



# CAP 4453 Robot Vision

Dr. Gonzalo Vaca-Castaño gonzalo.vacacastano@ucf.edu



### Administrative details

- Allow grader to review your homework:
  - moazam4453@gmail.com
- Homework 1 review

Any issues with hw3?



## Credits

- Some slides comes directly from:
  - Yogesh S Rawat (UCF)
  - Noah Snavely (Cornell)
  - Ioannis (Yannis) Gkioulekas (CMU)
  - Mubarak Shah (UCF)
  - S. Seitz
  - James Tompkin
  - Ulas Bagci
  - L. Lazebnik





# Short Review from last class



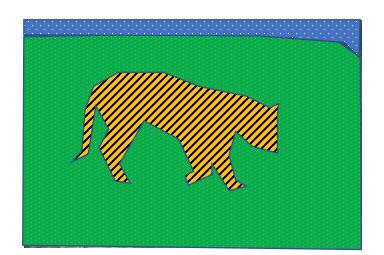
## Last 2 classes

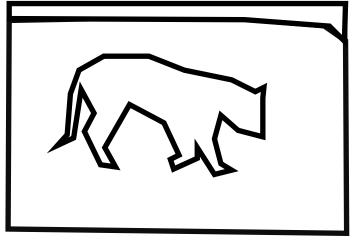
- Gradient operators
  - Prewit
  - Sobel
- Marr-Hildreth (Laplacian of Gaussian)
- Canny (Gradient of Gaussian)



# Regions ← Boundaries











# Robot Vision

7. Segmentation I



## Outline

- Image segmentation basics
- Thresholding based
  - Binarization
  - Otsu
- Region based
  - Merging
  - Splitting
- Clustering based
  - K-means (SLIC)



## Outline

- Image segmentation basics
- Thresholding based
  - Binarization
  - Otsu
- Region based
  - Merging
  - Splitting
- Clustering based
  - K-means (SLIC)



- Partition an image into a collection of set of pixels
  - Meaningful regions (coherent objects)
  - Linear structures (line, curve, ...)
  - Shapes (circles, ellipses, ...)



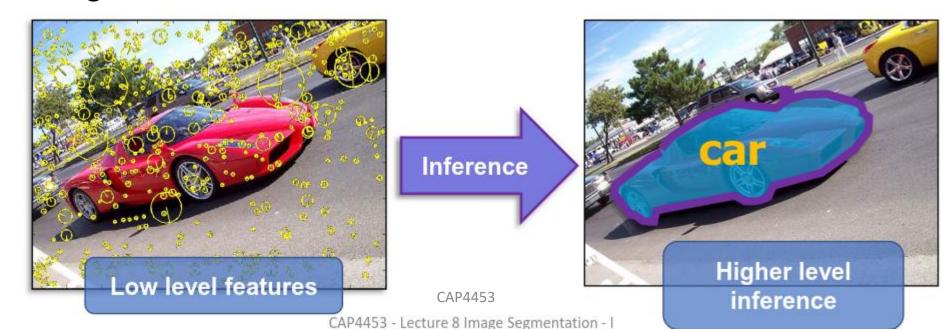


- Content base image retrieval
- Machine vision
- Medical imaging applications
- Object detection (face detection, ..)
- 3D reconstruction
- Object/motion tracking

• ...

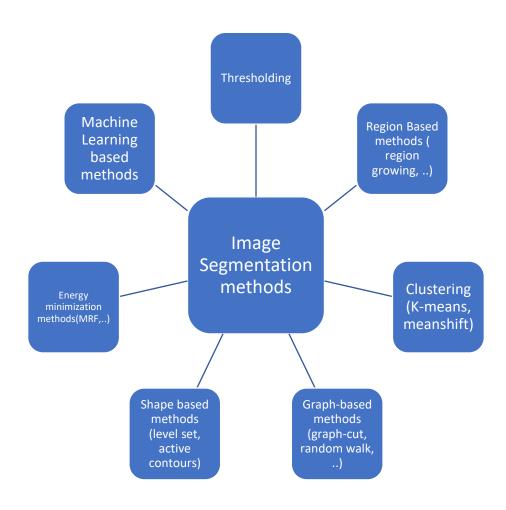


- In computer vision, image segmentation is one of the oldest and most widely studied problems
  - Early techniques -> region splitting or merging
  - Recent techniques -> Energy minimization, hybrid methods, and deep learning





# Image segmentation methods





• Image segmentation partitions an image into regions called segments.

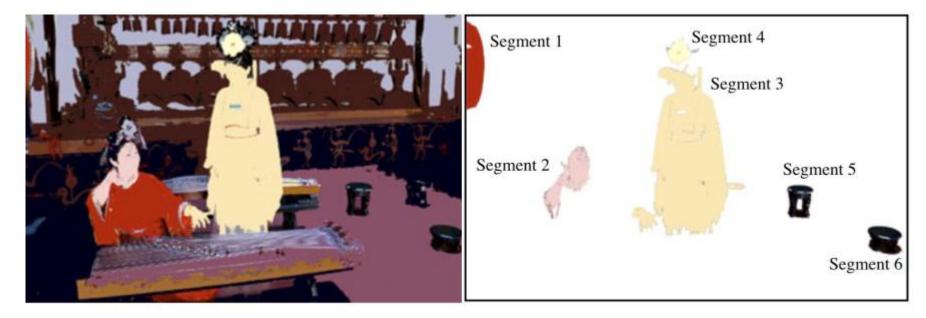
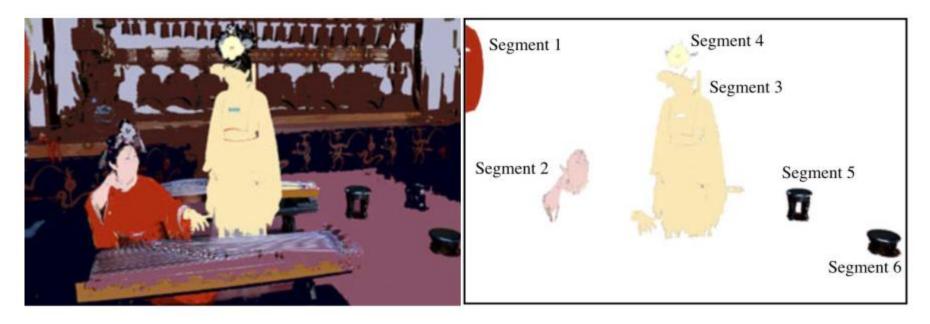




Image segmentation partitions an image into regions called segments.



