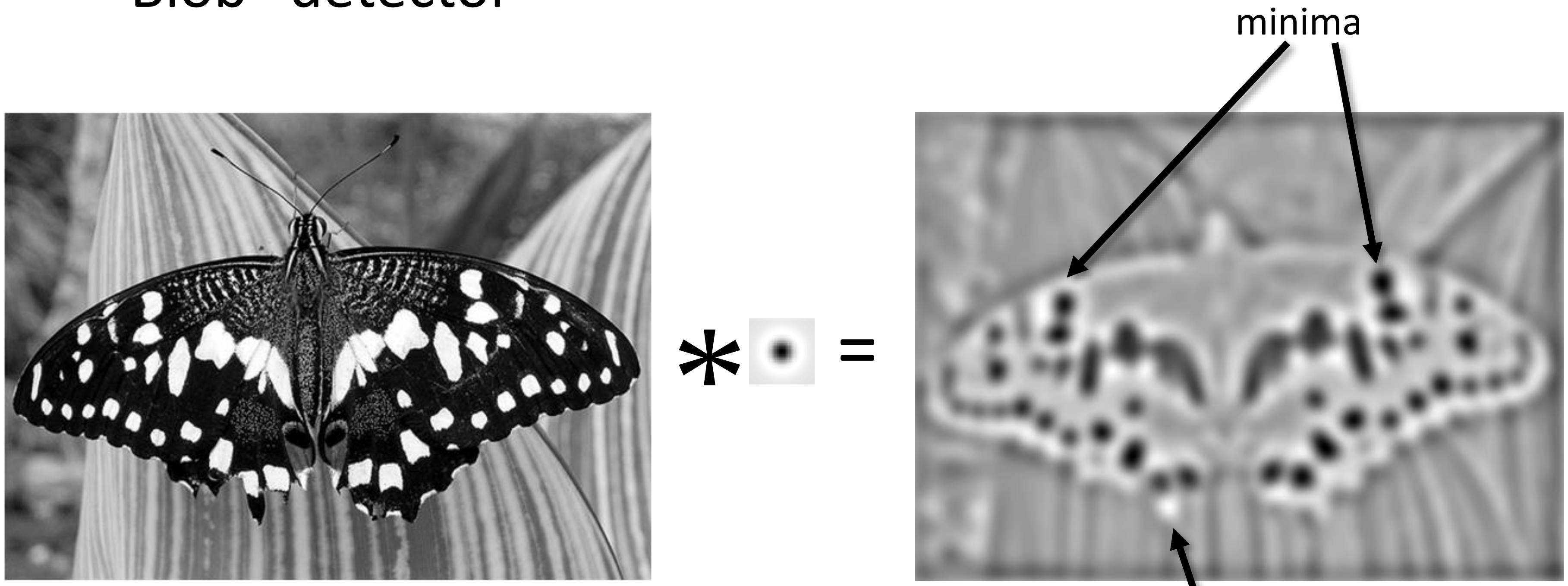




Blob detection

Laplacian of Gaussian

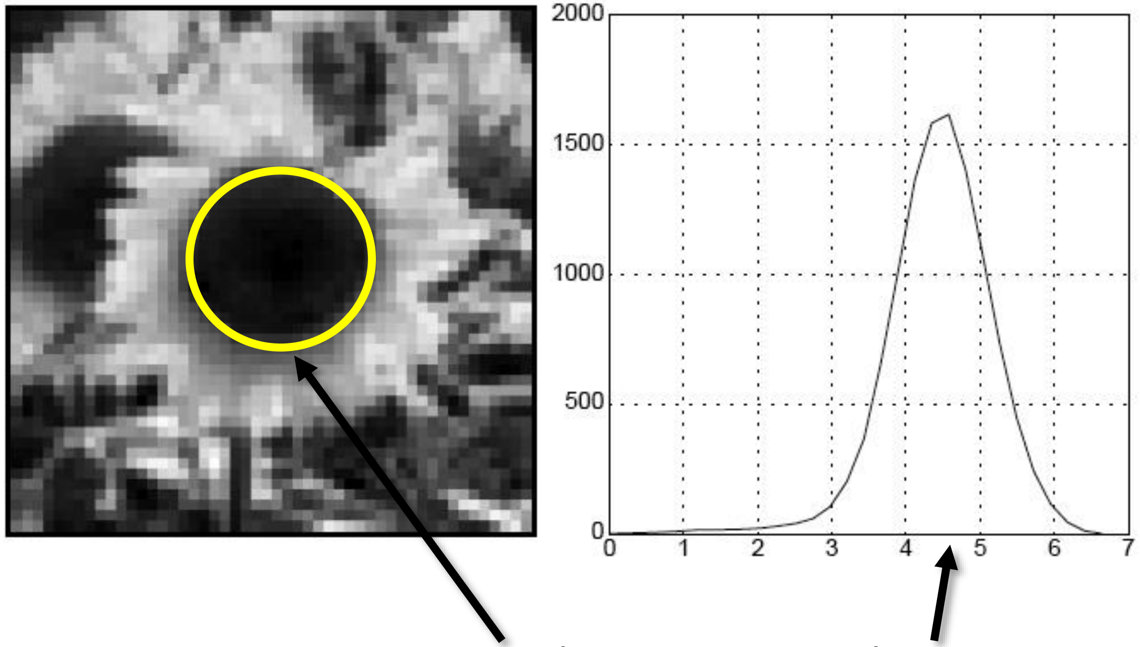
- “Blob” detector



- Find maxima *and minima* of LoG operator in space and scale

Characteristic scale

- We define the characteristic scale as the scale that produces peak of Laplacian response

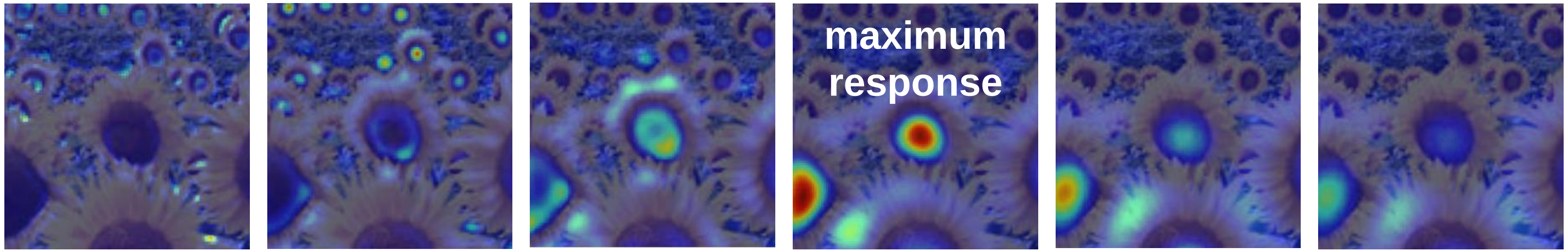


characteristic scale

T. Lindeberg (1998). ["Feature detection with automatic scale selection."](#) *International Journal of Computer Vision* **30** (2): pp 77--116.

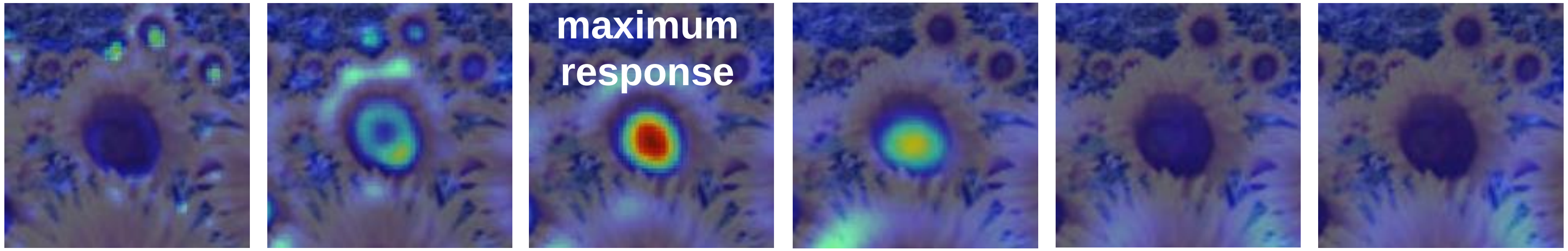
optimal scale

2.1 4.2 6.0 9.8 15.5 17.0



Full size image

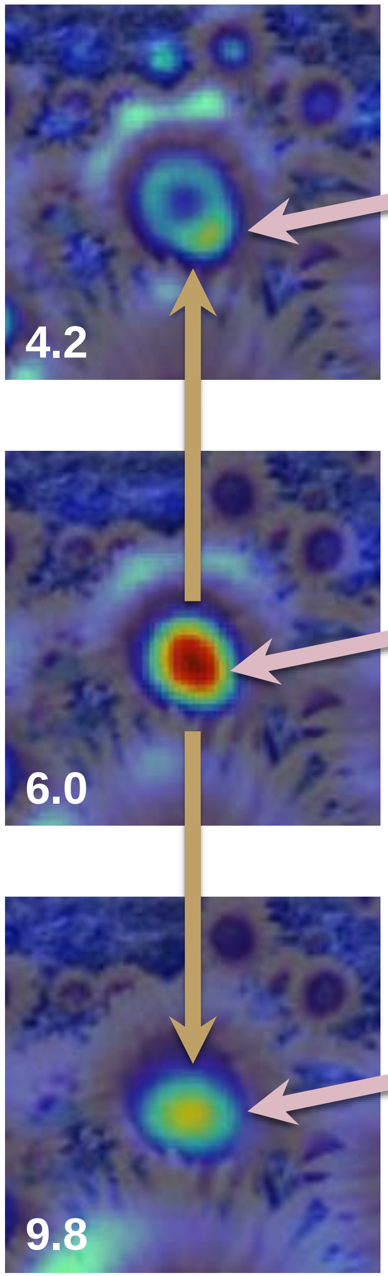
2.1 4.2 6.0 9.8 15.5 17.0



3/4 size image

local maximum

cross-scale maximum



local maximum

local maximum

Robot Vision

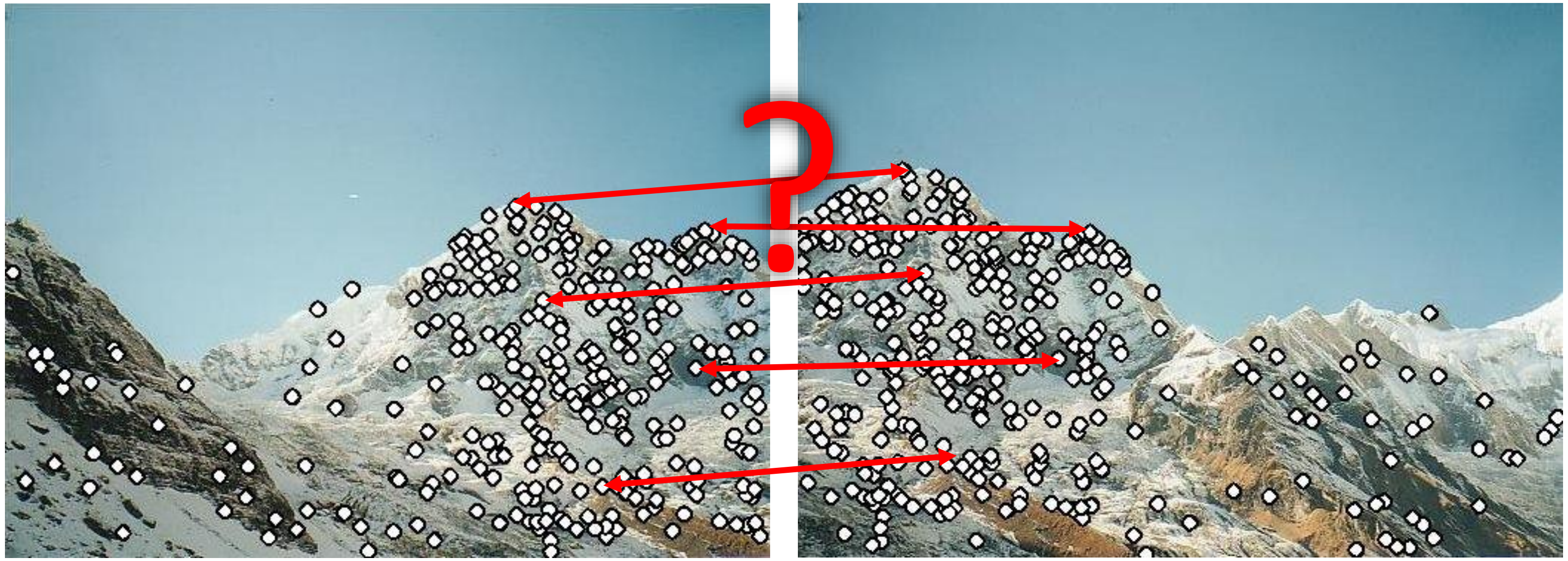
11. Feature points description

Outline

- Motivation
- Detecting key points
  - Harris corner detector
  - Blob detection
- Feature descriptors
  - HOG
  - MOPS
- SIFT

Matching feature points

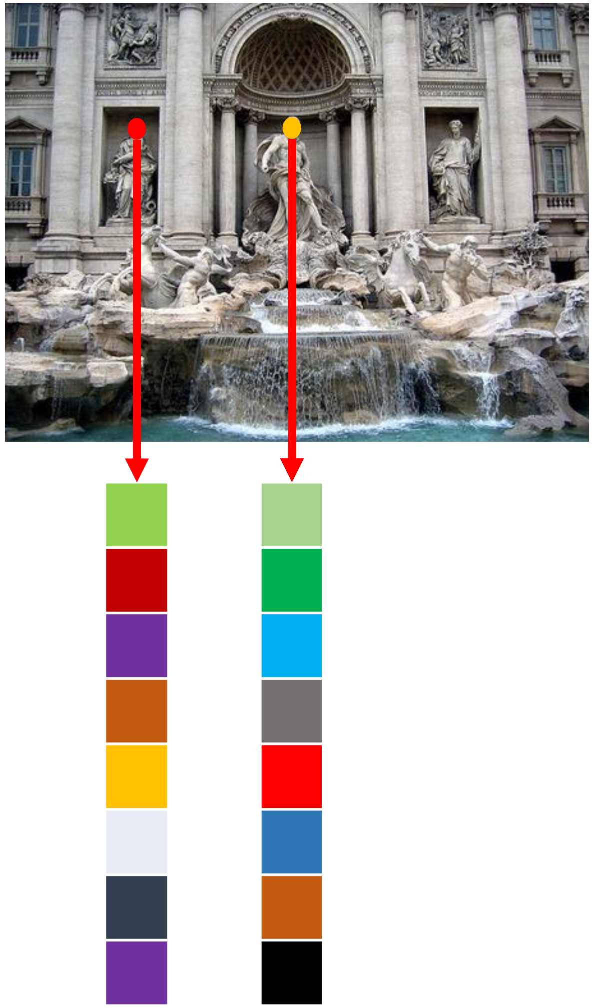
We know how to detect good points
Next question: **How to match them?**



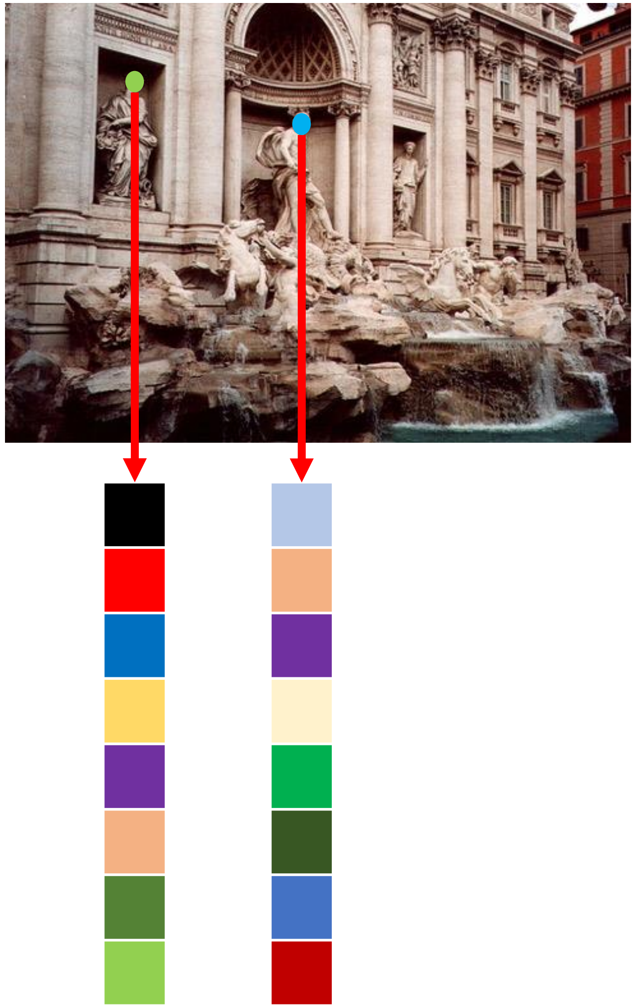
Two interrelated questions:

