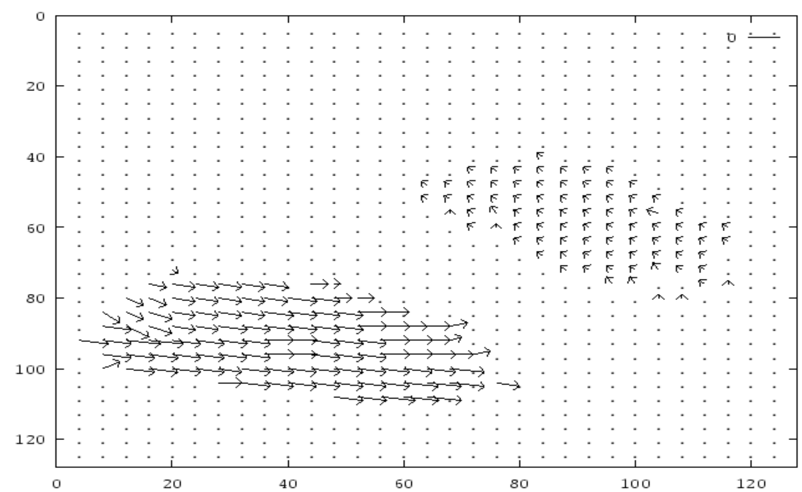
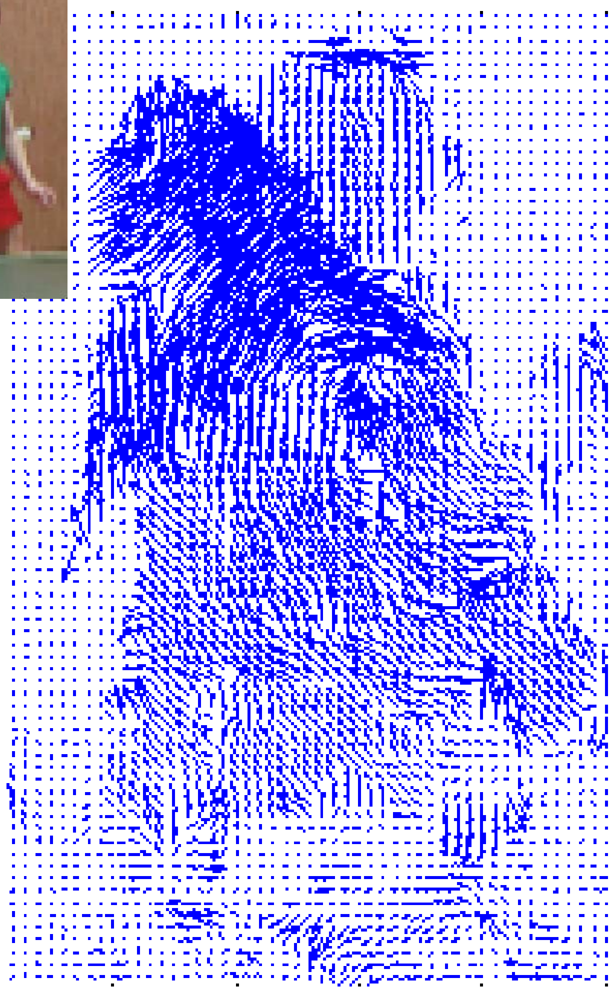
Computing Optical Flow