- Image segmentation creates segments of connected pixels by analyzing some similarity criteria:
  - intensity, color, texture, histogram, features



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 Image binarization applies often just one global threshold T for mapping a scalar image I into a binary image





 Image binarization applies often just one global threshold T for mapping a scalar image I into a binary image

$$J(x,y) = \begin{cases} 0 & \text{if } I(x,y) < T \\ 1 & \text{otherwise.} \end{cases}$$



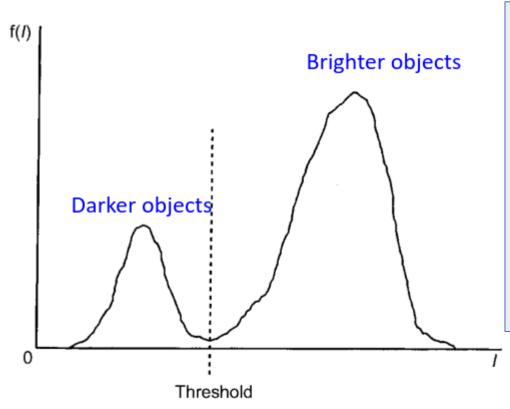
 Image binarization applies often just one global threshold T for mapping a scalar image I into a binary image

$$J(x,y) = \begin{cases} 0 & \text{if } I(x,y) < T \\ 1 & \text{otherwise.} \end{cases}$$

 The global threshold can be identified by an optimization strategy aiming at creating "large" connected regions and at reducing the number of small-sized regions, called artifacts.



 Thresholding: Most frequently employed method for determining threshold is based on histogram analysis of intensity levels



Peak on the left of the histogram corresponds to dark objects

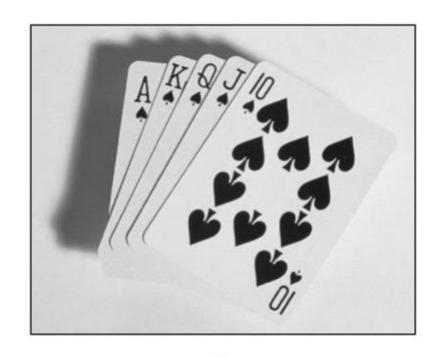
Peak on the right of the histogram corresponds to brigther objects

#### Difficulties

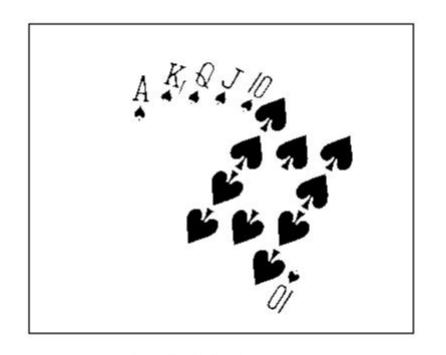
- 1. The valley may be so broad that it is difficult to locate a significant minimum
- 2. Number of minima due to type of details in the image
- 3. Noise
- 4. No visible valley
- 5. Histogram may be multi-modal