1. How do we *describe* each feature point?
2. How do we *match* descriptions?

Feature descriptor

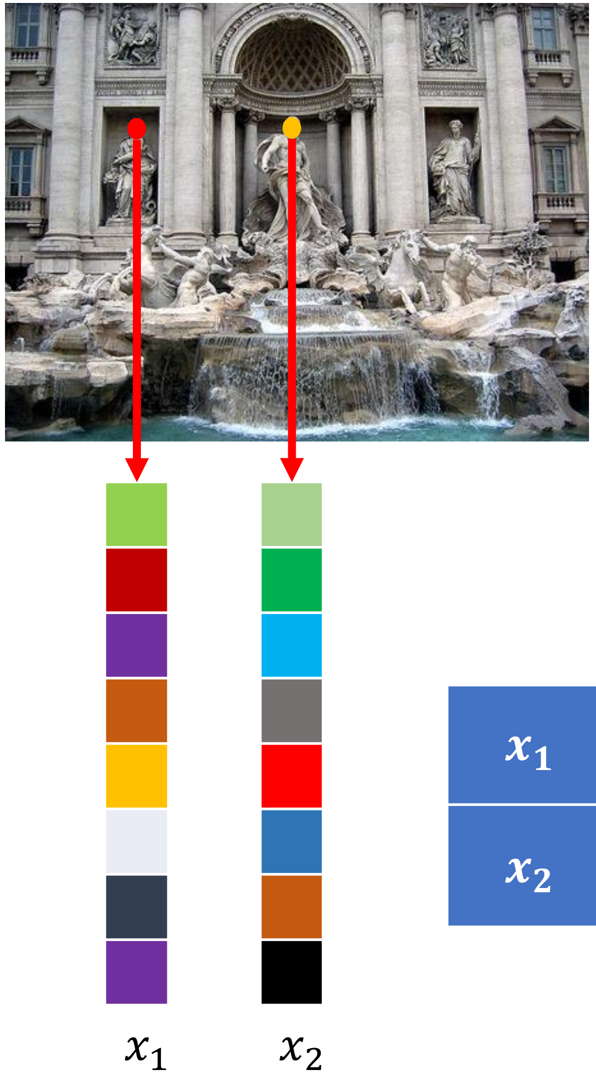


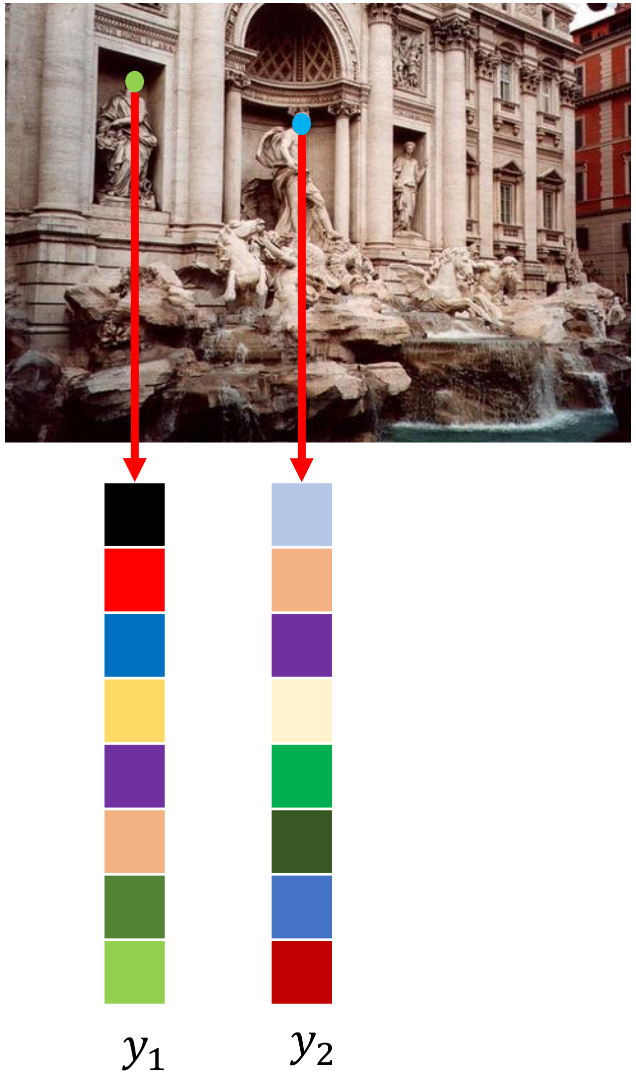
x_1 x_2



y_1 y_2

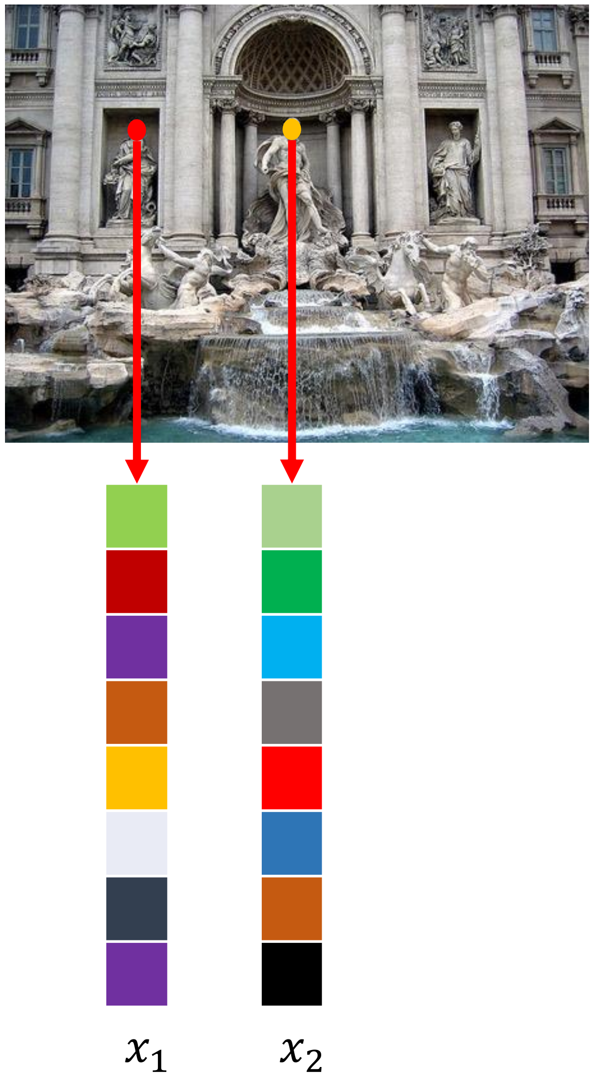
Feature matching





	y_1	y_2
x_1	$d(x_1, y_1)$	$d(x_1, y_2)$
x_2	$d(x_2, y_1)$	$d(x_2, y_2)$

Feature Descriptor

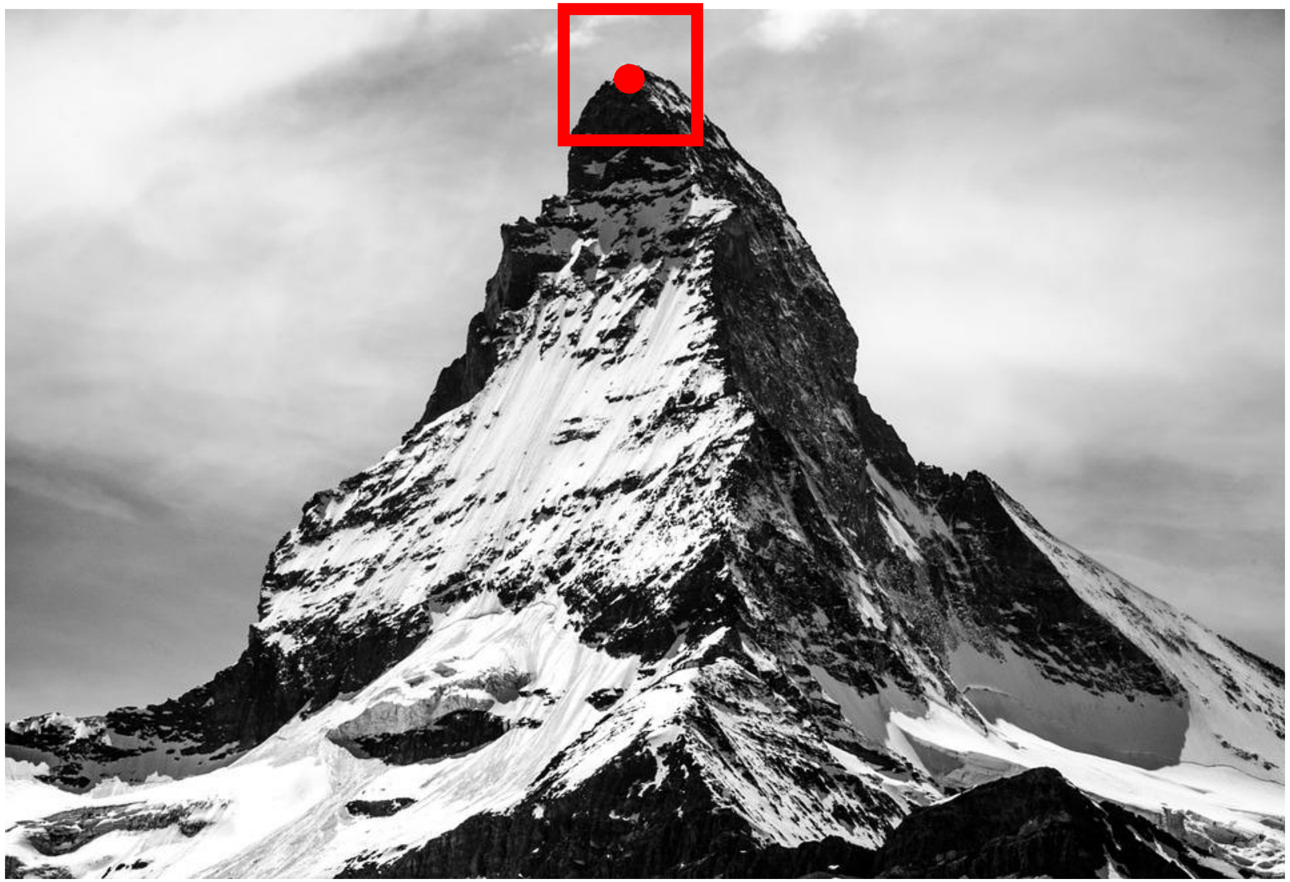


Feature detection and description

- Harris corner detection gives:
  - Location of each detected corner
  - Orientation of the corner (given by $\mathbf{x}_{max}$)
  - Scale of the corner (the image scale which gives the maximum response at this location)
- Want feature descriptor that is
  - Invariant to photometric transformations, translation, rotation, scaling
  - Discriminative

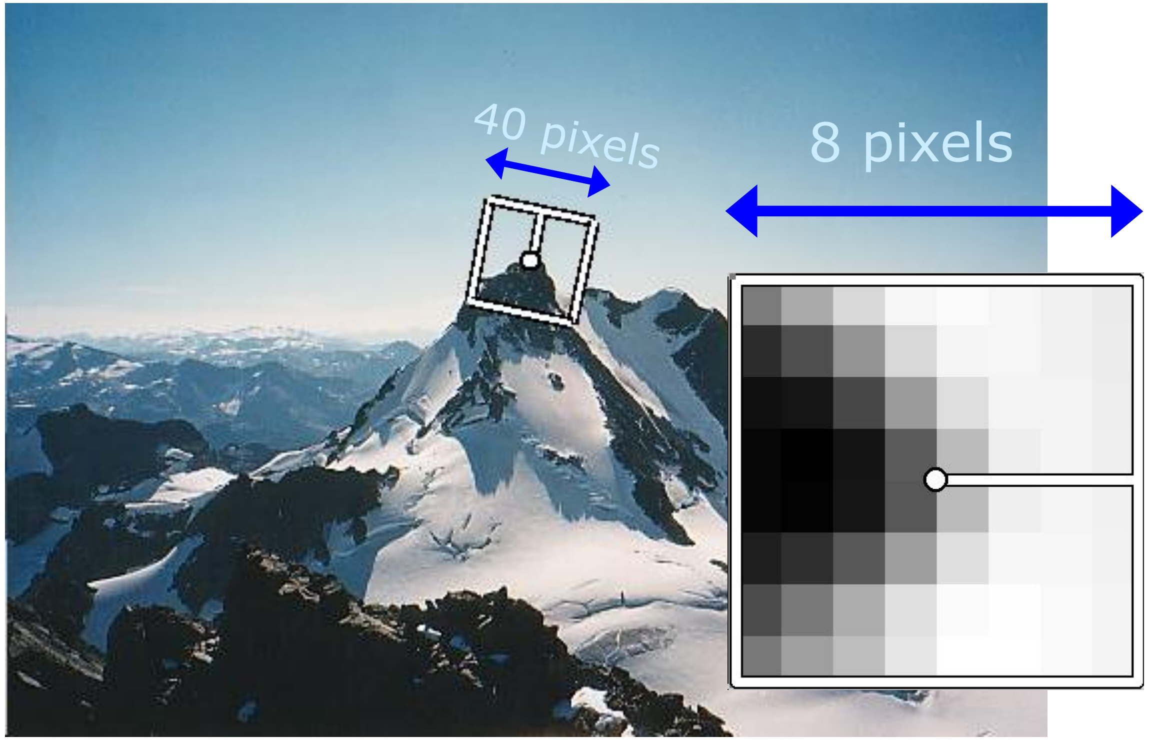
Multiscale Oriented PatcheS descriptor

- Describe a corner by the patch around that pixel
- Scale invariance by using scale identified by corner detector
- Rotation invariance by using orientation identified by corner detector
- Photometric invariance by subtracting mean and dividing by standard deviation



Multiscale Oriented PatcheS descriptor

- Take 40x40 square window around detected feature at the right scale
- Scale to 1/5 size (using prefiltering)
- Rotate to horizontal
- Sample 8x8 square window centered at feature
- Intensity normalize the window by subtracting the mean, dividing by the standard deviation in the window



Adapted from slide by Matthew Brown

MOPS

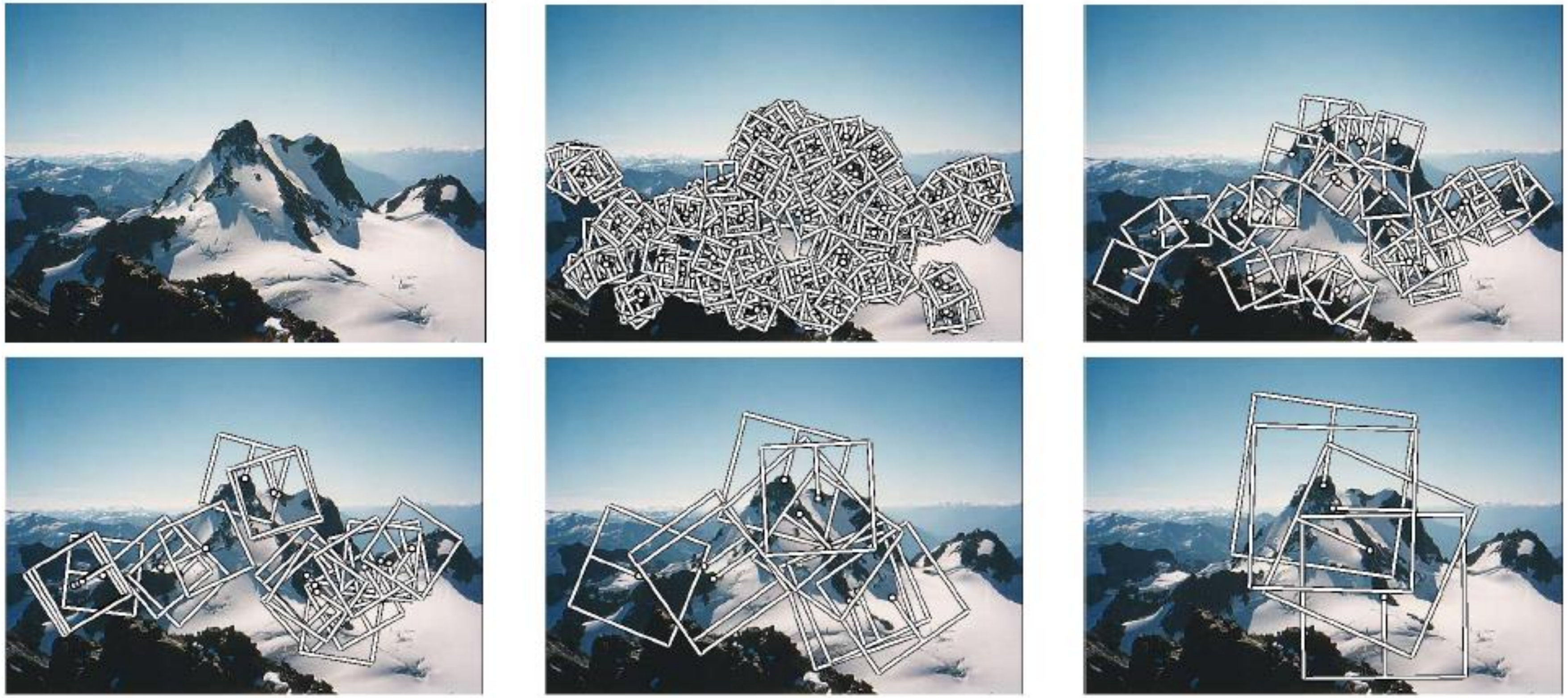


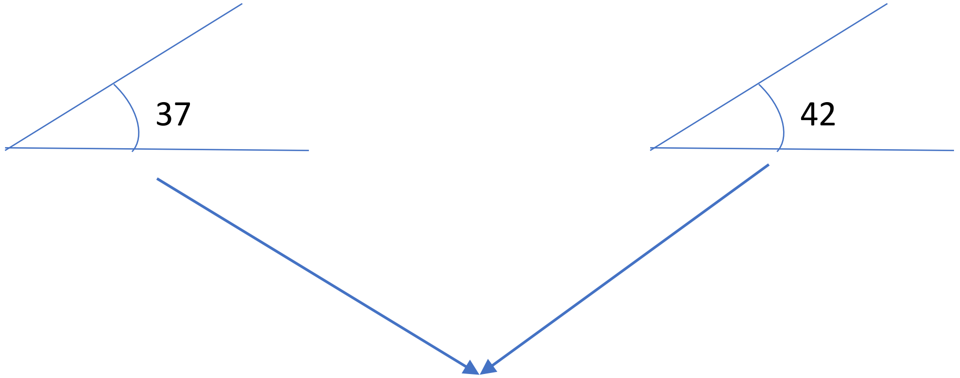
Figure 1. Multi-scale Oriented Patches (MOPS) extracted at five pyramid levels from one of the Matier images. The boxes show the feature orientation and the region from which the descriptor vector is sampled.