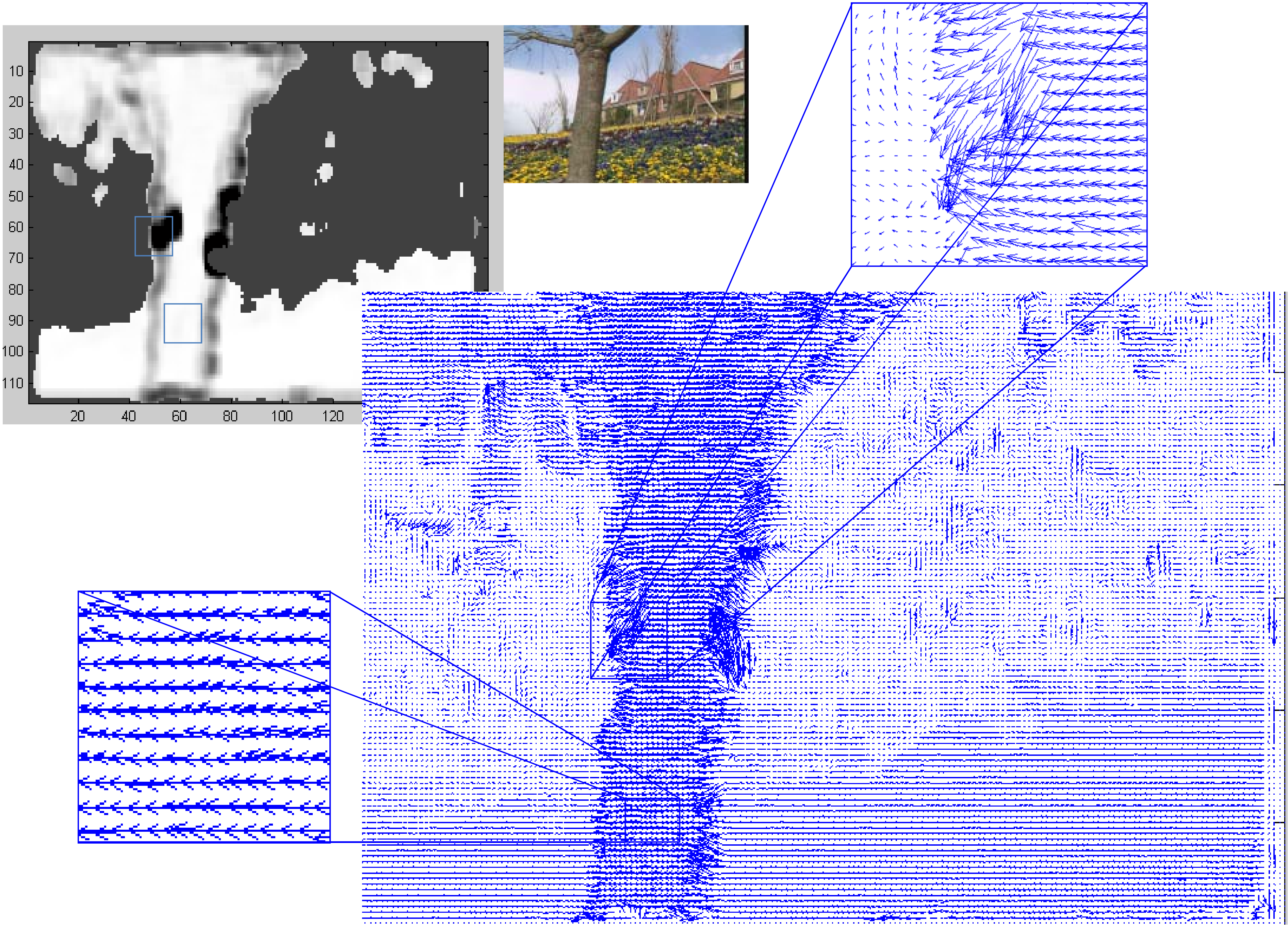
Lecture-6

Hamburg Taxi seq






CLAIROL
Lufthans
IBM
ATP Tour
EBEL


10
20
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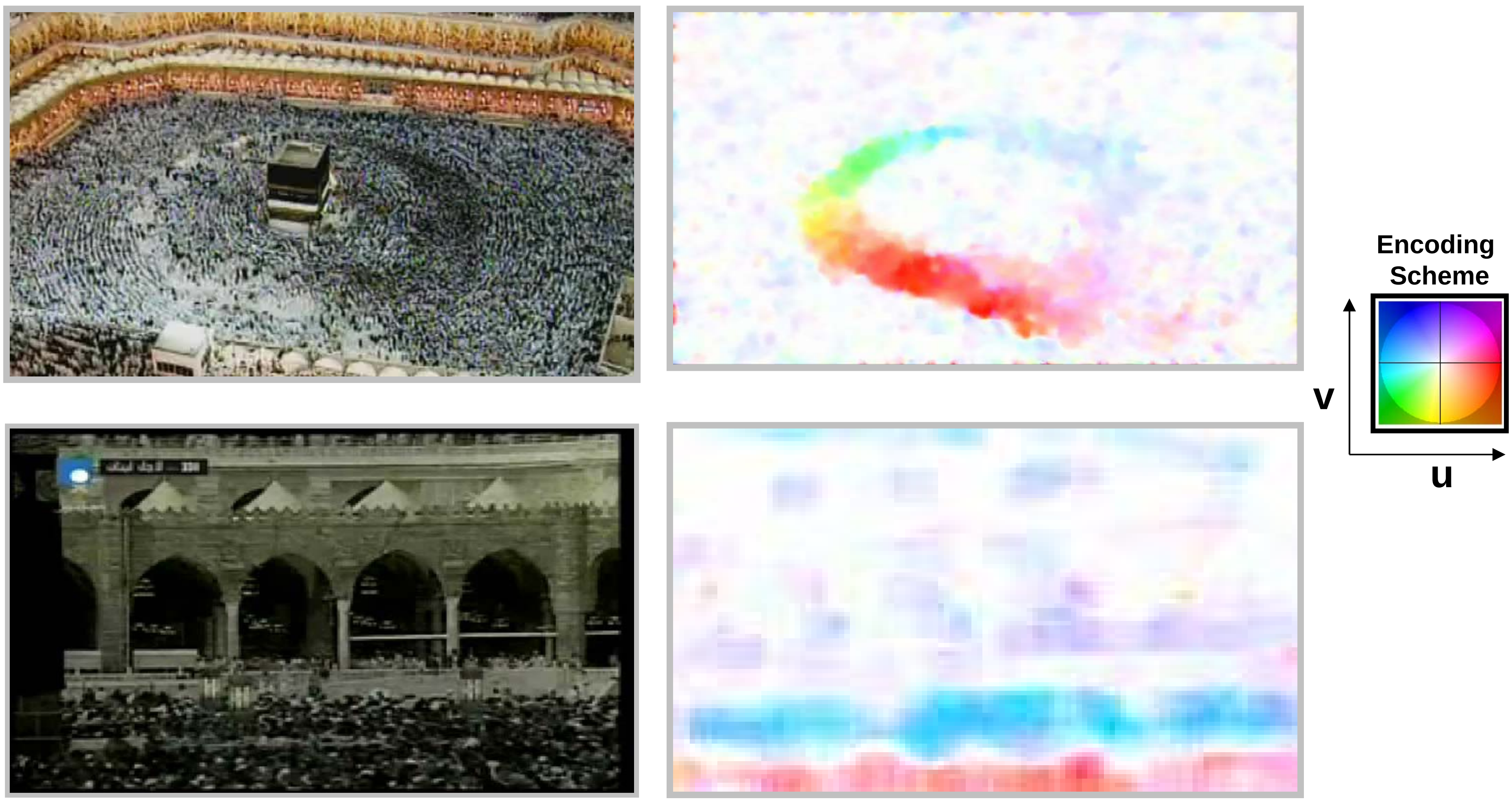
120