# Thresholding examples



Original Image



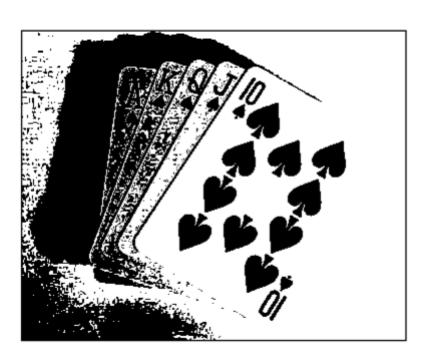
Thresholded Image







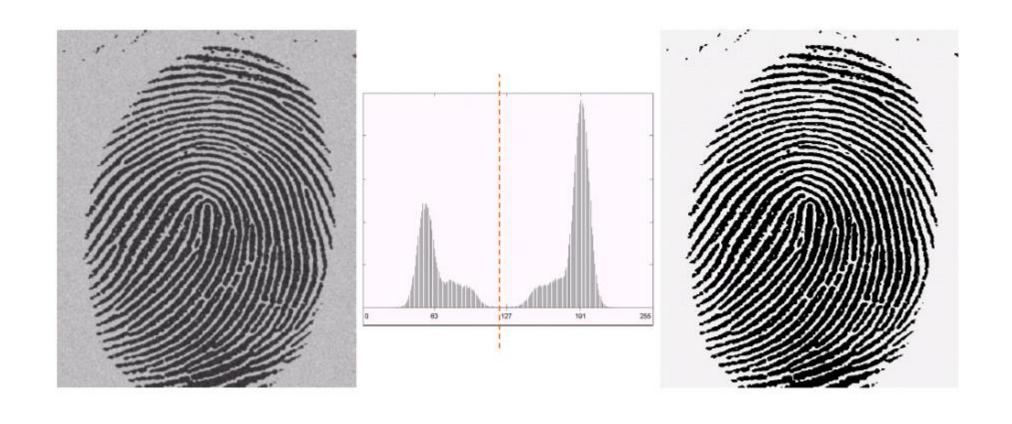
Threshold Too Low



Threshold Too High

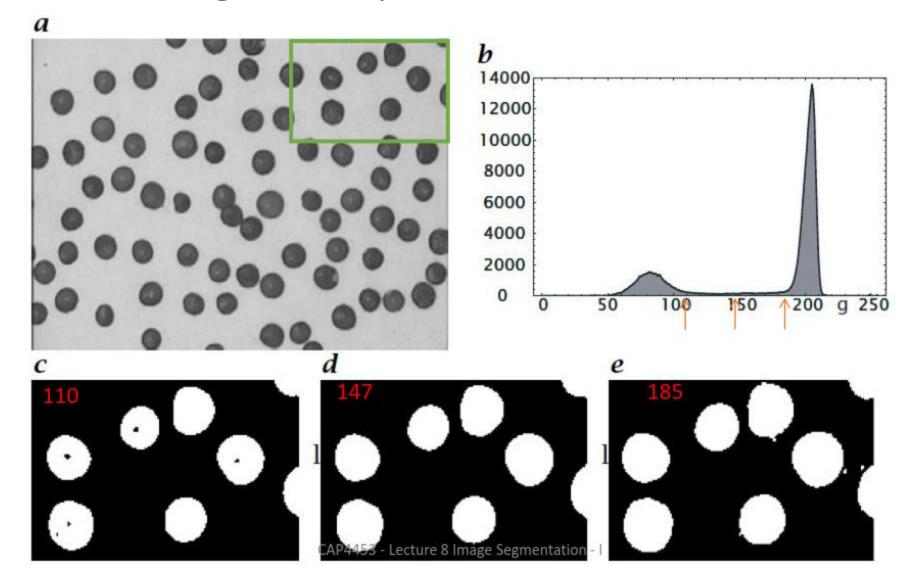


# Thresholding examples





# Thresholding examples





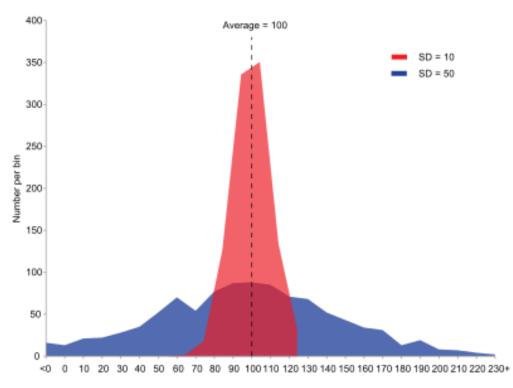
## Outline

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## Variance

$$\operatorname{Var}(X) = \operatorname{E}[(X - \mu)^2] = \sigma_X^2$$



where  $\mu$  is the expected value. That is,

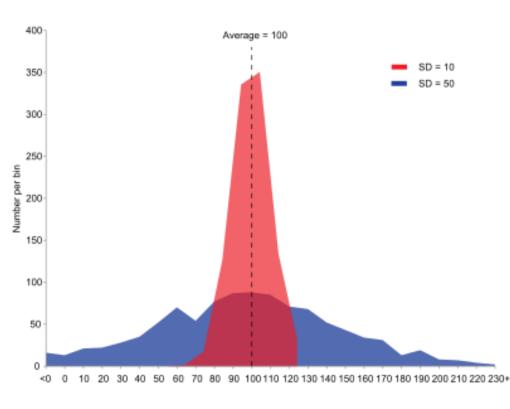
$$\mu = \sum_{i=1}^{n} p_i x_i.$$
  $p_i = \frac{\#values \ in \ i}{sum \ all \ values \ (area \ under \ curve)}$ 

<u>Variance - Wikipedia</u>



## Variance

$$\operatorname{Var}(X) = \operatorname{E}[(X - \mu)^{2}] = \sigma_{X}^{2}$$



#### Discrete random variable [edit]

If the generator of random variable X is discrete with probability mass function  $x_1\mapsto p_1, x_2\mapsto p_2, \ldots, x_n\mapsto p_n$ , then

$$\operatorname{Var}(X) = \sum_{i=1}^n p_i \cdot (x_i - \mu)^2,$$

where  $\mu$  is the expected value. That is,

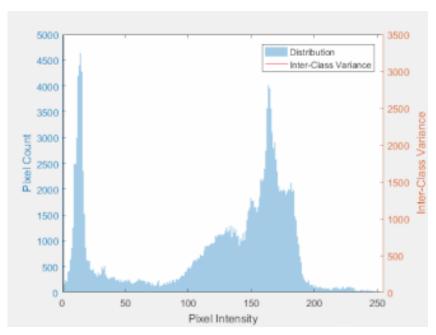
$$\mu = \sum_{i=1}^n p_i x_i.$$

<u>Variance - Wikipedia</u>



# Otsu thresholding

- Definition: The method uses grey-value histogram of the given image I as input and aims at providing the best threshold (foreground/background)
- Otsu's algorithm selects a threshold that maximizes the betweenclass variance  $\sigma_h^2$  or minimize within-class variance  $\sigma_w^2$
- For each threshold t in [0, 255], pixels can be separated into two classes, C1 and C2; those pixels whose Pi < t are put into C1, otherwise into C2
- The possibilities of C1 and C2 separated by t, denoted as W1 and W2, respectively. For example,
   W1 = (#pixels in C1) / (total pixels count).
- Given H, W1, and W2, for each t, compute the between-class variance  $\sigma_w^2$  or within-class variance  $\sigma_w^2$  ( $\sigma_b^2 \rightarrow$  red curve)
- optimal cut  $t^*$  corresponds to t whose  $\sigma_w^2$  is maximum or  $\sigma_w^2$  is minimum.





# Otsu thresholding

 Definition: The method uses grey-value histogram of the given image I as input and aims at providing the best threshold (foreground/background)

• Otsu's algorithm, selects a threshold that maximizes the between-

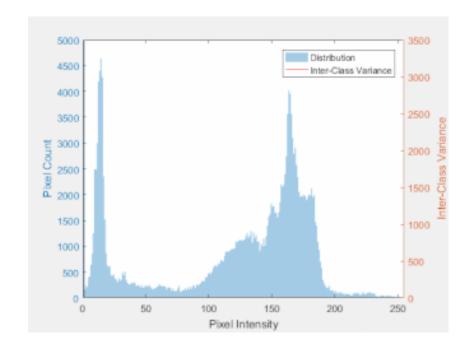
class variance  $\sigma_b^2$ 

#### Option 1: maximum of:

$$\sigma_b^2(t) = w_1(t)w_2(t)[\mu_1(t) - \mu_2(t)]^2$$

$$\mu_1(t) = \sum_{i=1}^t \frac{iP(i)}{w_1(t)}$$
  $w_1(t) = \sum_{i=1}^t P(i)$ 

$$\mu_2(t) = \sum_{i=t+1}^{I} \frac{iP(i)}{w_2(t)}$$
  $w_2(t) = \sum_{i=t+1}^{I} P(i)$ 





# Otsu thresholding

- Definition: The method uses grey-value histogram of the given image I as input and aims at providing the best threshold (foreground/background)
- Otsu's algorithm selects a threshold that maximizes the betweenclass variance  $\sigma_b^2$  or minimize within-class variance  $\sigma_w^2$

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#### Option 2: minimum of:

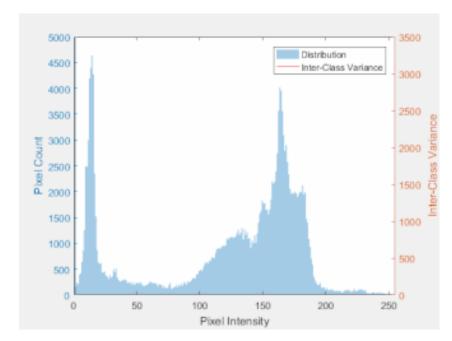
$$\sigma_w^2(t) = w_1(t)\sigma_1^2(t) + w_2(t)\sigma_2^2(t)$$

$$w_1(t) = \sum_{i=1}^t P(i) \qquad P(i) = \frac{n_i}{n}$$

$$w_2(t) = \sum_{i=t+1}^I P(i)$$

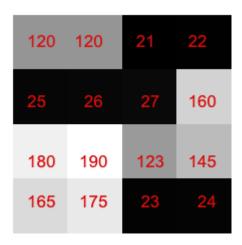
$$\sigma_1^2(t) = \sum_{i=1}^t [i - \mu_1(t)]^2 \frac{P(i)}{w_1(t)}$$

$$\sigma_2^2(t) = \sum_{i=t+1}^I [i - \mu_2(t)]^2 \frac{P(i)}{w_2(t)}$$



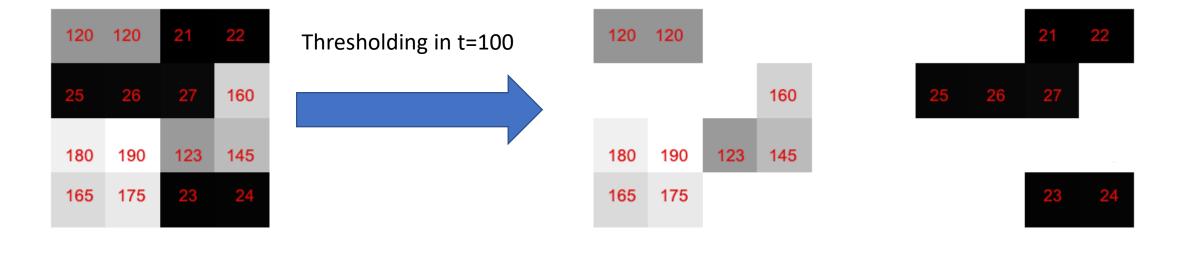


• Find Otsu threshold for this image



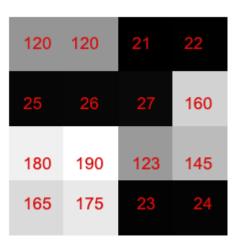
• By minimizing within-class variance



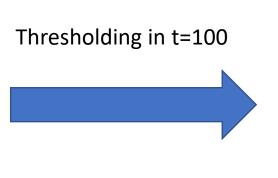


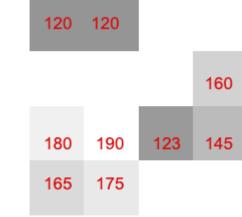
Foreground











Foreground

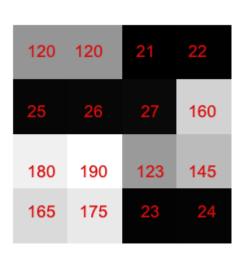
$$P_{FG} = 9$$
  $\omega_{fg}(t) = rac{P_{FG}(t)}{P_{all}} = 9/16 = 0.56$ 



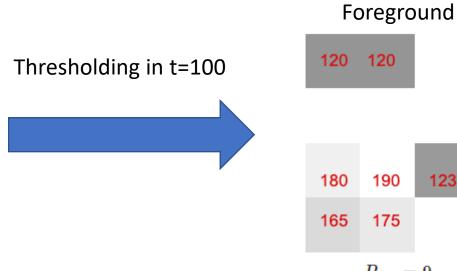


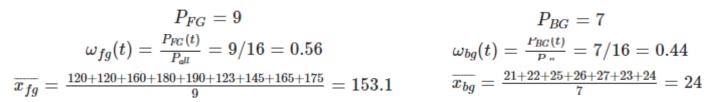
$$P_{BG}=7$$
  $\omega_{bg}(t)=rac{P_{BG}(t)}{P_{all}}=7/16=0.44$ 











160

145

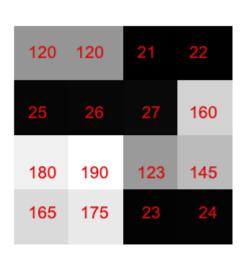
123



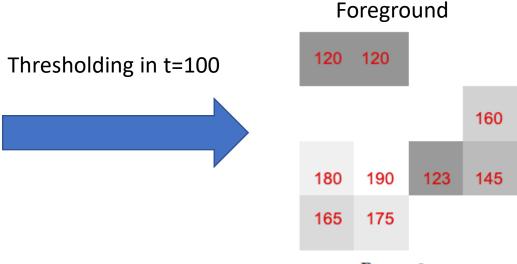


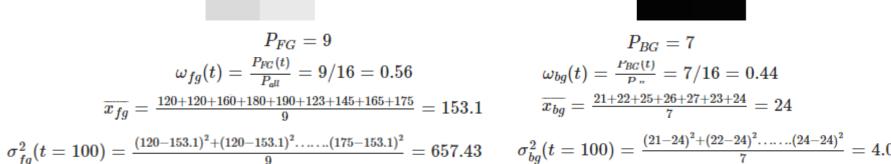
$$P_{BG}=7$$
  $\omega_{bg}(t)=rac{P_{BG}(t)}{P_{,,}}=7/16=0.44$   $=153.1$   $\overline{x_{bg}}=rac{21+22+25+26+27+23+24}{7}=24$ 















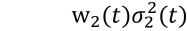
$$P_{BG}=7$$
 $\omega_{bg}(t)=rac{P_{BG}(t)}{P_{"}}=7/16=0.44$ 
 $\overline{x_{bg}}=rac{21+22+25+26+27+23+24}{7}=24$ 
 $c_{bg}^2(t=100)=rac{(21-24)^2+(22-24)^2......(24-24)^2}{7}=4.$ 

#### within-class variance



# Step by step example

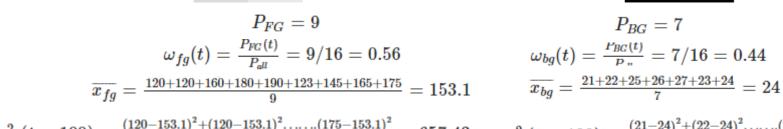
$$W_1(t)\sigma_1^2(t)$$
 +











Foreground





$$P_{FG} = 9$$

$$\omega_{fg}(t) = \frac{P_{FG}(t)}{P_{all}} = 9/16 = 0.56$$

$$\overline{x_{fg}} = \frac{120 + 120 + 160 + 180 + 190 + 123 + 145 + 165 + 175}{9} = 153.1$$

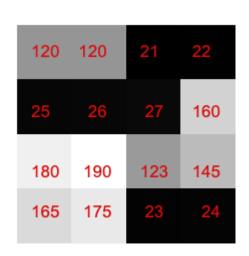
$$\sigma_{fg}^{2}(t = 100) = \frac{(120 - 153.1)^{2} + (120 - 153.1)^{2} \dots (175 - 153.1)^{2}}{9} = 657.43$$

$$\sigma_{bg}^{2}(t = 100) = \frac{(21 - 24)^{2} + (22 - 24)^{2} \dots (24 - 24)^{2}}{7} = 4.0$$