Towards a better feature descriptor

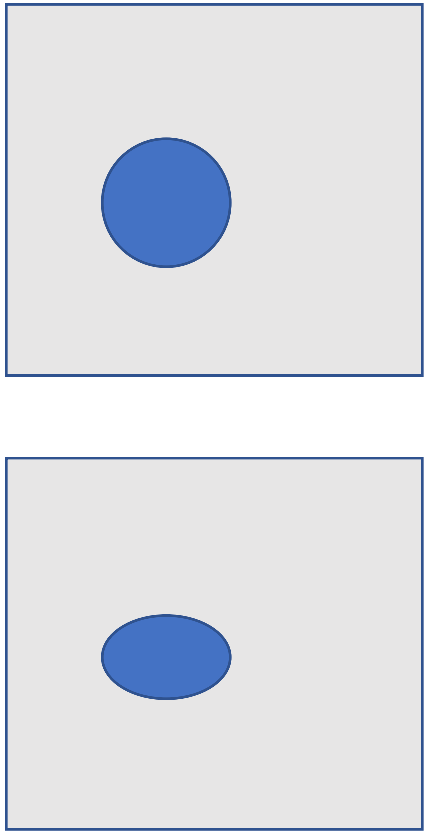
- Match *pattern of edges*
  - Edge orientation – clue to shape
  - Invariant to almost all photometric transformations
- Be resilient to *small deformations*
  - Deformations might move pixels around, but slightly
  - Deformations might change edge orientations, but slightly

Invariance to deformation

- Precise edge orientations are not resilient to out-of-plane rotations and deformations
- But we can *quantize* edge orientation: only record rough orientation



Between 30 and 45

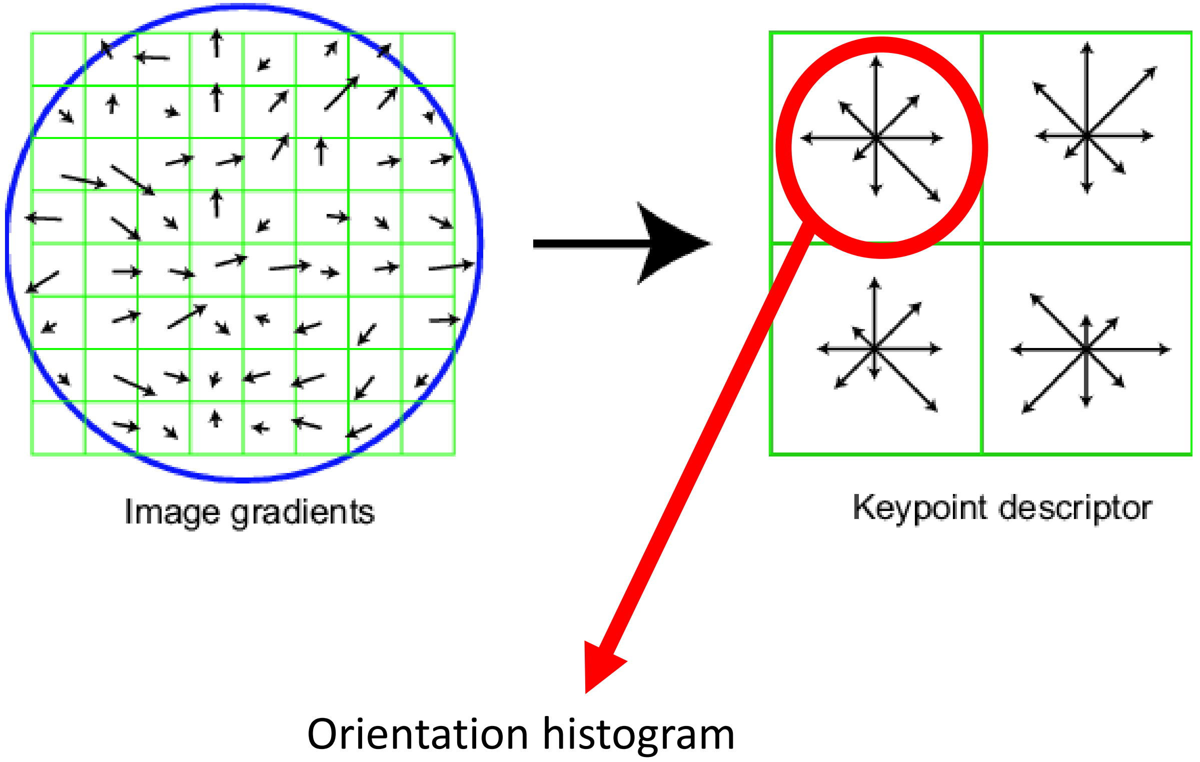


Invariance to deformation

$$g(\theta) = \begin{cases} 0 & \text{if } 0 < \theta < 2\pi/N \\ 1 & \text{if } 2\pi/N < \theta < 4\pi/N \\ 2 & \text{if } 4\pi/N < \theta < 6\pi/N \\ & \dots \\ N-1 & \text{if } 2(N-1)\pi/N \end{cases}$$

Invariance to deformation

- Deformation can also move pixels around
- Again, instead of precise location of each pixel, only want to record rough location
- Divide patch into a grid of *cells*
- Record *counts* of each orientation in each cell: *orientation histograms*

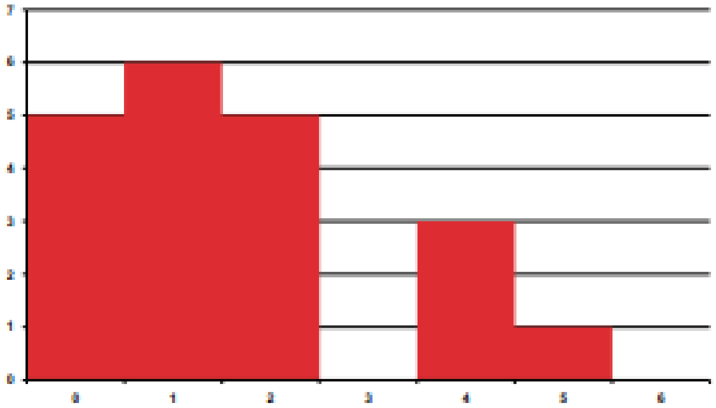


Histogram of Oriented Gradients (HOG)

- Revisiting histogram

0	1	1	2	4
2	1	0	0	2
5	2	0	0	4
1	1	2	4	1

image



histogram

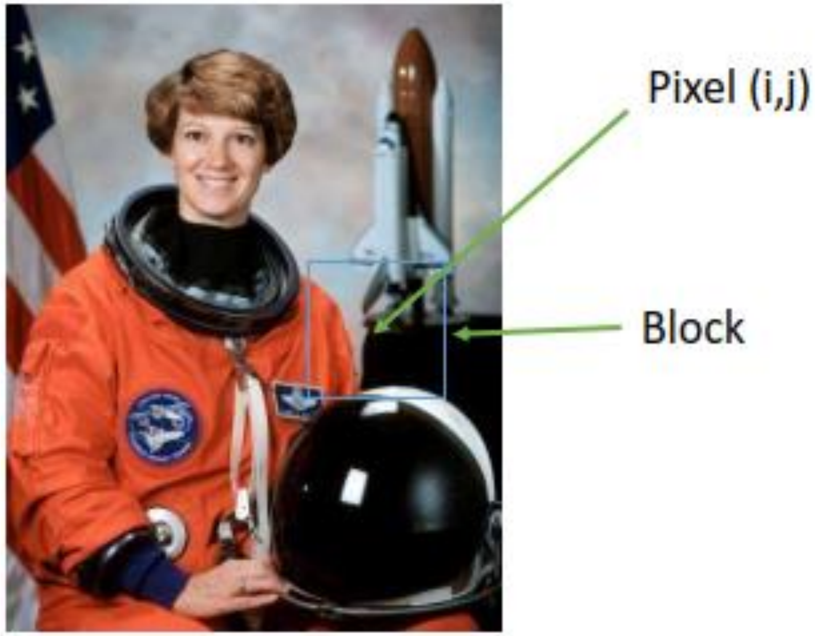
Histogram of Oriented Gradients (HOG)

- Given an image I, and a pixel location (i,j).
- We want to compute the HOG feature for that pixel.
- The main operations can be described as a sequence of five steps.

Pixel (i,j)

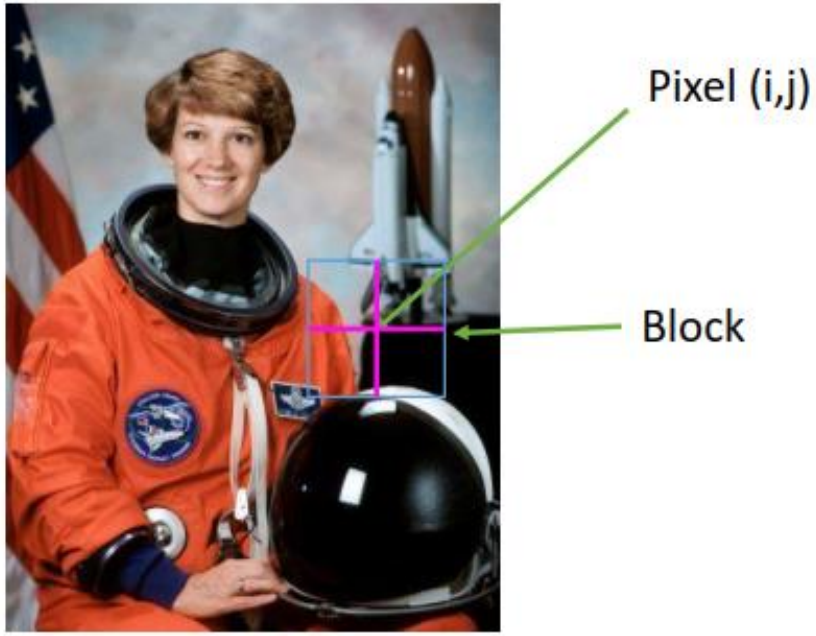
Histogram of Oriented Gradients (HOG)

- Step 1: Extract a square window (called “block”) of some size.



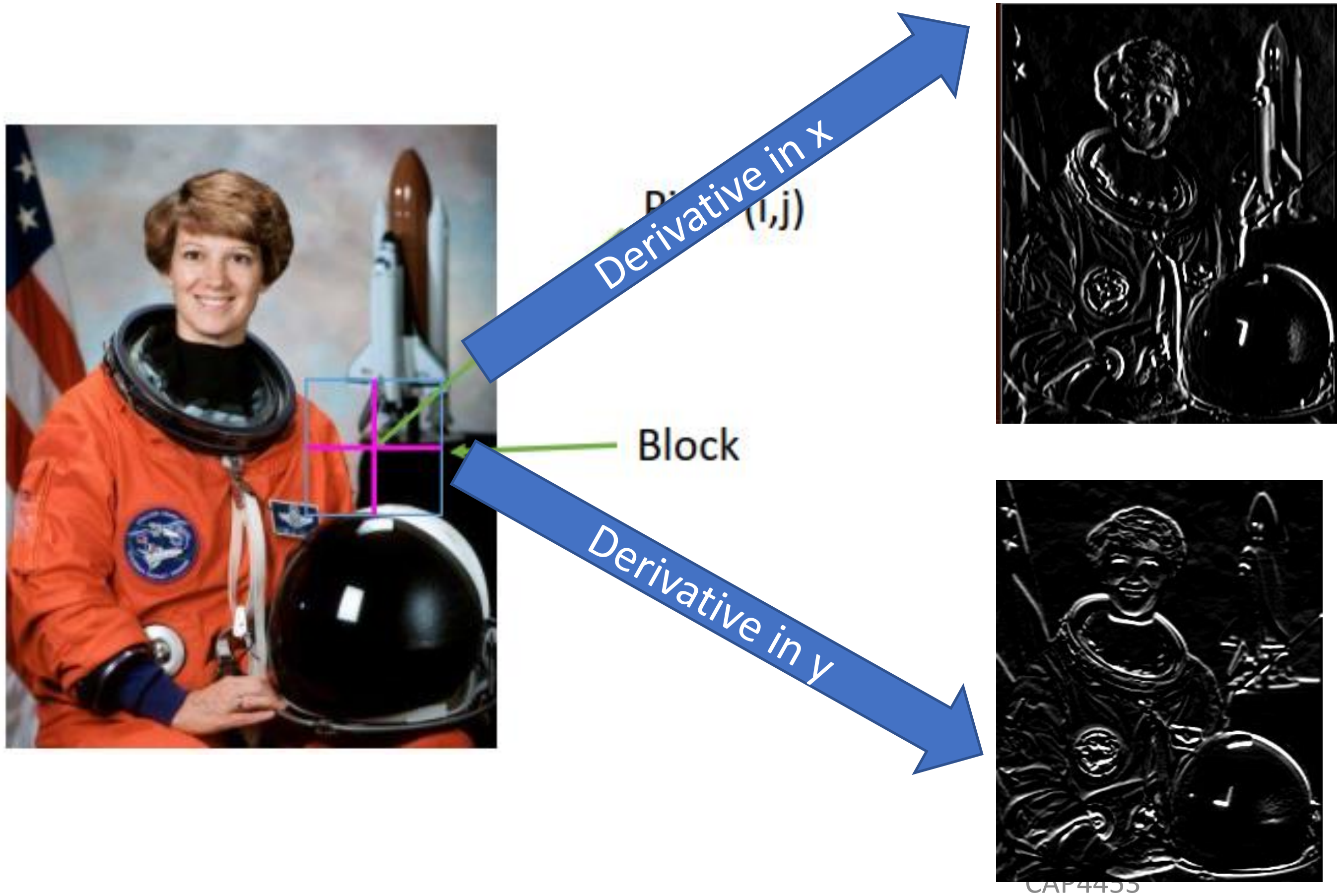
Histogram of Oriented Gradients (HOG)

- Step 2: Divide block into a square grid of sub-blocks (called “cells”) (2x2 grid in our example, resulting in four cells).



Histogram of Oriented Gradients (HOG)

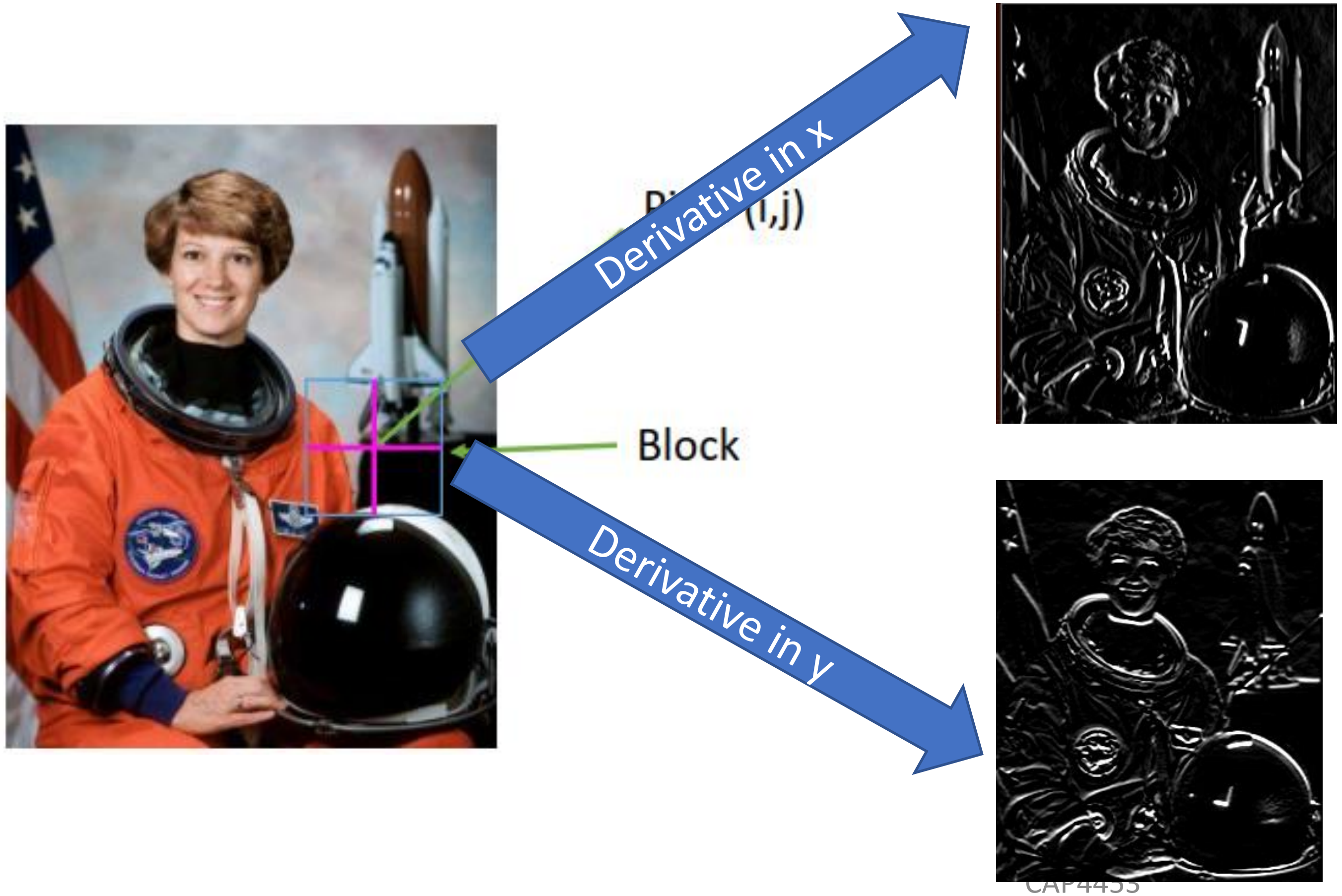
- Step 3: Compute orientation histogram of each cell.



Gradient direction $\theta = \tan^{-1}\dfrac{f_x}{f_y}$

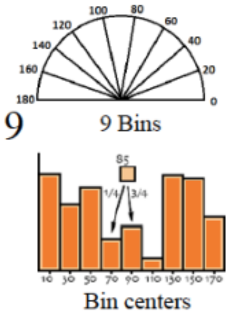
Histogram of Oriented Gradients (HOG)

- Step 3: Compute orientation histogram of each cell.



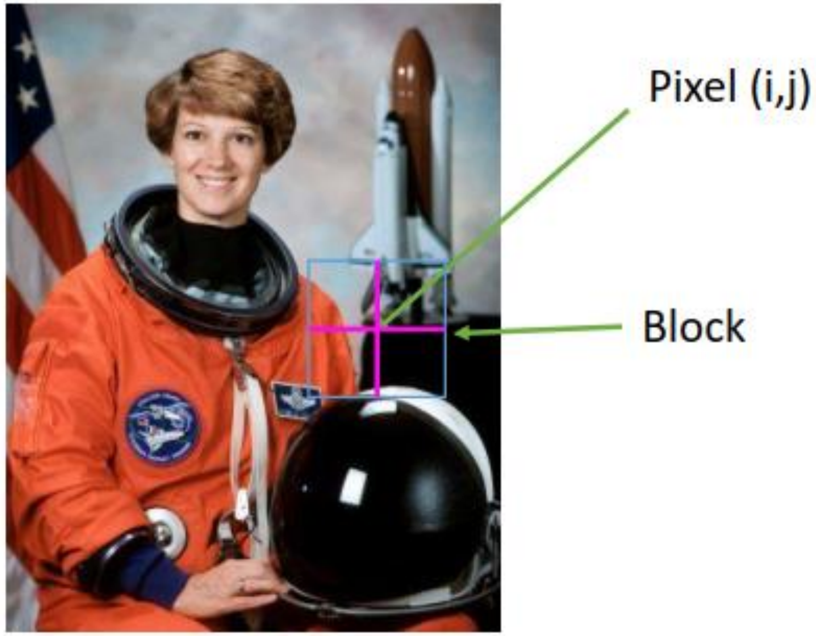
Gradient direction $$\theta = \tan^{-1}\frac{f_x}{f_y}$$

- Cell size is 8x8
- Quantize the gradient orientation into 9 bins (0-180)
  - The vote is the gradient magnitude



Histogram of Oriented Gradients (HOG)

- Step 4: Concatenate the four histograms of each block.



Histogram of Oriented Gradients (HOG)

Let vector v be concatenation of the four histograms from step 4.

- Step 5: Normalize v.

Here we have three options for how to do it:

- Option 1: Divide v by its Euclidean norm.
- Option 2: Divide v by its L1 norm (the L1 norm is the sum of all absolute values of v).
- Option 3:
  - Divide v by its Euclidean norm.
  - In the resulting vector, clip any value over 0.2
  - Then, renormalize the resulting vector by dividing again by its Euclidean norm.

Summary of HOG computation

- Step 1: Extract a square window (called “block”) of some size around the pixel location of interest.
- Step 2: Divide block into a square grid of sub-blocks (called “cells”) (2x2 grid in our example, resulting in four cells).
- Step 3: Compute orientation histogram of each cell.
- Step 4: Concatenate the four histograms.
- Step 5: normalize v using one of the three options described previously.

