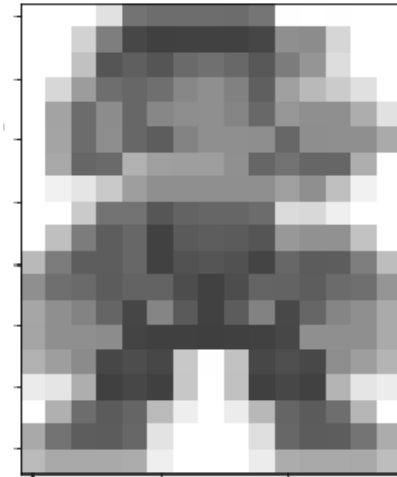


Homework 2

CAP 4453

Fall 2022

1. Considers the image below. What is the dimension of the matrix that represents the image? [10%]



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[255 255 255 225 115 109 109 109 108 117 254 255 254 255 255]
[255 255 210 126 71 64 65 65 65 70 143 140 214 255 255]
[255 255 193 85 95 85 106 112 104 87 124 149 221 255 255]
[255 215 156 110 103 111 135 143 129 95 159 185 203 224 255]
[255 163 108 140 103 135 141 143 132 104 150 143 143 173 225]
[255 164 108 141 103 93 131 143 143 141 103 142 143 143 170]
[255 168 102 106 177 159 158 158 144 102 114 102 102 174 255]
[255 240 229 201 158 143 143 143 143 159 143 191 241 255]
[255 255 202 115 114 86 99 102 102 108 219 216 239 255 255]
[255 191 124 93 103 65 90 93 91 85 152 145 145 194 255]
[185 123 93 93 103 65 82 85 81 68 103 92 93 126 189]
[143 111 106 92 99 99 77 65 77 100 99 93 107 111 149]
[166 143 131 101 70 133 90 65 92 130 72 102 133 143 170]
[165 143 143 140 68 64 64 63 64 65 71 143 143 143 170]
[177 158 139 71 77 71 200 254 194 71 78 72 141 158 180]
[235 229 174 64 71 64 199 255 193 64 71 65 180 229 236]
[255 176 109 91 101 186 238 255 236 184 101 91 110 182 255]
[169 114 93 92 103 227 255 255 255 223 103 92 93 116 172]
[186 168 168 169 172 239 255 255 255 236 172 168 169 168 189]
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2. If we filter using a 3x3 kernel and do not perform padding on the borders (convolution in the regions where filter and image are fully intersected), what will be the dimension of the previous image after filtering? [10%]
3. What will be the output dimension if the kernel is 5x5 (convolution in the regions where filter and image are fully intersected)? [10%]
4. Assuming the dimensions of the image are $M \times N$. can you come out with a general formula that tells you the dimension of the image after filtering if only consider 'valid' regions (where filter and image fully intersect)? [10%]

5. Compute the output of applying the filter $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$ on the first 5 valid columns of row 2 of "mario" (row 2 is the one that start with values 255 255 210 ..). Show your computations and write the obtained output. [10%]

Kernel estimation

$$G(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}}$$

6. Use the formula of 1D Gaussian function $G(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}}$ to find coefficients of a kernel of size 7 when $\sigma=1.4$. Hint: x is evaluated in interval $[-3 -2 -1 0 1 2 3]$ [10%]
7. The size of a gaussian kernel is usually chosen to have values in the order of 2 or 3 sigmas, since after that the values of the function are almost zero. In the extreme parts of this kernel (when x is either -3 or 3) how many sigmas it corresponds to? Is the chosen size of 7 a good value? [10%]
8. Approximate the obtained kernel as a fraction of integer numbers. Hint: use 64 as the denominator. [10%]
9. Compute a 7x7 Gaussian kernel using the 1D estimated kernel you estimated in the previous exercise. Remember, this is a separable filter and can be obtained using matrix multiplication. [10%]

$$G = K_{7 \times 1} * K_{1 \times 7}$$

10. In class we build a sharpen filter as the sum of original filter + detail. The detail part was built with the original function and a box filter. Create a new kernel for sharpening but this time uses a gaussian filter. [10%]