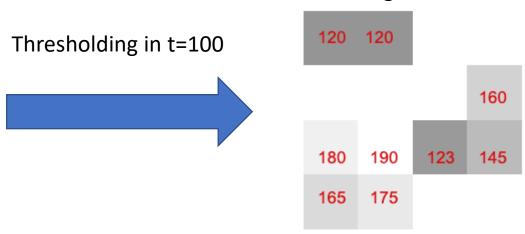
#### within-class variance

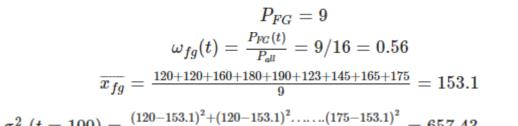
#### Step by step example

$$w_1(t)\sigma_1^2(t) + w_2(t)\sigma_2^2(t)$$









Foreground

#### Background





$$P_{FG} = 9$$

$$\omega_{fg}(t) = \frac{P_{FG}(t)}{P_{all}} = 9/16 = 0.56$$

$$\overline{x_{fg}} = \frac{120 + 120 + 160 + 180 + 190 + 123 + 145 + 165 + 175}{9} = 153.1$$

$$\sigma_{fg}^{2}(t = 100) = \frac{(120 - 153.1)^{2} + (120 - 153.1)^{2} \dots (175 - 153.1)^{2}}{9} = 657.43$$

$$P_{BG} = 7$$

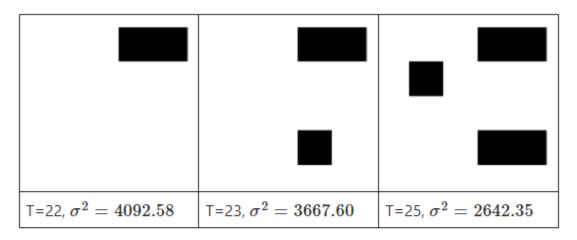
$$\omega_{bg}(t) = \frac{P_{BG}(t)}{P_{...}} = 7/16 = 0.44$$

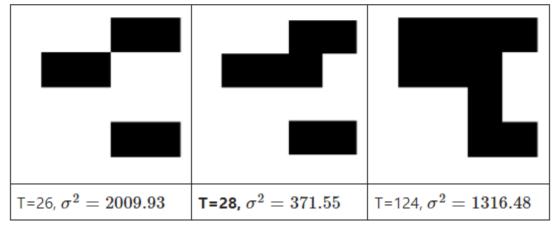
$$\overline{x_{bg}} = \frac{21 + 22 + 25 + 26 + 27 + 23 + 24}{7} = 24$$

$$\sigma_{bg}^{2}(t = 100) = \frac{(21 - 24)^{2} + (22 - 24)^{2} \dots (24 - 24)^{2}}{7} = 657.43$$



### Step by step (otsu thresholding)





#### Minimize within-class variance

- The value of variance remains the same from 28 and 120.
- within-class variance is least at t=28 or more precisely between 28 to 120.
- Otsu threshold = 28.

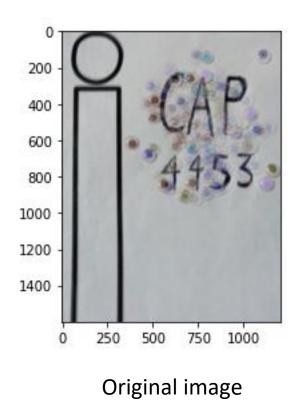


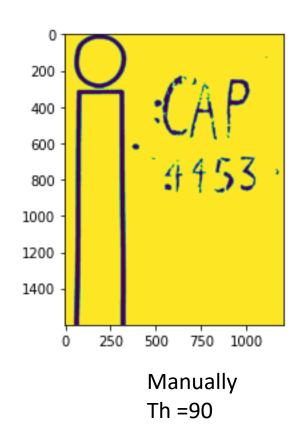
#### Otsu threshold implementation

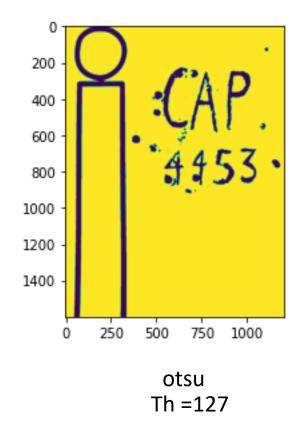
```
1 # Set total number of bins in the histogram
                                                                        P : A 0
   bins_num = 256
    # Get the image histogram
   hist, bin edges = np.histogram(image, bins=bins num)
   # Get normalized histogram if it is required
 8 if is_normalized:
        hist = np.divide(hist.ravel(), hist.max())
   # Calculate centers of bins
12 bin_mids = (bin_edges[:-1] + bin_edges[1:]) / 2.
14 # Iterate over all thresholds (indices) and get the probabilities w1(t), w2(t)
15 | weight1 = np.cumsum(hist)
   weight2 = np.cumsum(hist[::-1])[::-1]
18 # Get the class means mu0(t)
19 mean1 = np.cumsum(hist * bin mids) / weight1
20 # Get the class means mu1(t)
21 mean2 = (np.cumsum((hist * bin mids)[::-1]) / weight2[::-1])[::-1]
22
   inter_class_variance = weight1[:-1] * weight2[1:] * (mean1[:-1] - mean2[1:]) **
   # Maximize the inter class variance function val
26 index of max val = np.argmax(inter class variance)
27
28 threshold = bin_mids[:-1][index_of_max_val]
29 print("Otsu's algorithm implementation thresholding result: ", threshold)
```



#### Otsu threshold implementation









#### Otsu threshold implementation

```
# Applying Otsu's method setting the flag value into cv.THRESH_OTSU.
# Use a bimodal image as an input.
# Optimal threshold value is determined automatically.
otsu_threshold, image_result = cv2.threshold(
    image, 0, 255, cv2.THRESH_BINARY + cv2.THRESH_OTSU,
)
print("Obtained threshold: ", otsu_threshold)
```

```
Obtained threshold: 132.0
```