Histogram of Oriented Gradients (HOG)

- Parameters and design options:
- Angles range from 0 to 180 or from 0 to 360 degrees?
  - In the Dalal & Triggs paper, a range of 0 to 180 degrees is used, and
  - HOGs are used for detection of pedestrians.
- Number of orientation bins.
  - Usually 9 bins, each bin covering 20 degrees.
- Cell size.
  - Cells of size 8x8 pixels are often used.
- Block size.
  - Blocks of size 2x2 cells (16x16 pixels) are often used.
- Usually a HOG feature has 36 dimensions.
  - 4 cells * 9 orientation bins.

Histogram of Oriented Gradients (HOG)

Input image

Histogram of Oriented Gradients

Scale Invariant Feature Transform (SIFT)

A feature detector and a feature descriptor

Scale Invariant Feature Transform (SIFT)

- Lowe., D. 2004, IJCV

cited > 58K

Distinctive Image Features from Scale-Invariant Keypoints

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Received January 10, 2003; Revised January 7, 2004; Accepted January 22, 2004

Abstract. This paper presents a method for extracting distinctive invariant features from images that can be used to perform reliable matching between different views of an object or scene. The features are invariant to image scale and rotation, and are shown to provide robust matching across a substantial range of affine distortion, change in 3D viewpoint, addition of noise, and change in illumination. The features are highly distinctive, in the sense that a single feature can be correctly matched with high probability against a large database of features from many images. This paper also describes an approach to using these features for object recognition. The recognition proceeds by matching individual features to a database of features from known objects using a fast nearest-neighbor algorithm, followed by a Hough transform to identify clusters belonging to a single object, and finally performing verification through least-squares solution for consistent pose parameters. This approach to recognition can robustly identify objects among clutter and occlusion while achieving near real-time performance.

Keywords: invariant features, object recognition, scale invariance, image matching

1. Introduction

Image matching is a fundamental aspect of many problems in computer vision, including object or scene recognition, solving for 3D structure from multiple images, stereo correspondence, and motion tracking. This paper describes image features that have many properties that make them suitable for matching differing images of an object or scene. The features are invariant to image scaling and rotation, and partially invariant to change in illumination and 3D camera viewpoint. They are well localized in both the spatial and frequency domains, reducing the probability of disruption by occlusion, clutter, or noise. Large numbers of features can be extracted from typical images with efficient algorithms. In addition, the features are highly distinctive, which allows a single feature to be correctly matched with high probability against a large database of features, providing a basis for object and scene recognition.

The cost of extracting these features is minimized by taking a cascade filtering approach, in which the more expensive operations are applied only at locations that pass an initial test. Following are the major stages of computation used to generate the set of image features:

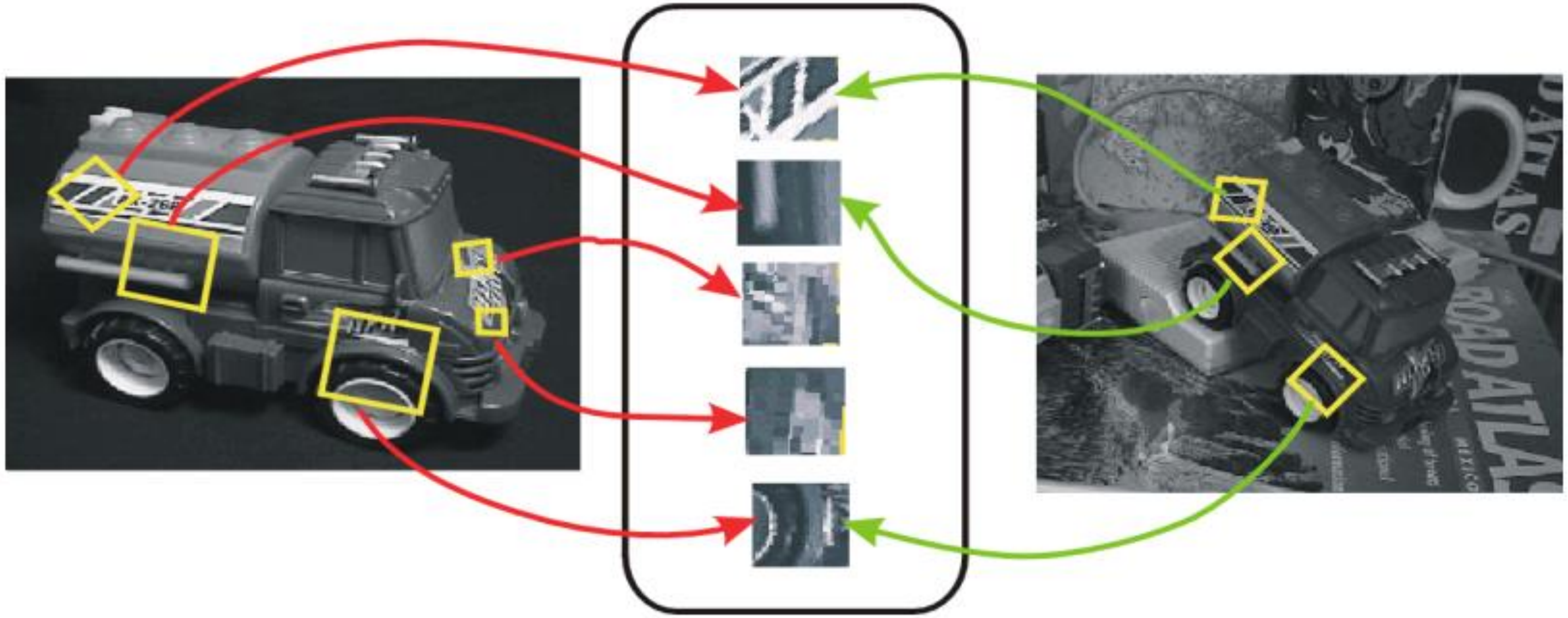
1. *Scale-space extrema detection:* The first stage of computation searches over all scales and image locations. It is implemented efficiently by using a difference-of-Gaussian function to identify potential interest points that are invariant to scale and orientation.
2. *Keypoint localization:* At each candidate location, a detailed model is fit to determine location and scale. Keypoints are selected based on measures of their stability.
3. *Orientation assignment:* One or more orientations are assigned to each keypoint location based on local image gradient directions. All future operations are performed on image data that has been transformed relative to the assigned orientation, scale, and location for each feature, thereby providing invariance to these transformations.

Scale Invariant Feature Transform (SIFT)

- Image content is transformed into local feature coordinates
- Invariant to
  - translation
  - rotation
  - scale, and
  - other imaging parameters

Scale Invariant Feature Transform (SIFT)

- Image content is transformed into local feature coordinates

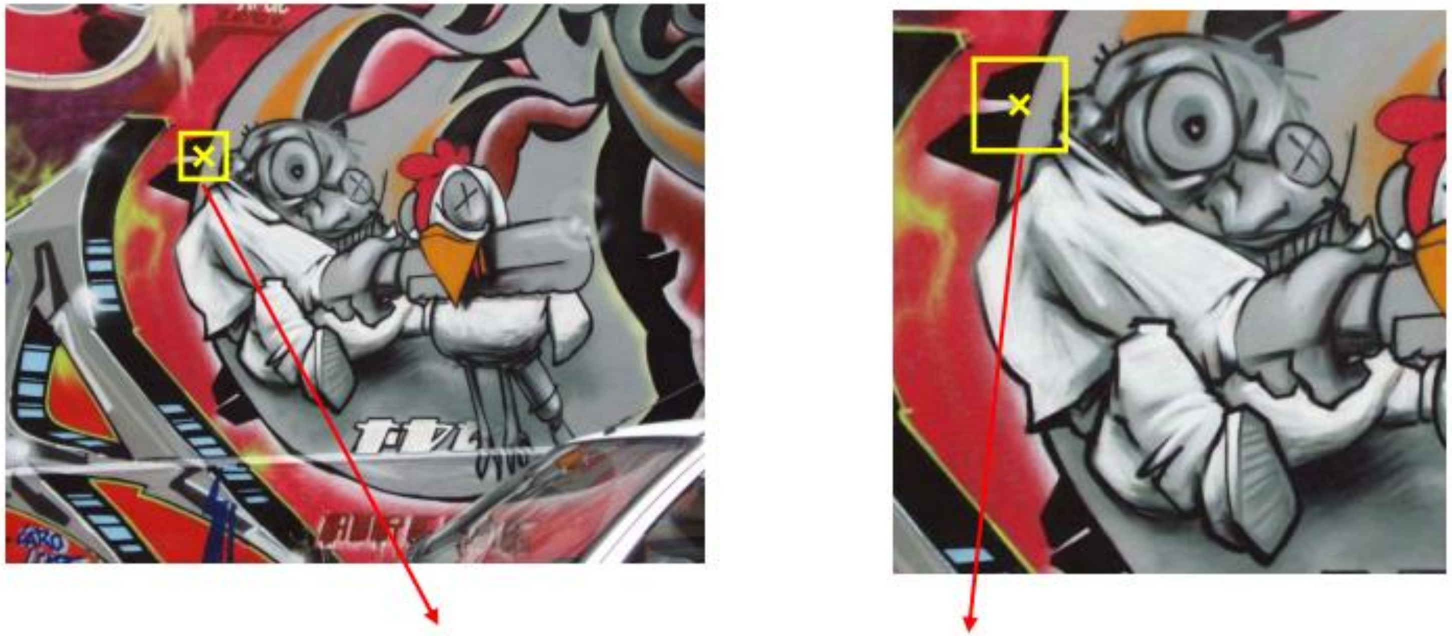


Scale Invariant Feature Transform (SIFT)

- Procedure at High Level

Scale-Space Extrema Detection	Search over multiple scales and image locations
KeyPoint Localization	Fit a model to determine location and scale. Select KeyPoints based on a measure of stability.
Orientation Assignment	Compute best orientation(s) for each keyPoint region.
KeyPoint Description	Use local image gradients at selected scale and rotation to describe each keyPoint region.

SIFT. Automatic scale selection

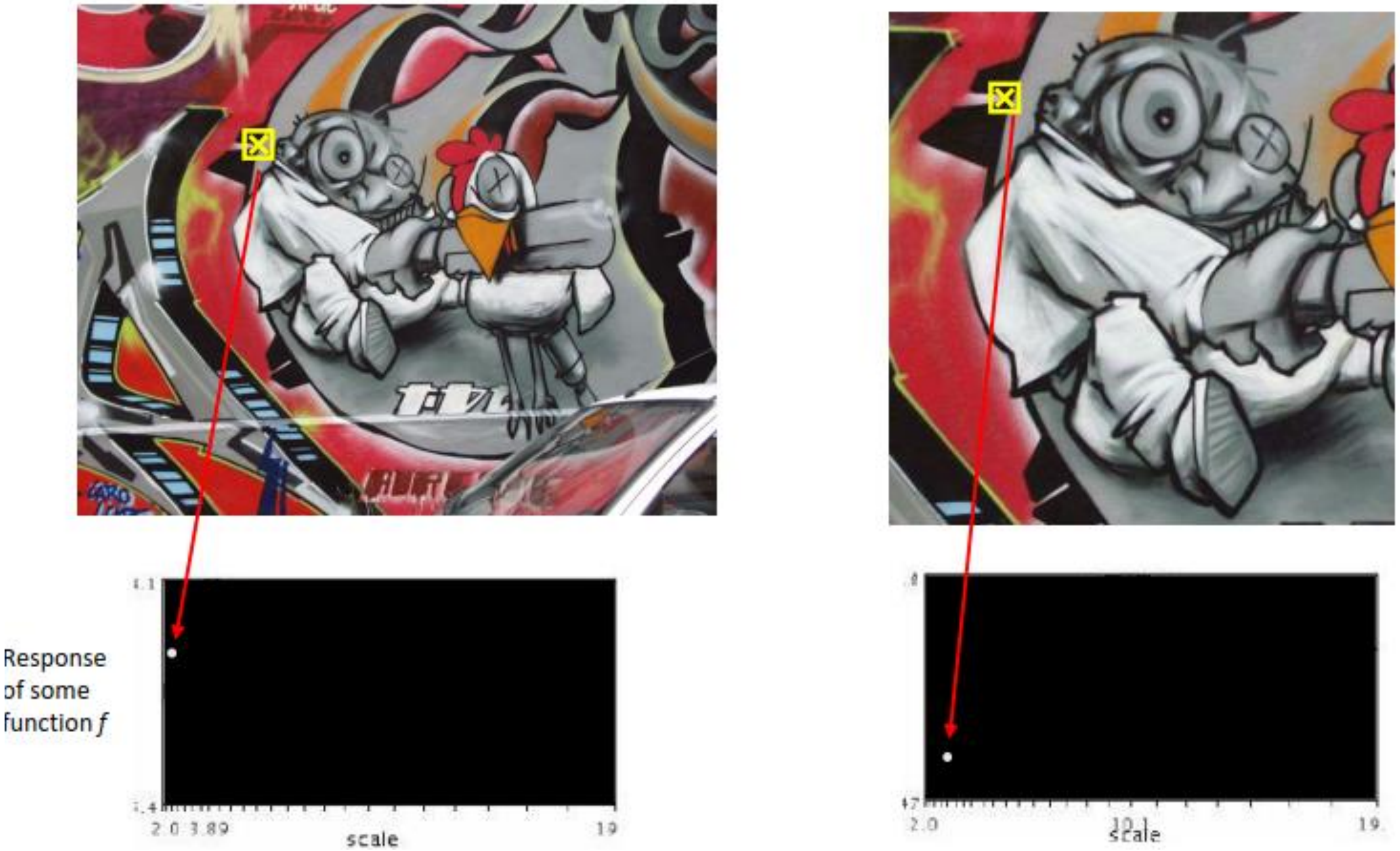


$$f(I_{i_1\ldots i_m}(x,\sigma)) = f(I_{i_1\ldots i_m}(x',\sigma'))$$

How to find patch sizes at which *f* response is equal?

What is a good *f* ?

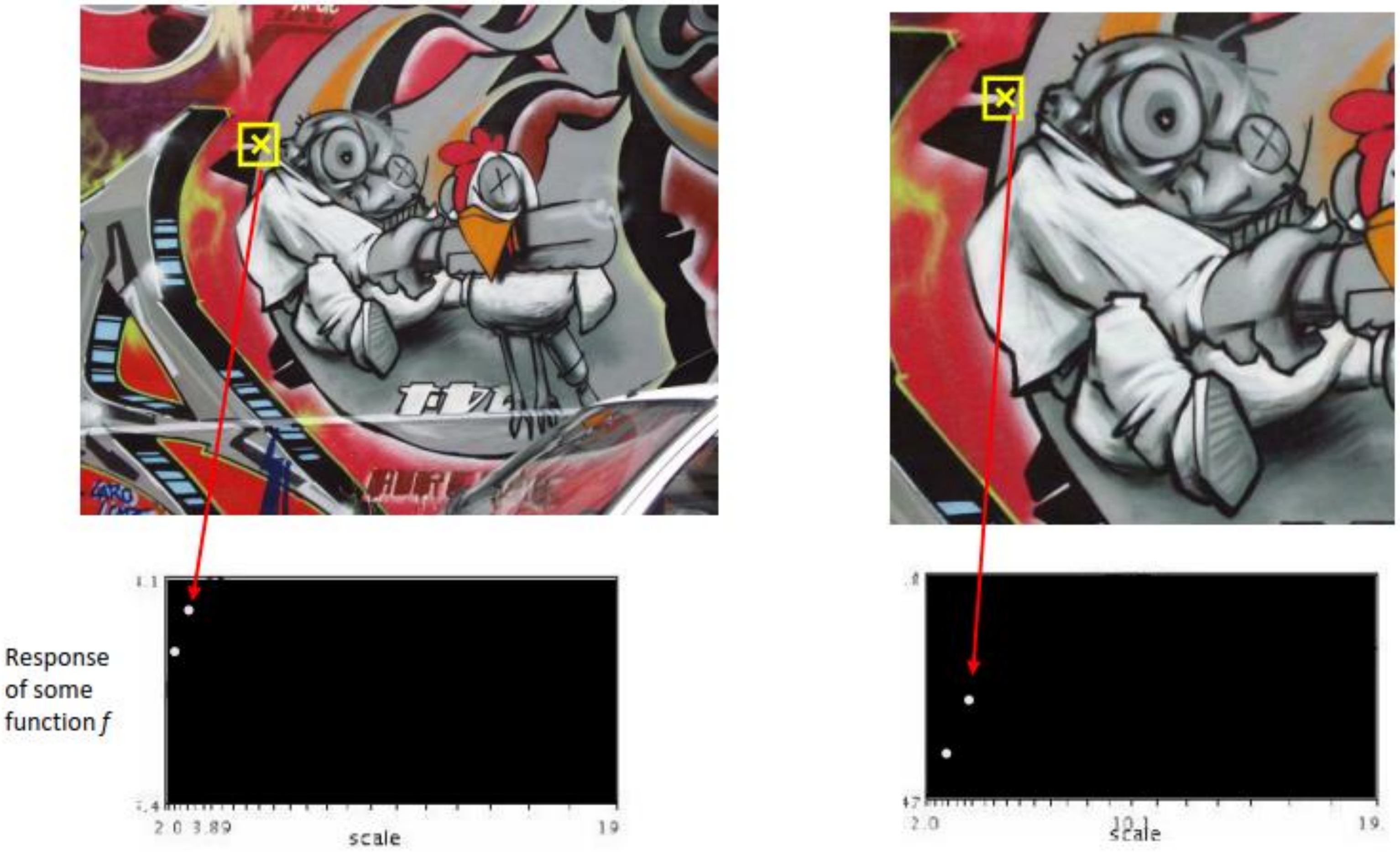
SIFT. Automatic scale selection



$f(I_{i_1 \dots i_m}(x, \sigma))$ $f(I_{i_1 \dots i_m}(x', \sigma'))$

Function responses for increasing scale (scale signature)

SIFT. Automatic scale selection



$f(I_{i_1 \ldots i_m}(x, \sigma))$ $\qquad$ $f(I_{i_1 \ldots i_m}(x', \sigma'))$

Function responses for increasing scale (scale signature)

SIFT. Automatic scale selection

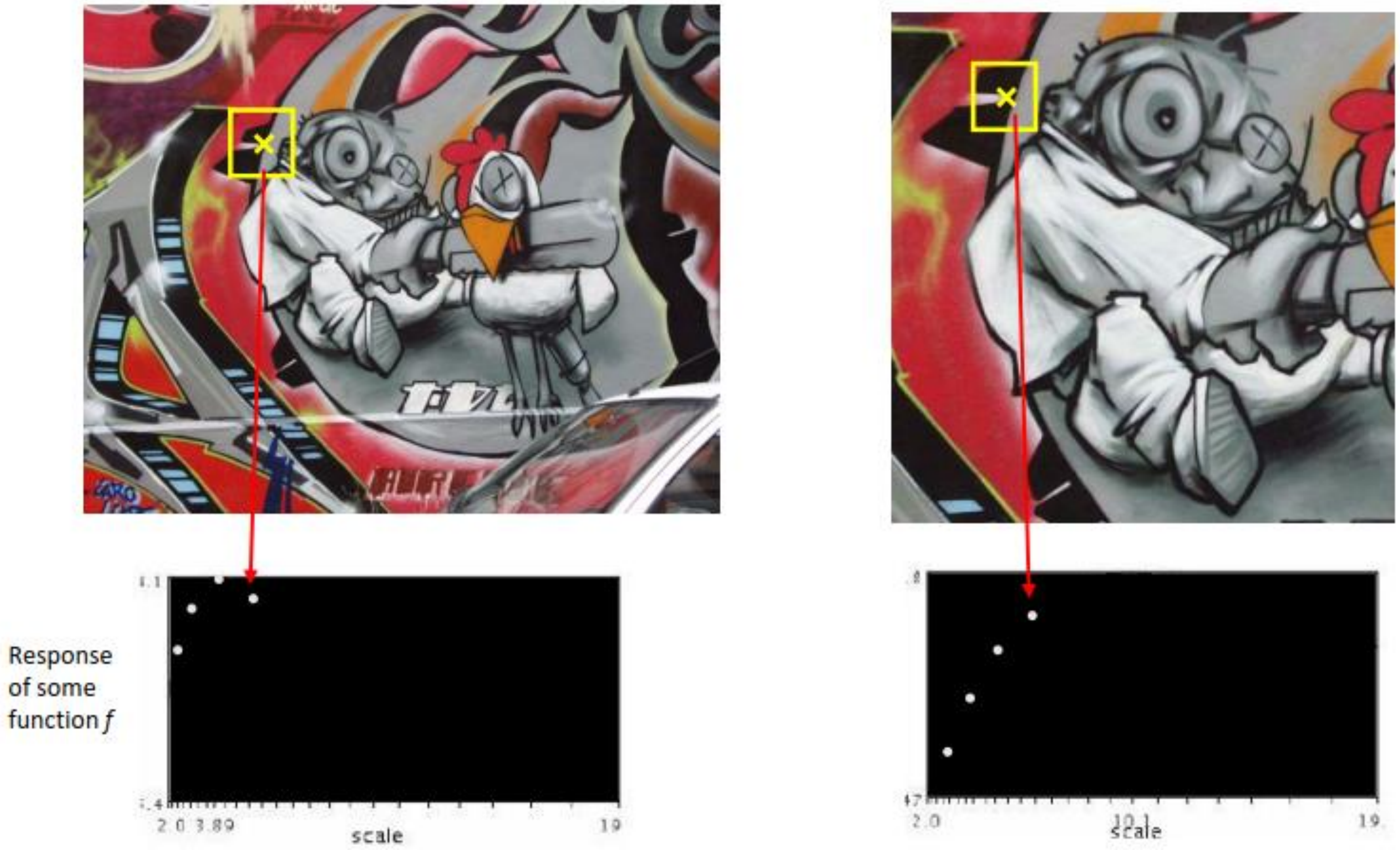
Response of some function f

$f(I_{i_1 \ldots i_m}(x, \sigma))$

$f(I_{i_1 \ldots i_m}(x', \sigma'))$

Function responses for increasing scale (scale signature)

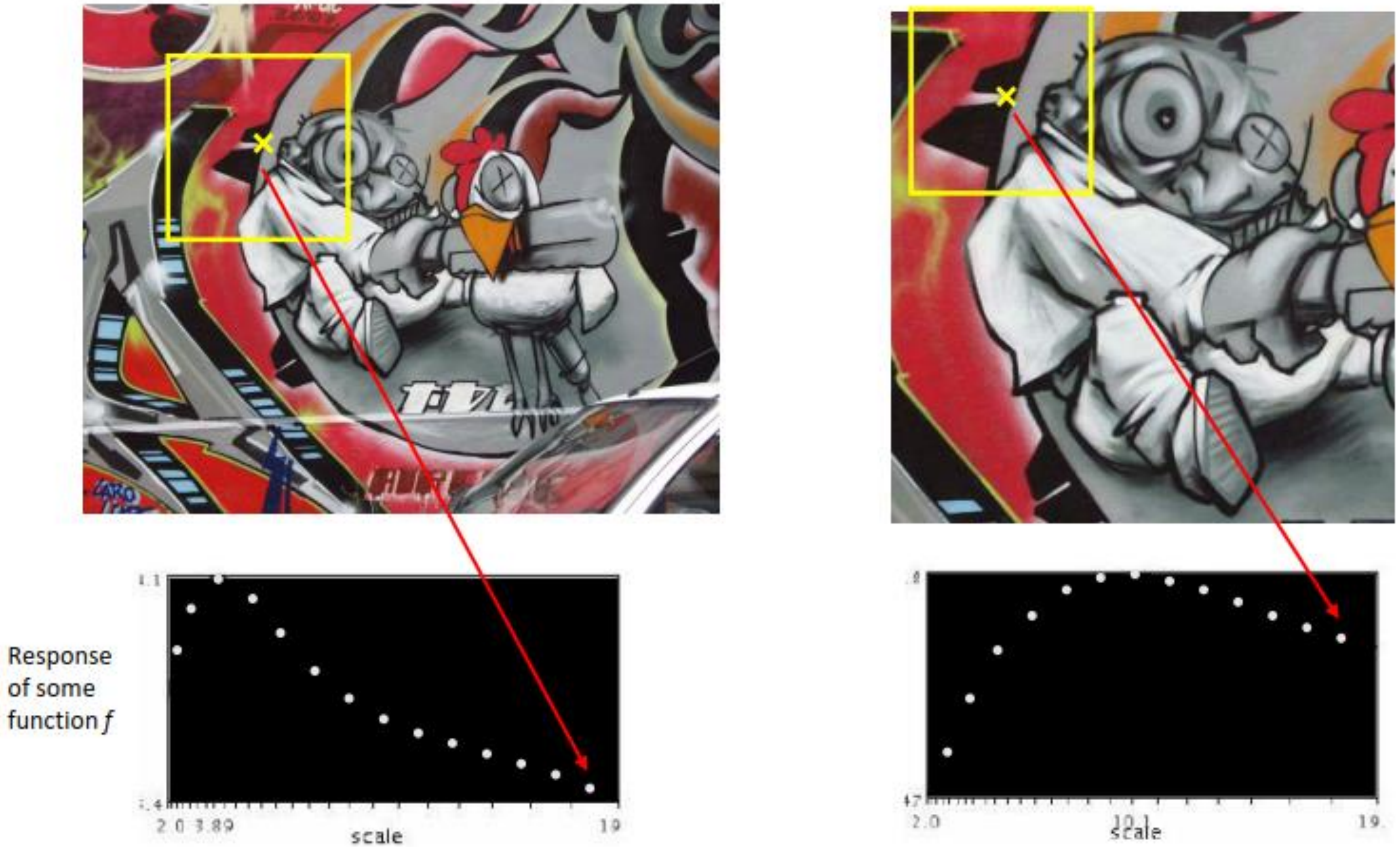
SIFT. Automatic scale selection



$f(I_{i_1 \dots i_m}(x,\sigma))$ $\qquad$ $f(I_{i_1 \dots i_m}(x',\sigma'))$

Function responses for increasing scale (scale signature)

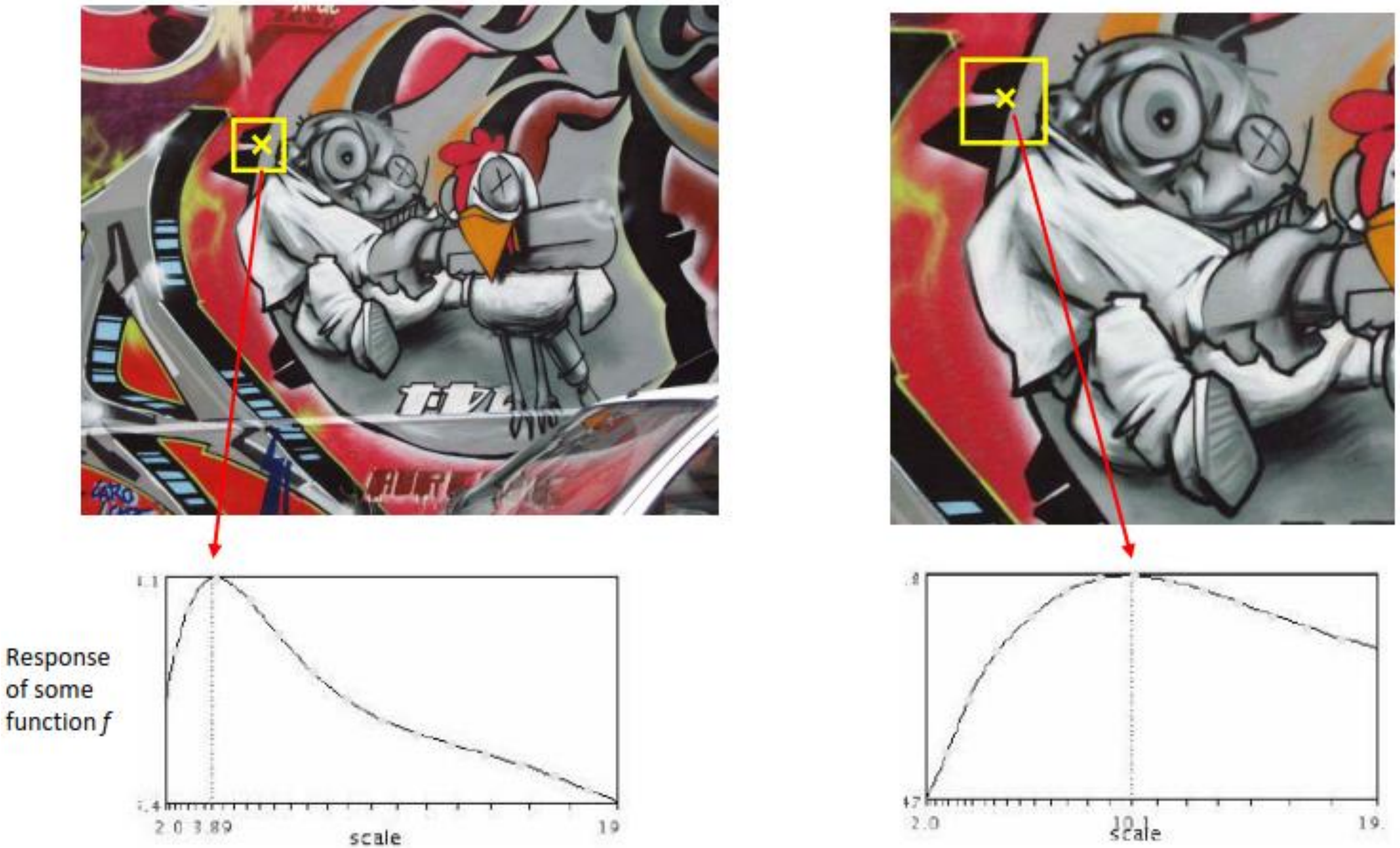
SIFT. Automatic scale selection



$f(I_{i_1 \ldots i_m}(x, \sigma))$ $f(I_{i_1 \ldots i_m}(x', \sigma'))$

Function responses for increasing scale (scale signature)

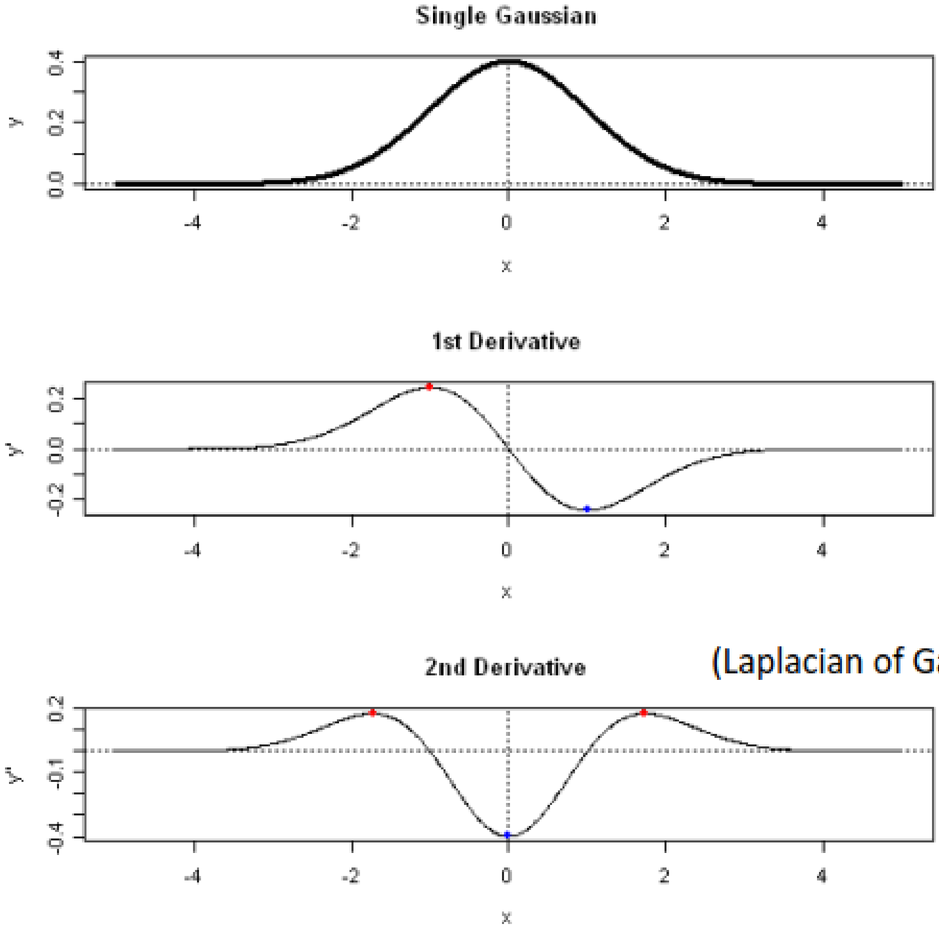
SIFT. Automatic scale selection



$f(I_{i_1 \ldots i_m}(x, \sigma))$ $\qquad$ $f(I_{i_1 \ldots i_m}(x', \sigma'))$

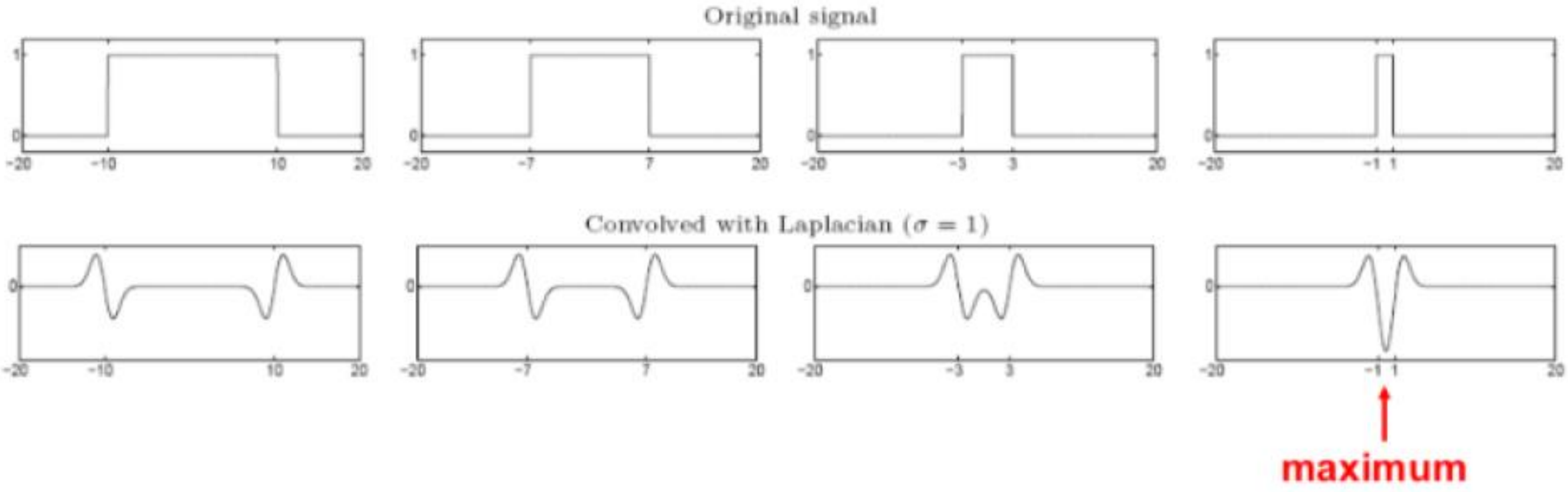
Function responses for increasing scale (scale signature)

What is a useful signature function f?