# Otsu thresholding example





#### The math!

$$\begin{aligned} \text{Var}(X) &= \text{E} \big[ (X - \text{E}[X])^2 \big] \\ &= \text{E} \big[ X^2 - 2X \, \text{E}[X] + \text{E}[X]^2 \big] \\ &= \text{E} \big[ X^2 \big] - 2 \, \text{E}[X] \, \text{E}[X] + \text{E}[X]^2 \\ &= \text{E} \big[ X^2 \big] - \text{E}[X]^2 \end{aligned}$$



$$\sigma_{total}^2 = E[(X - E[X])^2] = E[X_{total}^2] - \mu_{total}^2$$
 (1)

$$P_{i} = \frac{n_{i}}{n_{total}} \qquad \text{When i<=t} \qquad P_{i}^{1} = \frac{n_{i}}{n_{1}} \qquad P_{i} = \frac{n_{1}P_{i}^{1}}{n_{total}} = w_{1}(t)P_{i}^{1}$$
 
$$\qquad P_{i}^{2} = \frac{n_{i}}{n_{2}} \qquad P_{i} = \frac{n_{2}P_{i}^{2}}{n_{total}} = w_{2}(t)P_{i}^{2}$$

$$E[X_{total}^{2}] = \sum_{i=1}^{T} P_{i} x_{i}^{2} = \sum_{i=1}^{t} P_{i} x_{i}^{2} + \sum_{i=t+1}^{T} P_{i} x_{i}^{2} = w_{1}(t) \sum_{i=1}^{t} P_{i}^{1} x_{i}^{2} + w_{2}(t) \sum_{i=t+1}^{T} P_{i}^{1} x_{i}^{2} = w_{1}(t) E[X_{1}^{2}] + w_{2}(t) E[X_{2}^{2}]$$
 (2)

$$\mu_{total}^2 = (w_1(t)\mu_1 + w_2(t)\mu_2)^2 = w_1^2\mu_1^2 + 2w_1w_2\mu_1\mu_2 + w_2^2\mu_2^2 = w_1(1 - w_2)\mu_1^2 + 2w_1w_2\mu_1\mu_2 + w_2(1 - w_1)\mu_2^2$$
 (3)

#### The math!

$$\begin{split} \operatorname{Var}(X) &= \operatorname{E} \big[ (X - \operatorname{E}[X])^2 \big] \\ &= \operatorname{E} \big[ X^2 - 2X \operatorname{E}[X] + \operatorname{E}[X]^2 \big] \\ &= \operatorname{E} \big[ X^2 \big] - 2 \operatorname{E}[X] \operatorname{E}[X] + \operatorname{E}[X]^2 \\ &= \operatorname{E} \big[ X^2 \big] - \operatorname{E}[X]^2 \end{split}$$



$$\sigma_{total}^{2} = E[(X - E[X])^{2}] = E[X_{total}^{2}] - \mu_{total}^{2}$$

$$E[X_{total}^{2}] = w_{1}(t)E[X_{1}^{2}] + w_{2}(t)E[X_{2}^{2}]$$

$$\mu_{total}^{2} = w_{1}(1 - w_{2})\mu_{1}^{2} + 2w_{1}w_{2}\mu_{1}\mu_{2} + w_{2}(1 - w_{1})\mu_{2}^{2}$$
(3)

$$\sigma_{total}^{2} = w_{1}E[X_{1}^{2}] + w_{2}E[X_{2}^{2}] - [w_{1}\mu_{1}^{2} + w_{1}w_{2}\mu_{1}^{2} + 2w_{1}w_{2}\mu_{1}\mu_{2} + w_{2}\mu_{2}^{2} - w_{1}w_{2}\mu_{2}^{2}]$$

$$\sigma_{total}^{2} = w_{1}E[X_{1}^{2}] - w_{1}\mu_{1}^{2} + w_{2}E[X_{2}^{2}] - w_{2}\mu_{2}^{2} + [-w_{1}w_{2}\mu_{1}^{2} - 2w_{1}w_{2}\mu_{1}\mu_{2} + w_{1}w_{2}\mu_{2}^{2}]$$

$$\sigma_{total}^{2} = w_{1}(t)(E[X_{1}^{2}] - \mu_{1}^{2}) + w_{2}(t)(E[X_{2}^{2}] - \mu_{2}^{2}) + w_{1}(t)w_{2}(t)(\mu_{2}^{2} - \mu_{1}^{2})$$
(4)

#### The math!

fixed

$$egin{aligned} ext{Var}(X) &= ext{E}ig[(X - ext{E}[X])^2ig] \ &= ext{E}ig[X^2 - 2X \, ext{E}[X] + ext{E}[X]^2ig] \ &= ext{E}ig[X^2ig] - 2 \, ext{E}[X] \, ext{E}[X] + ext{E}[X]^2 \ &= ext{E}ig[X^2ig] - ext{E}[X]^2 \end{aligned}$$



$$\sigma_{total}^{2} = E[(X - E[X])^{2}] = E[X_{total}^{2}] - \mu_{total}^{2}$$
 (1) 
$$\sigma_{total}^{2} = w_{1}(t)(E[X_{1}^{2}] - \mu_{1}^{2}) + w_{2}(t)(E[X_{2}^{2}] - \mu_{2}^{2}) + \sigma_{total}^{2} = w_{1}(t)\sigma_{1}^{2}(t) + w_{2}(t)\sigma_{2}^{2}(t) + w_{3}(t)\sigma_{2}^{2}(t)$$
 within-class variance

Minimize

$$w_1(t)w_2(t)(\mu_2^2 - \mu_1^2)$$

$$w_1(t)w_2(t)(\mu_2^2(t) - \mu_1^2(t))$$

between-class variance

Maximize

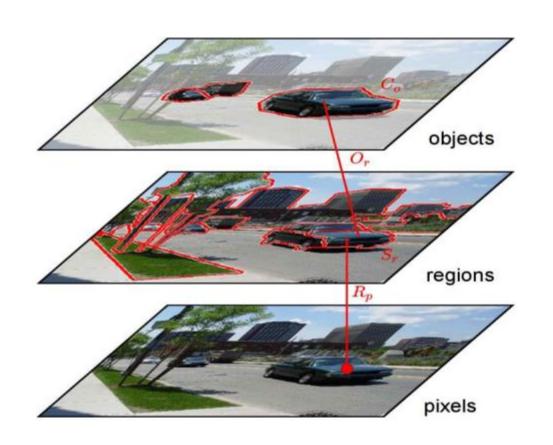


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- Region based
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  - Splitting
- Clustering based
  - K-means (SLIC)



#### Region based segmentation



#### Region:

A group of connected pixels with <u>similar</u> properties

Closed boundaries

Computation of regions is based on <u>similarity</u>

Regions may correspond to Objects in a scene or parts of objects

Spatial proximity + similarity



#### Region growing

• For segment generation in grey-level or color images, we may start at one seed pixel (x,y,I(x,y)) and add recursively adjacent pixels that satisfy a "similarity criterion" with pixels contained in the so-far grown region around the seed pixel.