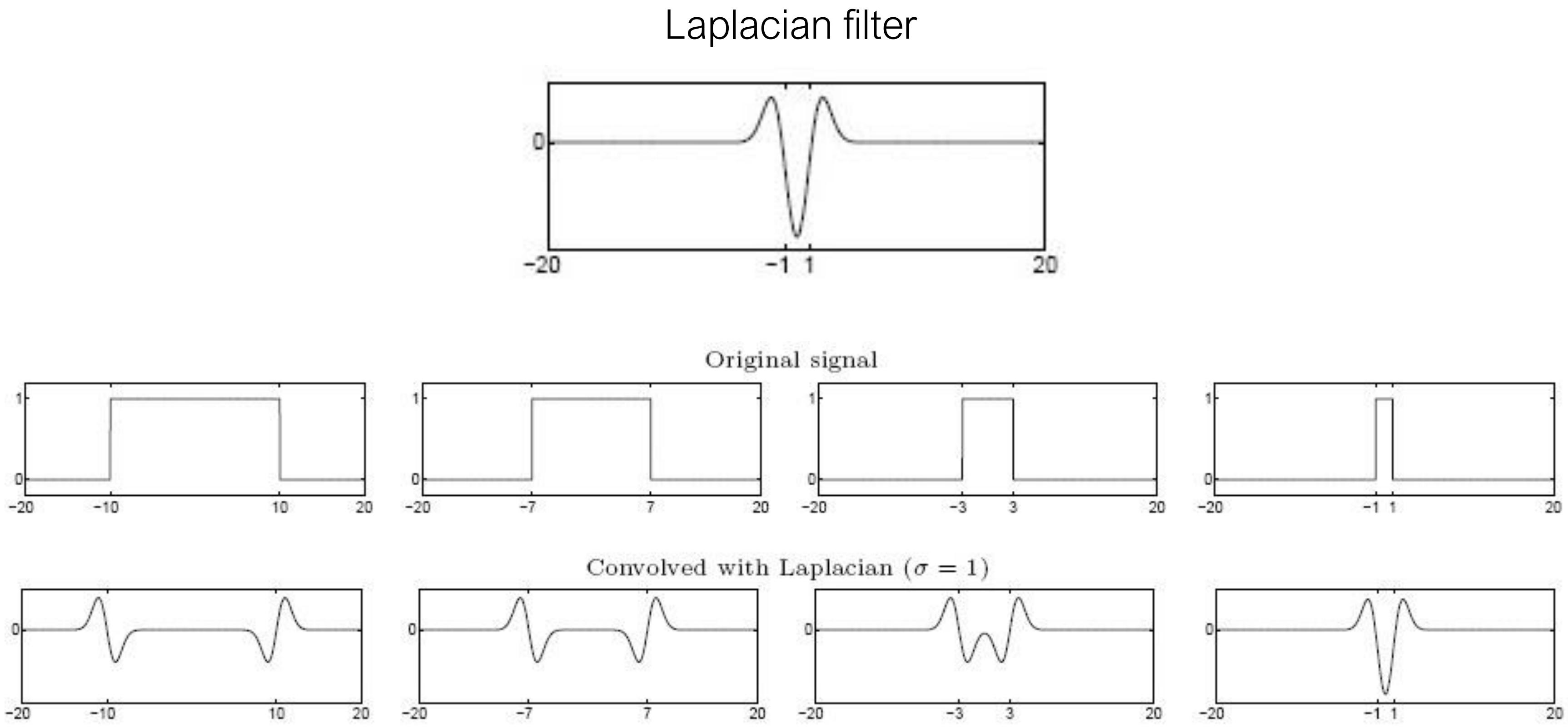


(Laplacian of Gaussian)

Blob detection



Formally…

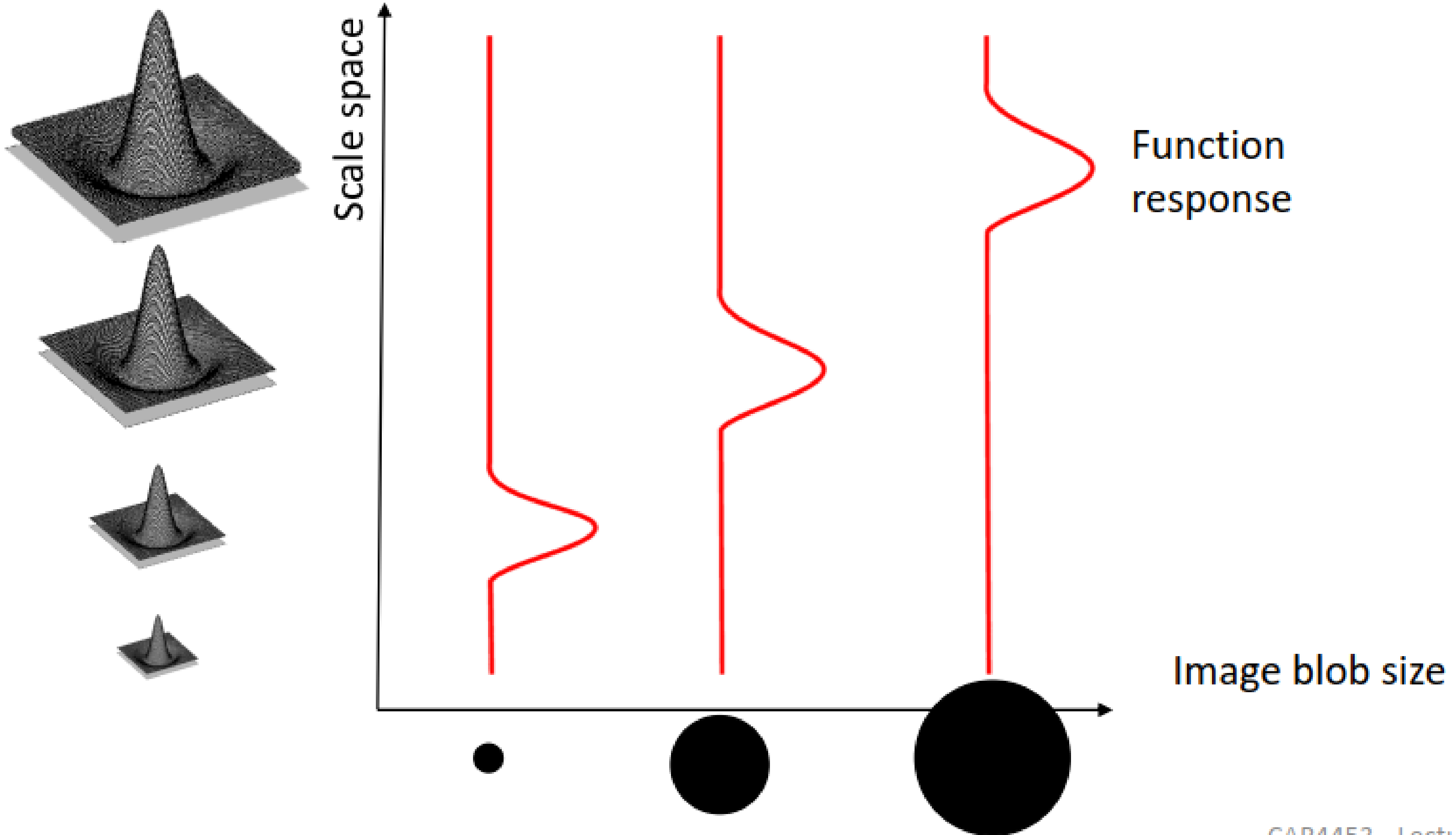


Highest response when the signal has the same **characteristic scale** as the filter

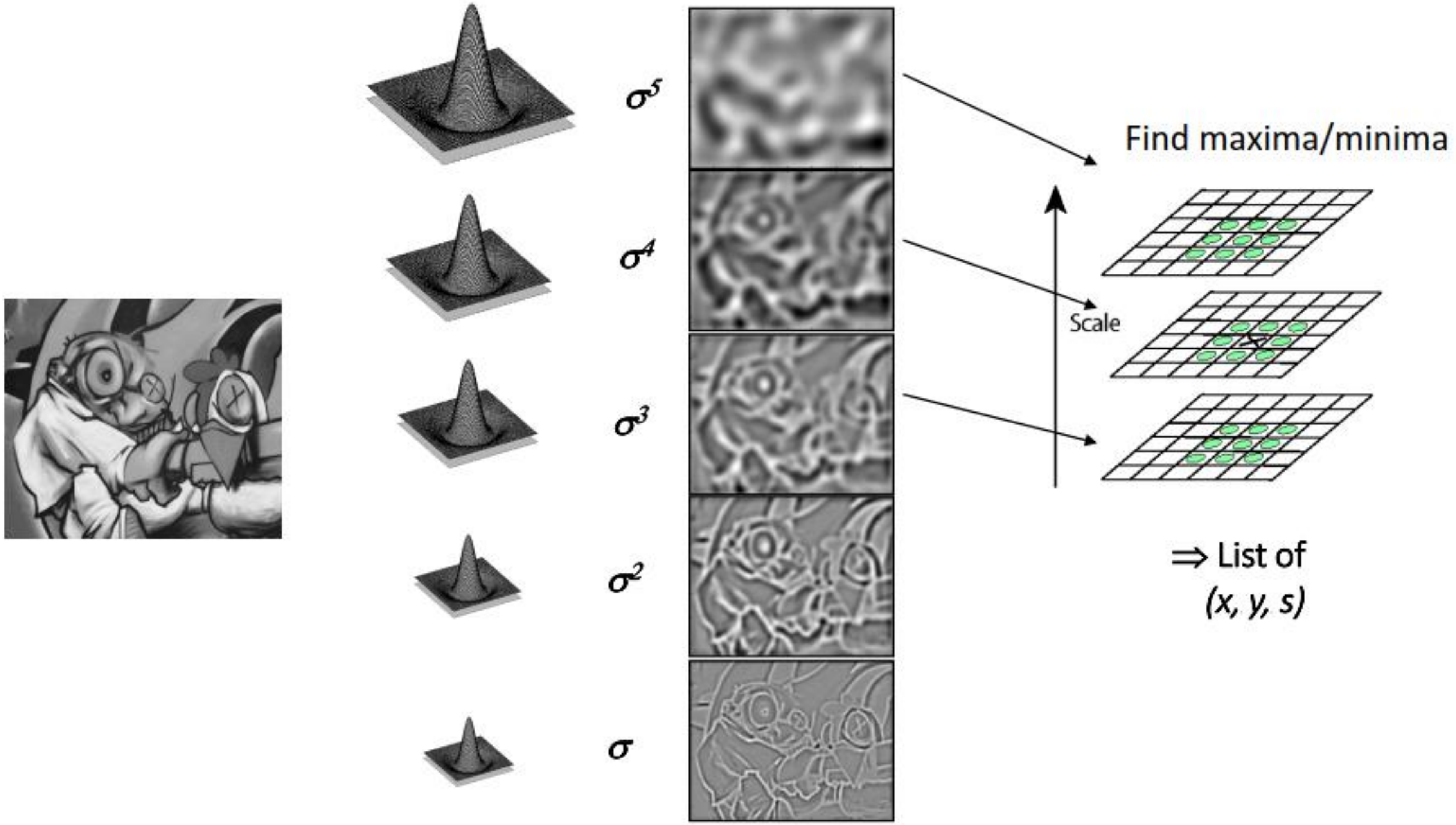
What is a useful signature function f?

"Blob" detector is common for corners

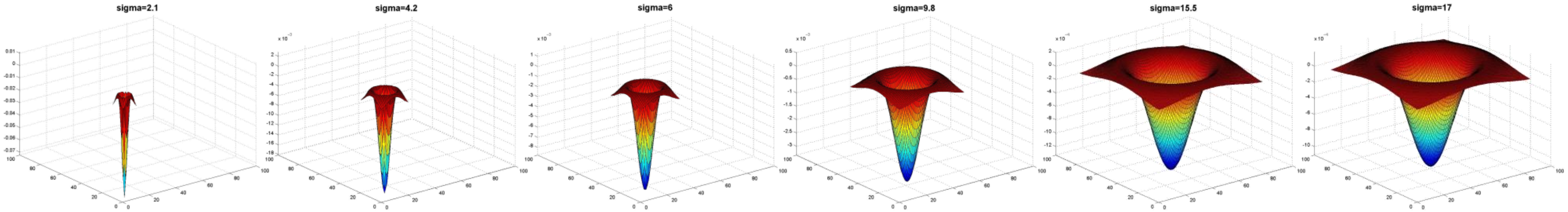
- Laplacian (2nd derivative) of Gaussian (LoG)



Find local maxima in position-scale space



What happens if you apply different Laplacian filters?



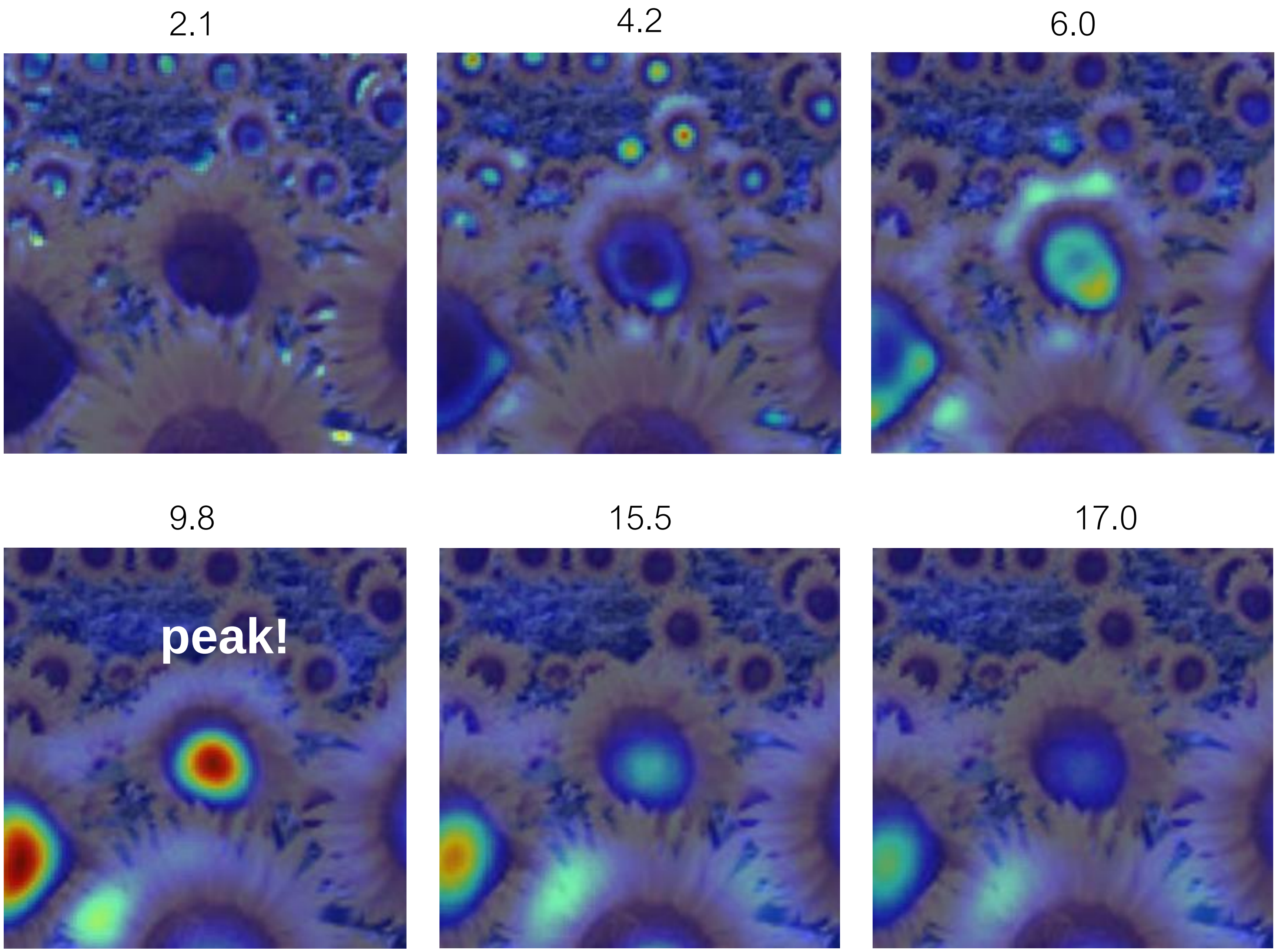
Full size

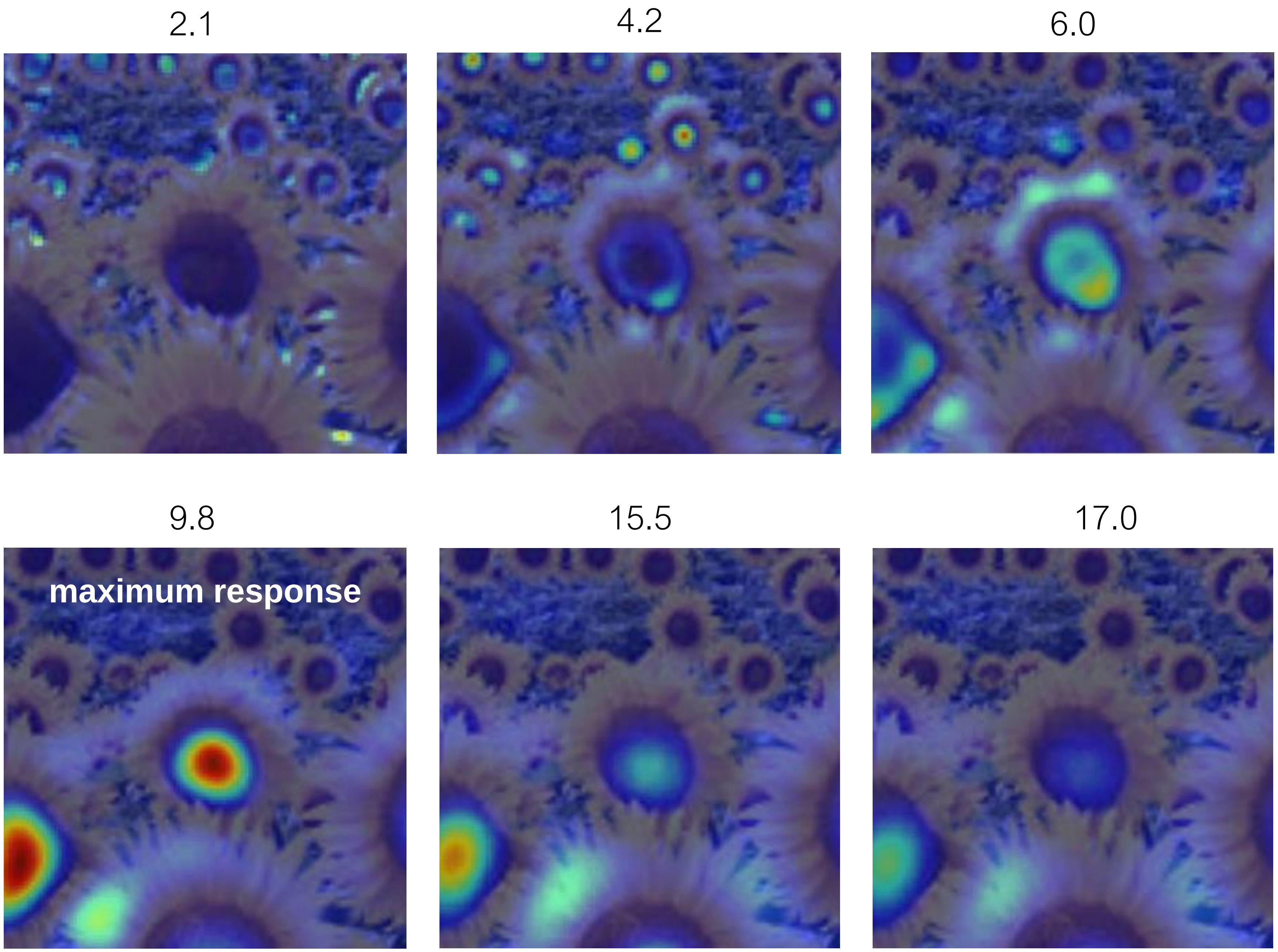
3/4 size

What happened when you applied different Laplacian filters?

Full size

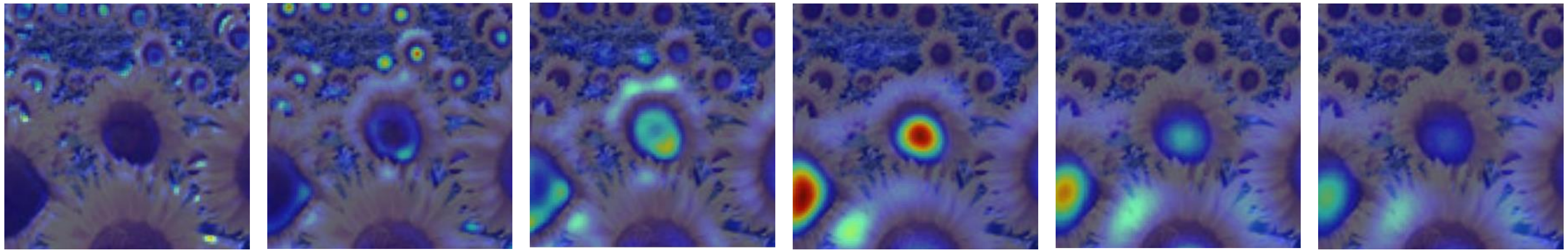
3/4 size


2.1
4.2
6.0
9.8
peak!
15.5
17.0


2.1
4.2
6.0
9.8
maximum response
15.5
17.0

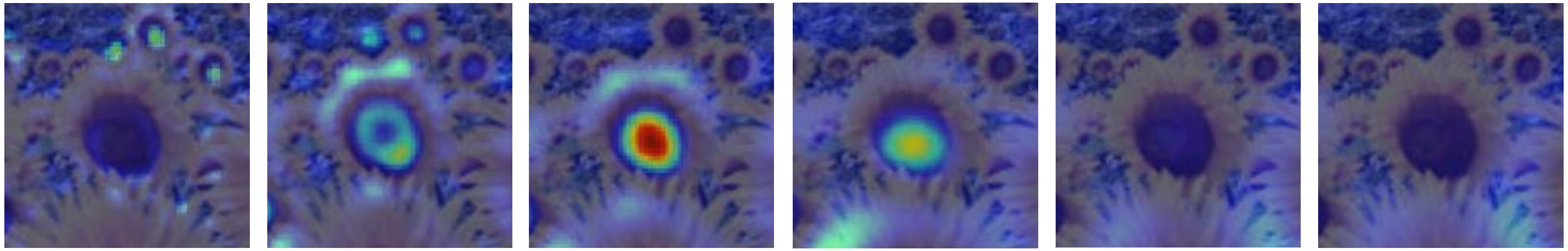
optimal scale

2.1 4.2 6.0 9.8 15.5 17.0



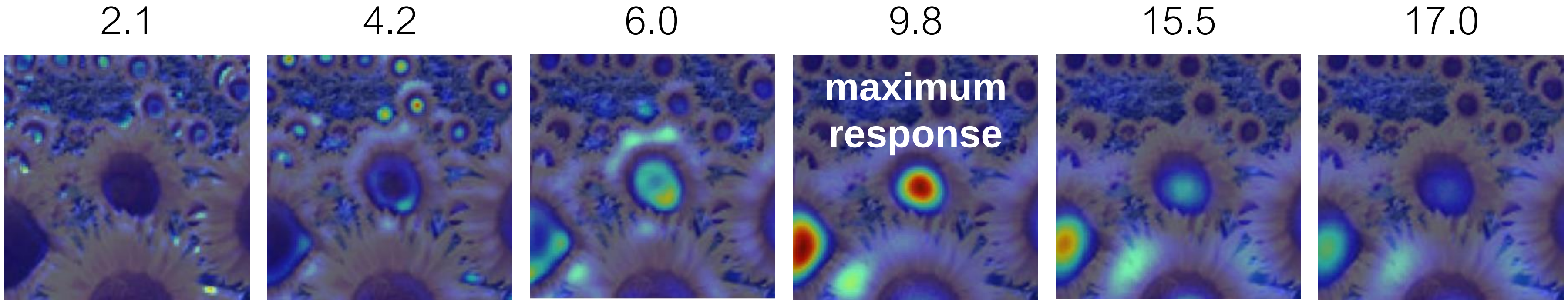
Full size image

2.1 4.2 6.0 9.8 15.5 17.0

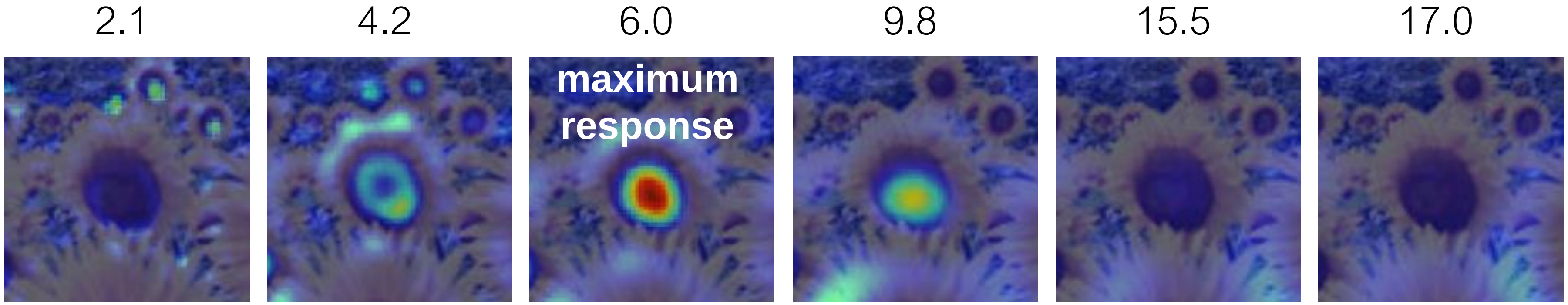


3/4 size image

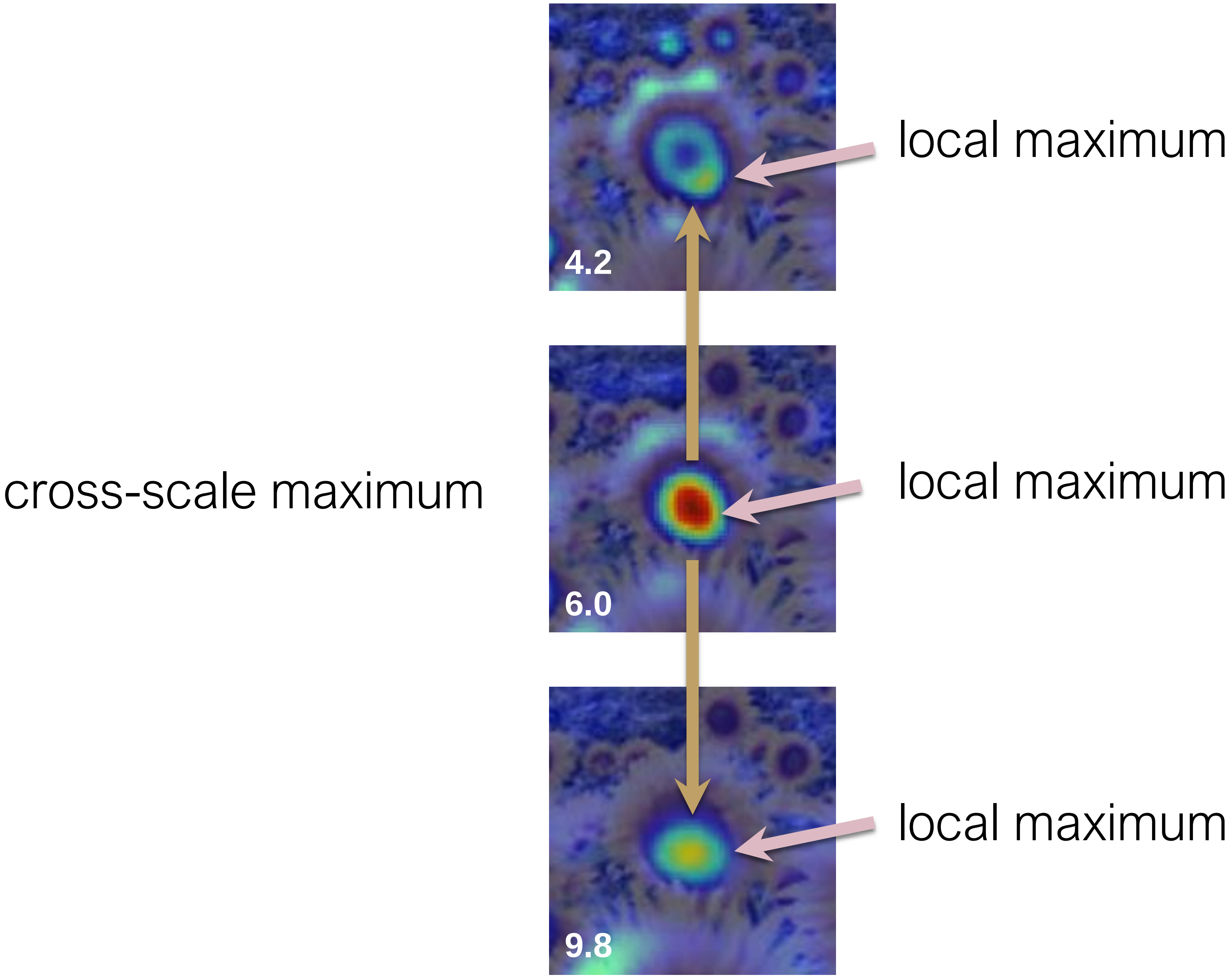
optimal scale



Full size image



3/4 size image


local maximum
4.2
cross-scale maximum
local maximum
6.0
local maximum
9.8

Scale Invariant Detection

- Functions for determining scale $f = \text{Kernel} * \text{Image}$

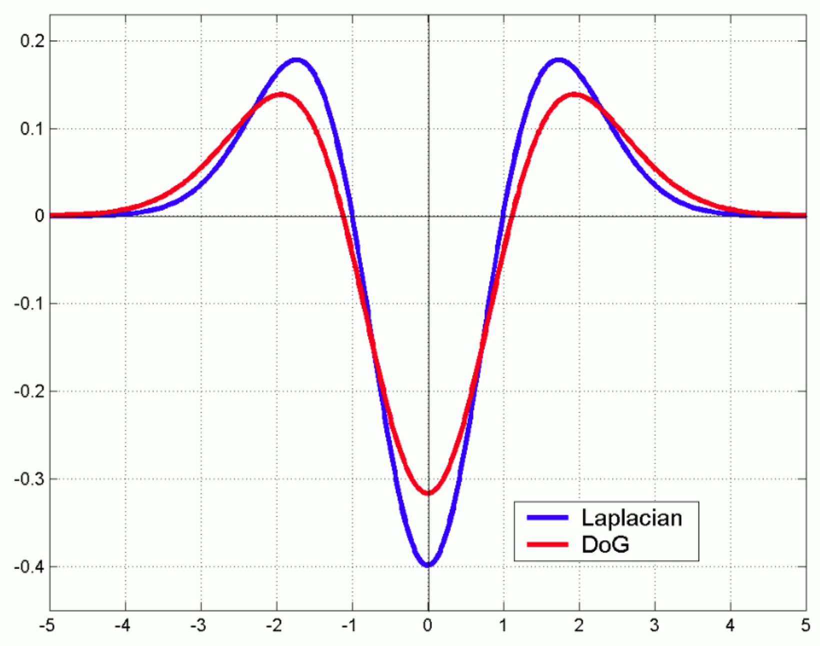
Kernels:

$$\nabla^2 g = \frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2}$$

(Laplacian)

$$DoG = G(x, y, k\sigma) - G(x, y, \sigma)$$

(Difference of Gaussians)



where Gaussian

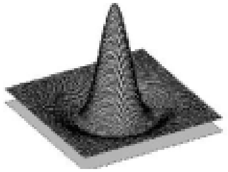
$$G(x, y, \sigma) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

Note: The LoG and DoG operators are both rotation equivariant

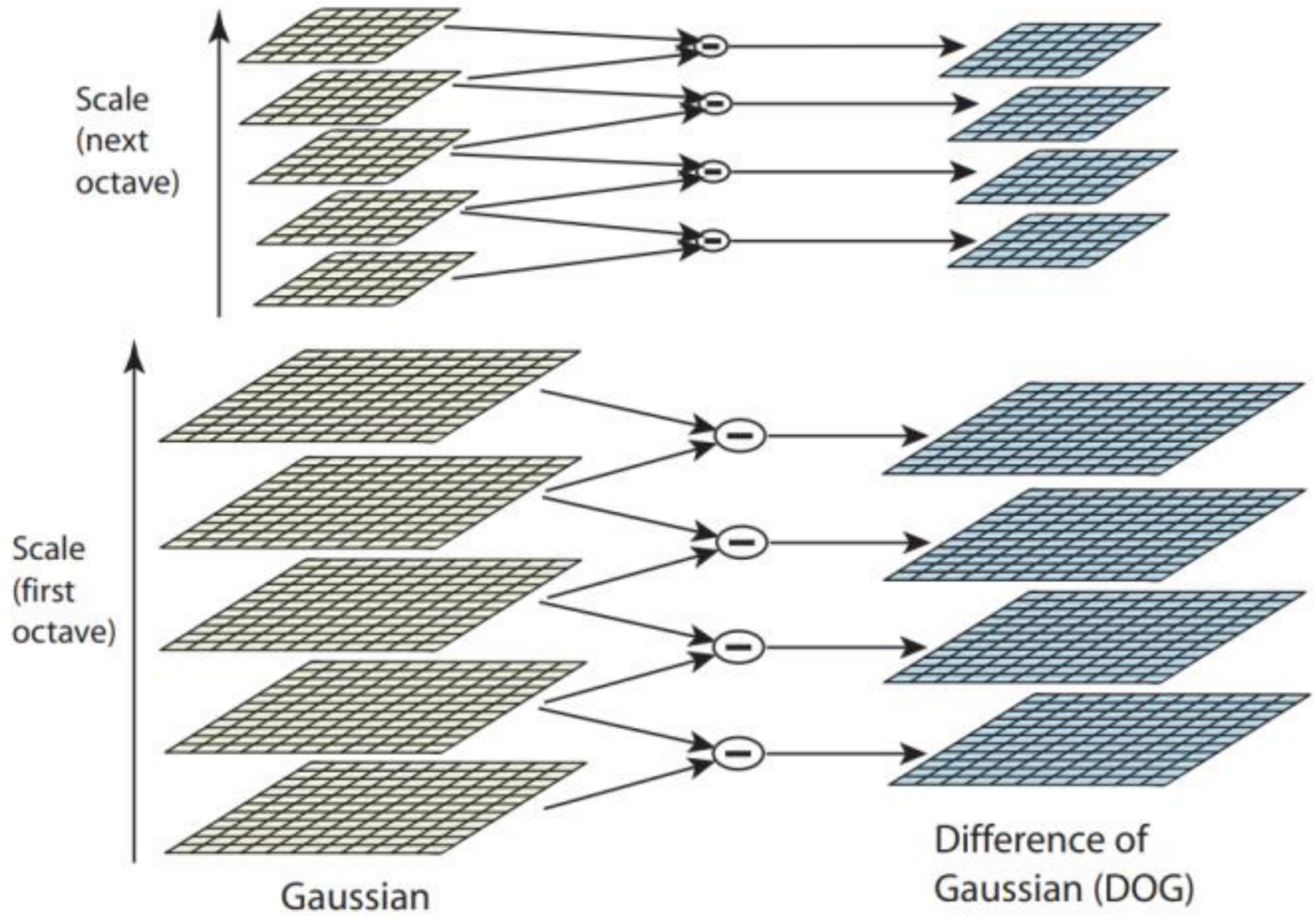
Alternative to compute Laplacian of Gaussian

- Approximate LoG with Difference-of-Gaussian (DoG).

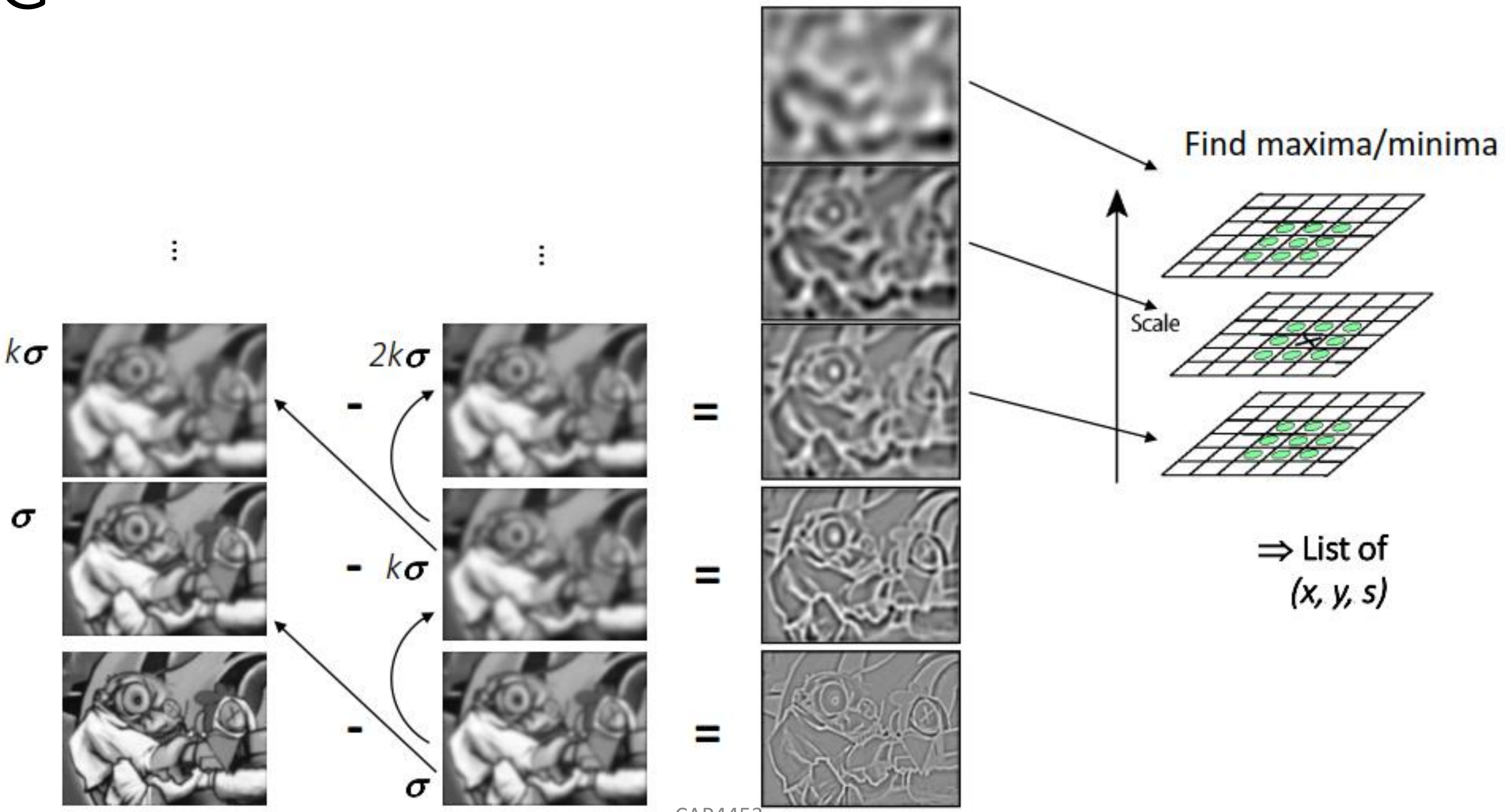
1. Blur image with σ Gaussian kernel
2. Blur image with kσ Gaussian kernel
3. Subtract 2. from 1.