- Defining similarity criteria alone is not an effective basis for segmentation
- It is necessary to consider the adjacency spatial relationship between pixels



#### Region growing

- Algorithm
- 1. The absolute intensity difference between candidate pixel and the seed pixel must lie within a specified range
- 2. The absolute intensity difference between a candidate pixel and the running average intensity of the growing region must lie within a specified range;
- 3. The difference between the standard deviation in intensity over a specified local neighborhood of the candidate pixel and that over a local neighborhood of the candidate pixel must (or must not) exceed a certain threshold



#### 1. Chose the seed pixel

0	0	5	6	7
1	1	5	8	7
0	1	6	2	7
2	.0	7	6	6
0	1	5	6	5
		(a)		



- 1. Chose the seed pixel
- 2. Check the neighboring pixels and add them to the region if they are similar to the seed

0	0	5	6	7
1	1	5	8	7
0	1	6	2	7
2	0	7	6	6
0	1	5	6	5



- 1. Chose the seed pixel
- 2. Check the neighboring pixels and add them to the region if they are similar to the seed

.0	- 1	5	6	5	a	à	ь	ь	ь
2	0	7	6	6	a	2	ь	b	b
0	1	6	2	7	a	а	ь	b	b
1	1	5	8	7	a	a	ь	b	ь
0	0	5	6	7	a	2	ь	ь	ь

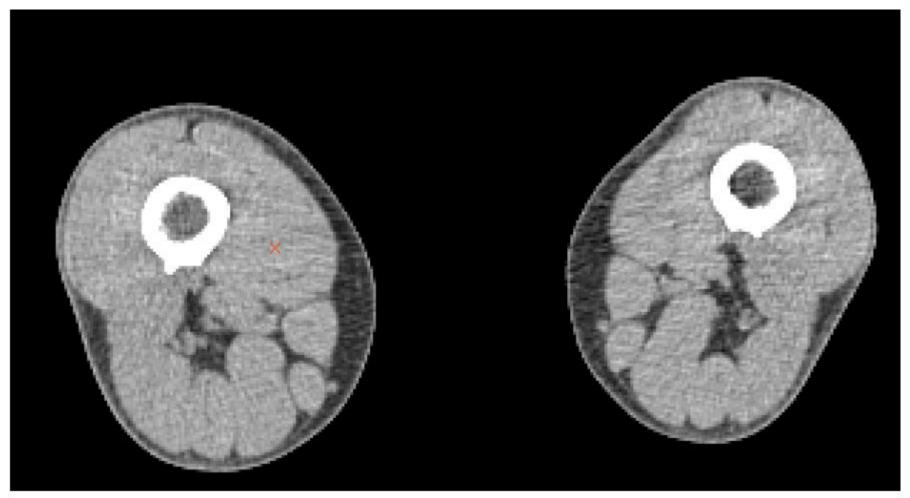


- 1. Chose the seed pixel
- 2. Check the neighboring pixels and add them to the region if they are similar to the seed
- 3. Repeat step 2 for each of the newly added pixels; stop if no more pixels can be added

0	0	5	6	7	a		ь	ь	ь
1	1	5	8	7	a	а	ь	b	b
0	1	6	2	7	a	а	ь	ь	b
2	0	7	6	6	a	2	ь	ь	b
0	1	5	6	5	a	a	ь	ь	ь
(a)					a vibus is		(b)		



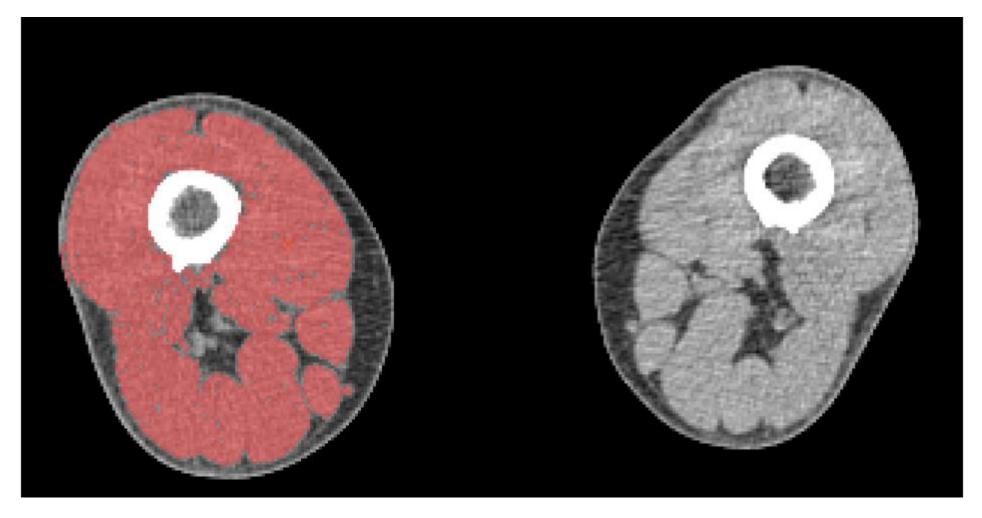
### Ex: Muscle/Bone Segmentation in CT Scans