Scale-Space

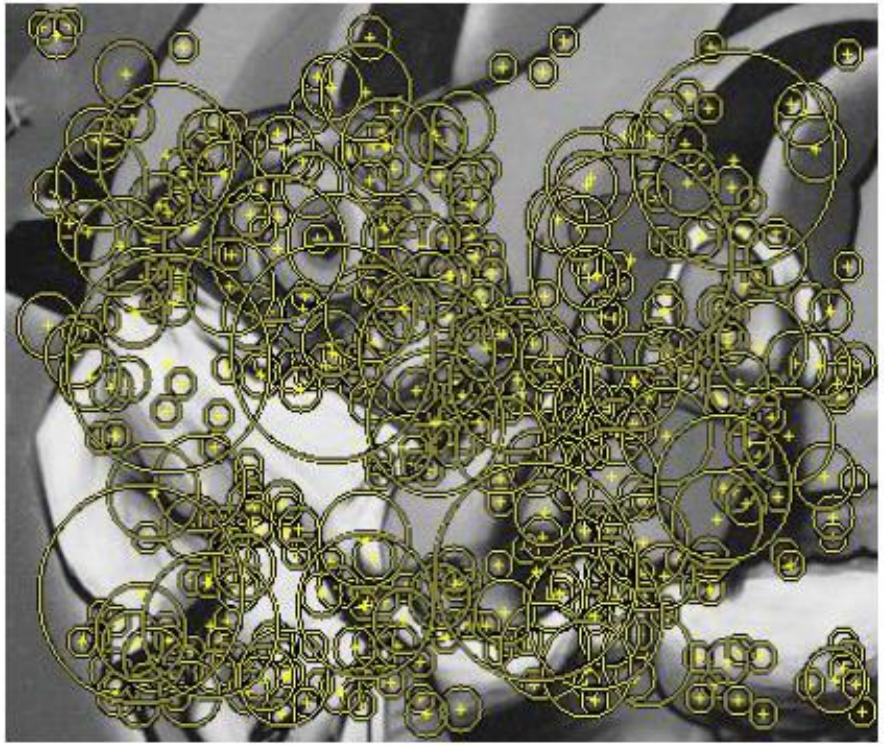

Scale
(next
octave)
Scale
(first
octave)
Gaussian
Difference of
Gaussian (DOG)

Find local maxima in position-scale space of DoG



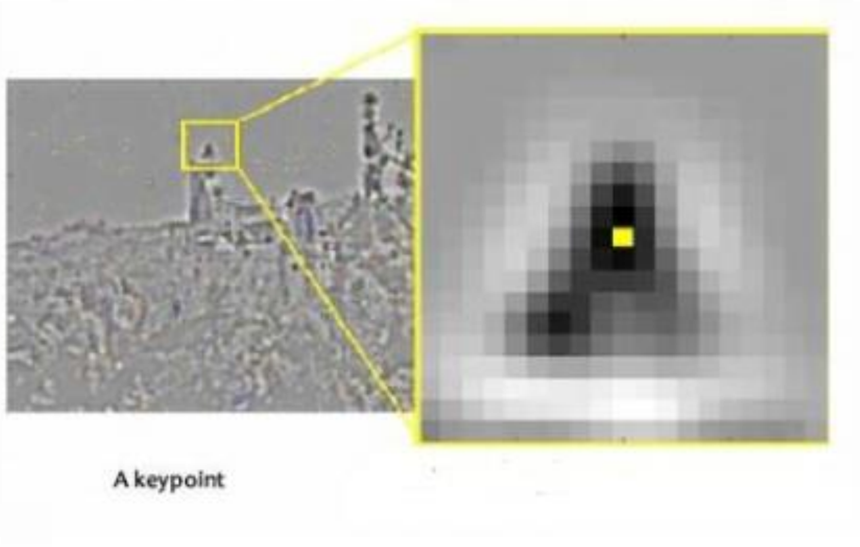
Results: Difference of Gaussians

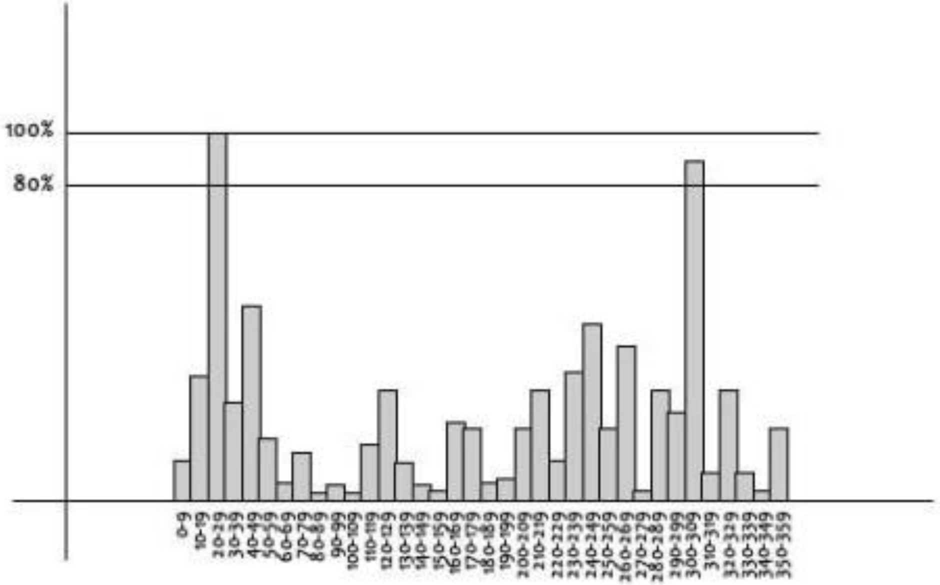
- Larger circles = larger scale
- Descriptors with maximal scale response



SIFT Orientation estimation

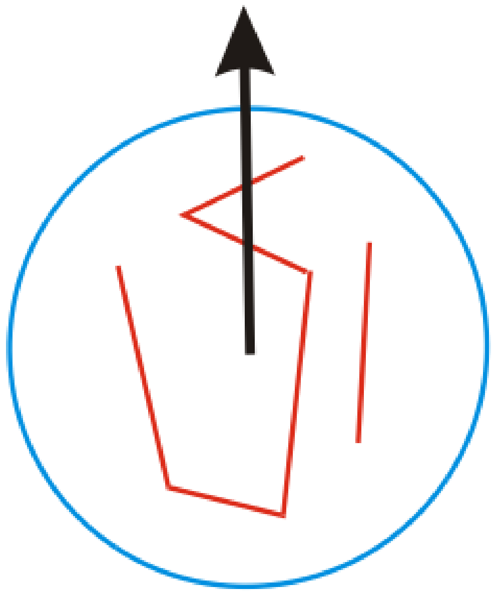
- Compute gradient orientation histogram
- Select dominant orientation ϴ

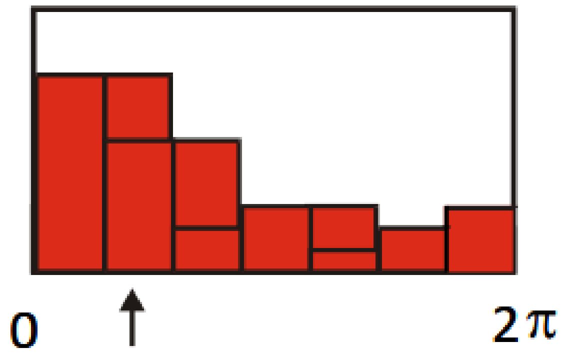




SIFT Orientation Normalization

- Compute gradient orientation histogram
- Select dominant orientation ϴ
- Normalize: rotate to fixed orientation





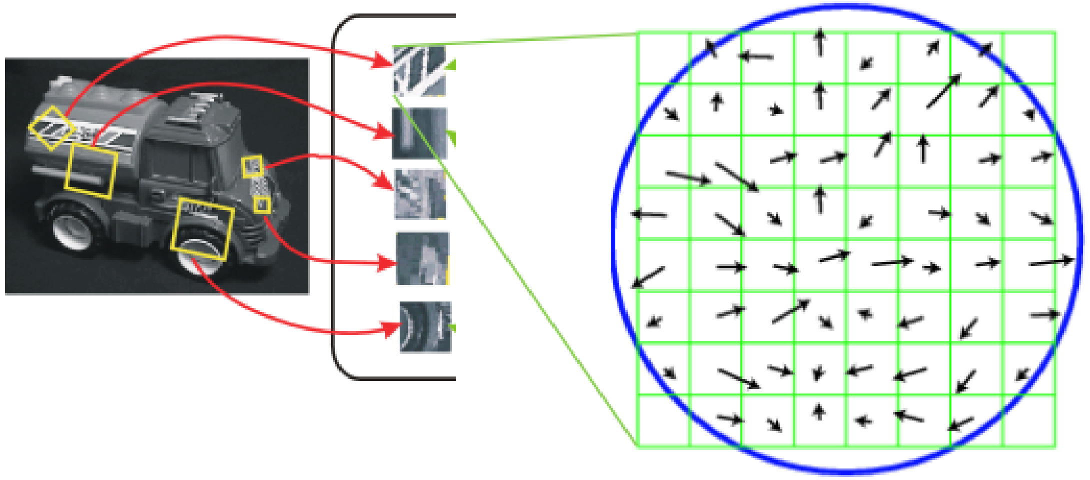
SIFT Detector

- In addition to position x,y of the feature,

  - Scale σ (determined by smoothing value)
  - Orientation of dominant gradient θ

SIFT descriptor

- Compute on local 16 x 16 window around detection.
- Rotate and scale window according to discovered orientation ϴ and scale σ (gain invariance).
- Compute gradients weighted by a Gaussian of variance half the window (for smooth falloff).

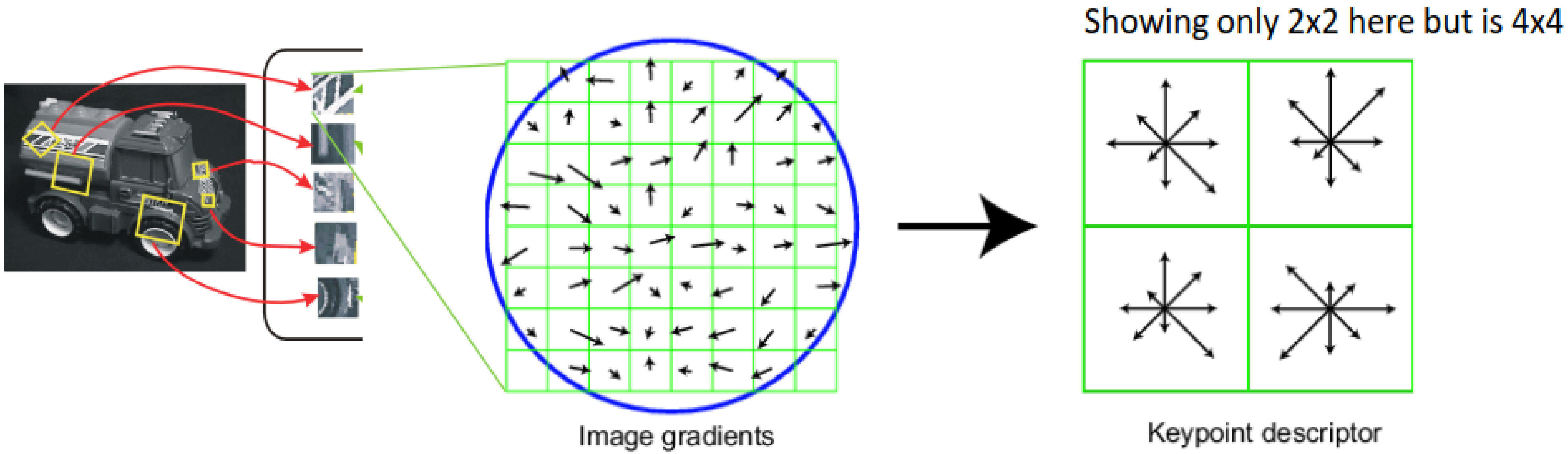


Actually 16x16, only showing 8x8

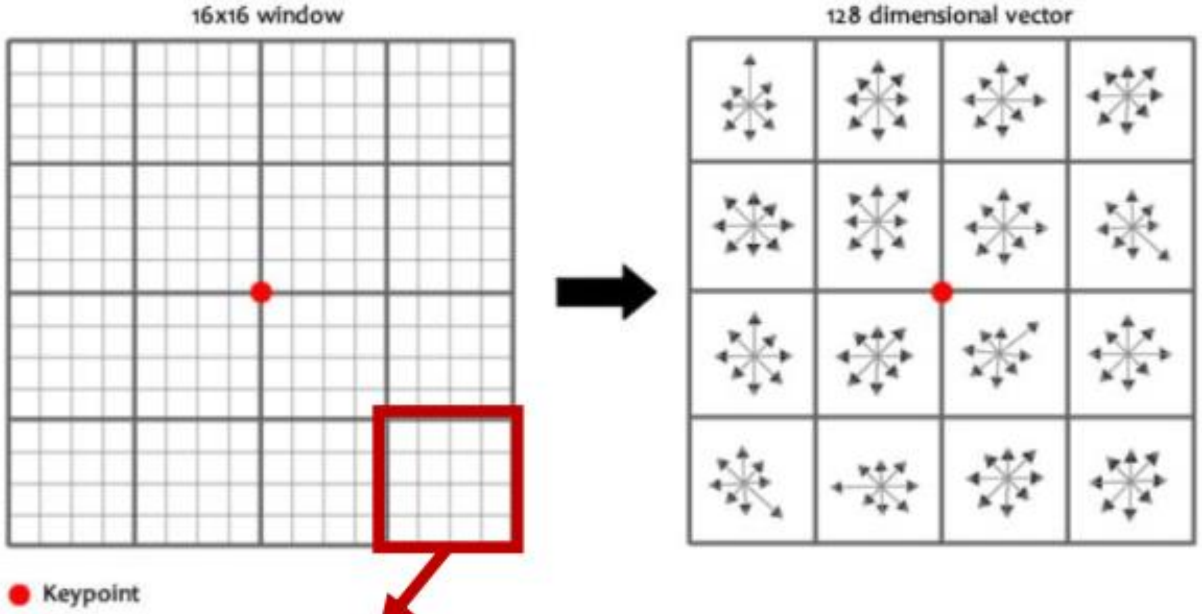
Image gradients

SIFT descriptor

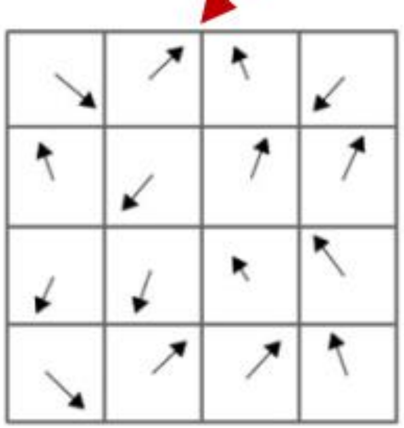
- 4x4 array of gradient orientation histograms weighted by gradient magnitude.
- Bin into 8 orientations x 4x4 array = 128 dimensions.

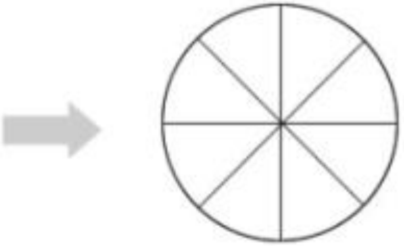


SIFT Descriptor Extraction



Gradient magnitude and orientation



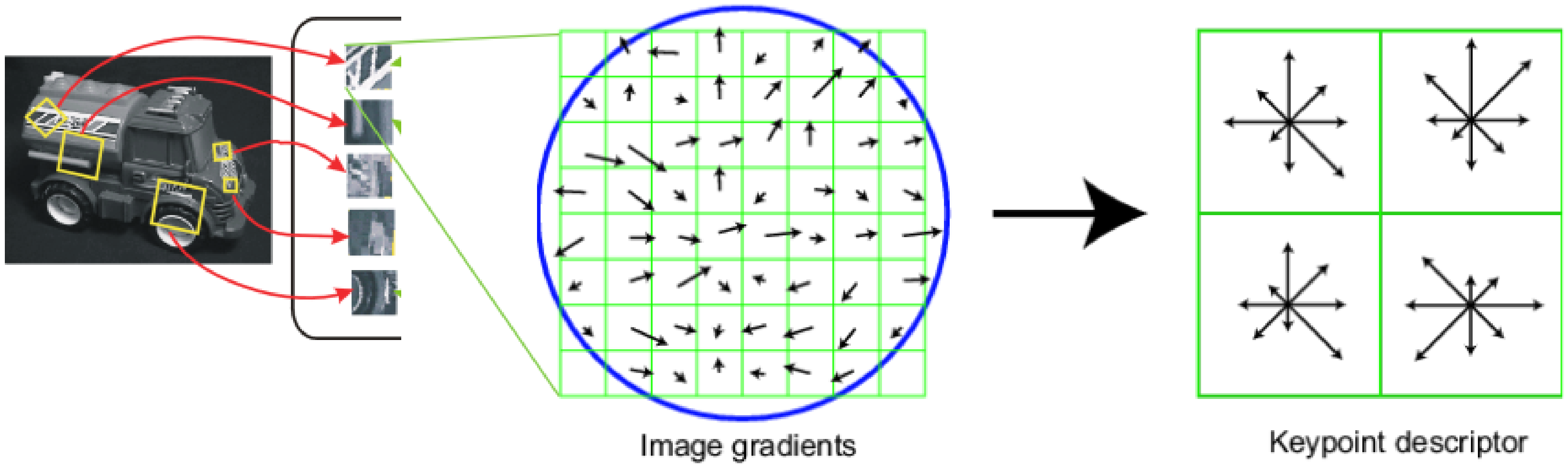


8 bin 'histogram' - add magnitude amounts!

Utkarsh Sinha

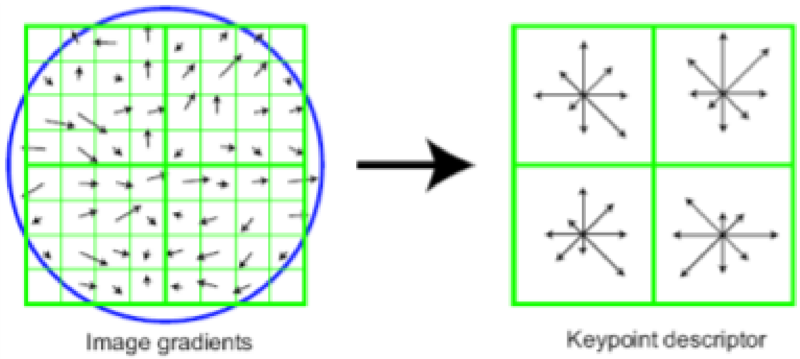
Reduce effect of illumination

- 128-dim vector normalized to 1
- Threshold gradient magnitudes to avoid excessive influence of high gradients
  - After normalization, clamp gradients > 0.2
  - Renormalize



Review: Local Descriptors

- Most features can be thought of as
  - templates,
  - histograms (counts),
  - or combinations
- The ideal descriptor should be
  - Robust and Distinctive
  - Compact and Efficient
- Most available descriptors focus on edge/gradient information
  - Capture texture information
  - Color rarely used



References

Basic reading:

- Szeliski textbook, Sections 4.1.

Questions?