CAP4453

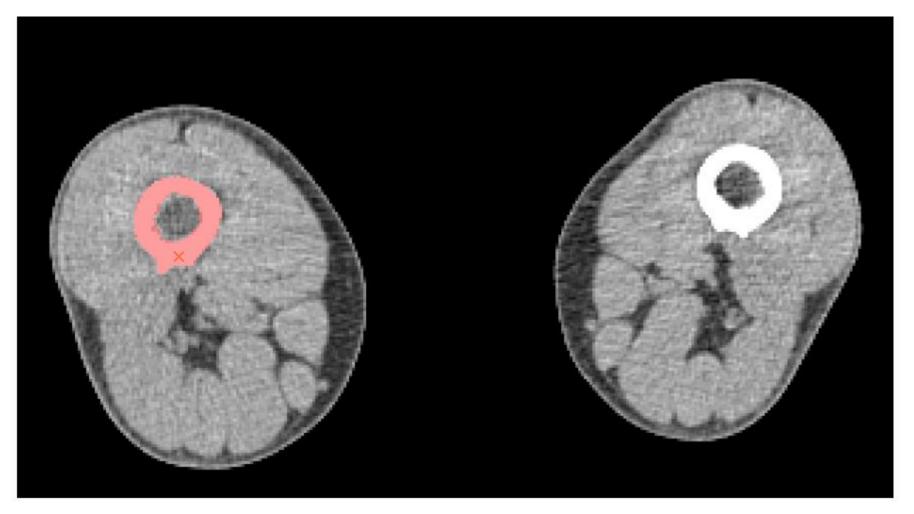


# Ex: Muscle/Bone Segmentation in CT Scans





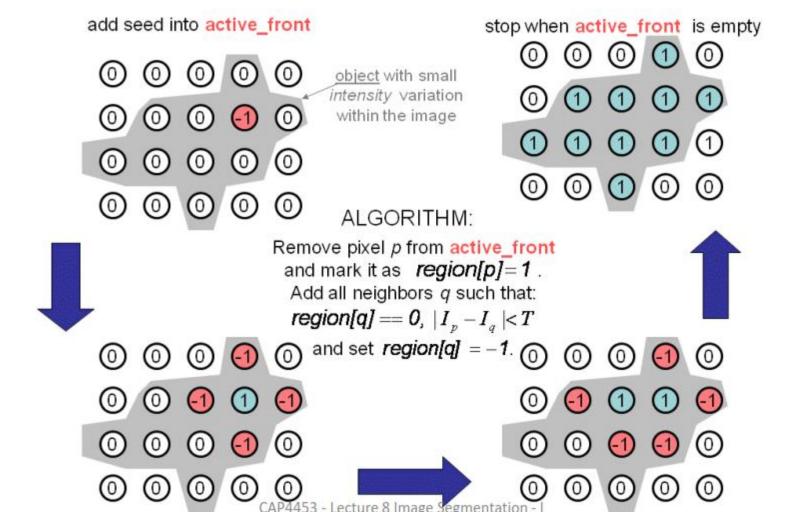
# Ex: Muscle/Bone Segmentation in CT Scans





### Region Growing Implementation

growRegion: red nodes are the "active\_front" (queue or stack)





#### Outline

- Image segmentation basics
- Thresholding based
  - Binarization
  - Otsu
- Region based
  - Merging
  - Splitting
- Clustering based
  - K-means (SLIC)



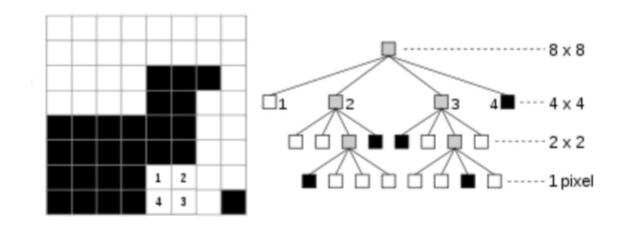
#### Region splitting:

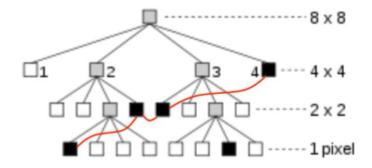
 Unlike region growing, which starts from a set of seed points, region splitting starts with the whole image as a single region and subdivides it into subsidiary regions recursively while a condition of homogeneity is not satisfied.

#### Region merging:

- Region merging is the opposite of splitting, and works as a way of avoiding over-segmentation
- Start with small regions (2x2 or 4x4 regions) and merge the regions that have similar characteristics (such as gray level, variance).







RAG with adjacency relations (in red) for big black region.

 RAG: region adjacency graph

 Quadtree for splitting (top-down) procedure

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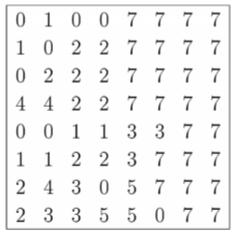


#### Algorithm:

- If a region R is inhomogeneous (P (R)=FALSE), then R is split into four sub-regions.
- If two adjacent regions R<sub>i</sub>,R<sub>j</sub> are homogeneous (P(R<sub>i</sub>UR<sub>j</sub>)=TRUE), they are then merged.
- The algorithm stops when no further splitting or merging is possible.

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original image

0	1	0	0	7	7	7	7
1	0	2	2	7	7	7	7
0	2	2	2	7	7	7	7
4	4	2	2	7	7	7	7
0	0	1	1	3	3	7	7
1	1	2	2	3	7	7	7
2	4	3	0	5	7	7	7
2	3	3	5	5	0	7	7

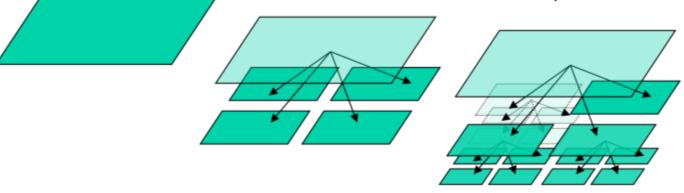
split I

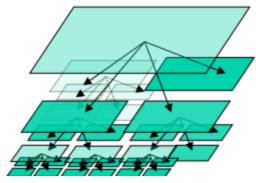
0	1	0	0	7	7	7	7
1	0	2	2	7	7	7	7
0	2	2	2	7	7	7	7
4	4	2	2	7	7	7	7
0	0	1	1	3	3	7	7
1	1	2	2	3	7	7	7
2	4	3	0	5	7	7	7
2	3	3	5	5	0	7	7
		-	hlit	2			

split 2

0	1	0	0	7	7	7	7
1	0	2	2	7	7	7	7
0	2	2	2	7	7	7	7
4	4	2	2	7	7	7	7
0	0	1	1	3	3	7	7
1	1	2	2	3	7	7	7
2	4	3	0	5	7	7	7
2	3	3	5	5	0	7	7
				-			

split 3





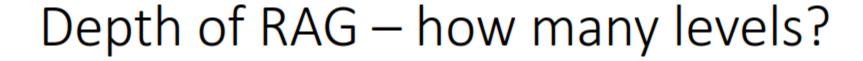


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2	2	2	2	1	1	1	1
2	2	2	2	1	1	1	1
2	2	3	3	2	2	2	2
2	2	3	3	3	2	2	2
1	1	1	1	2	2	2	2
1	1	1	1	2	2	2	2
1	1	1	1	2	2	2	2
1	1	1	1	2	2	2	2



# Depth of RAG – how many levels? 4

2	2	2	2	1	1	1	1
2	2	2	2	1	1	1	1
2	2	3	3	2	2	2	2
2	2	3	3	3	2	2	2
1	1	1	1	2	2	2	2
1	1	1	1	2	2	2	2
1	1	1	1	2	2	2	2
1	1	1	1	2	2	2	2



# Questions?