Optical Flow Field Examples

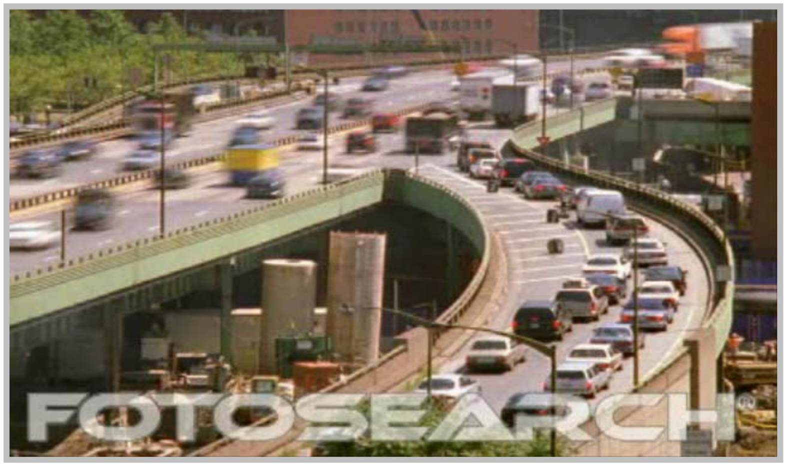
Optical Flow - Examples

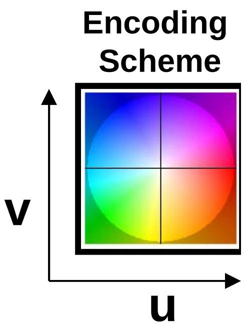


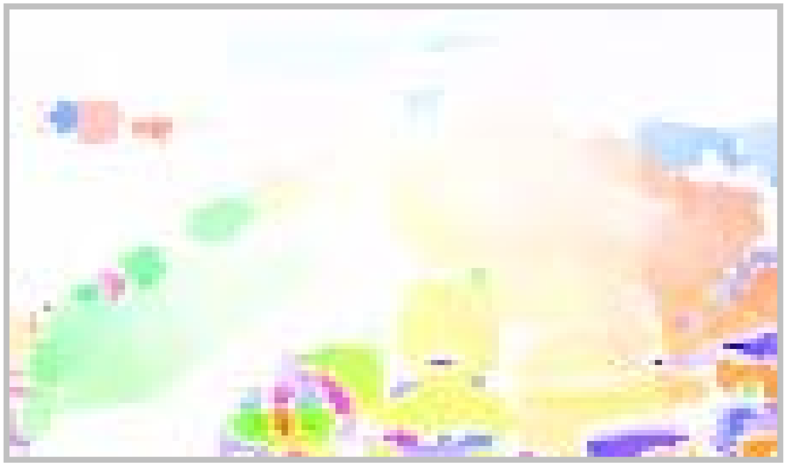
Videos

Color Coded Optical Flows

Optical Flow - Examples



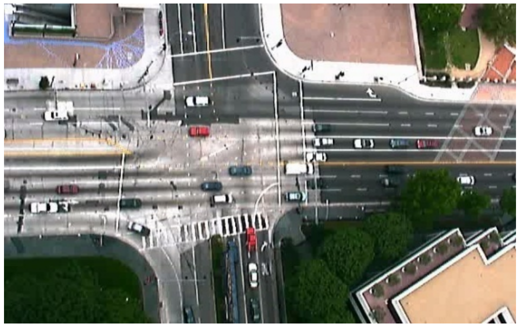


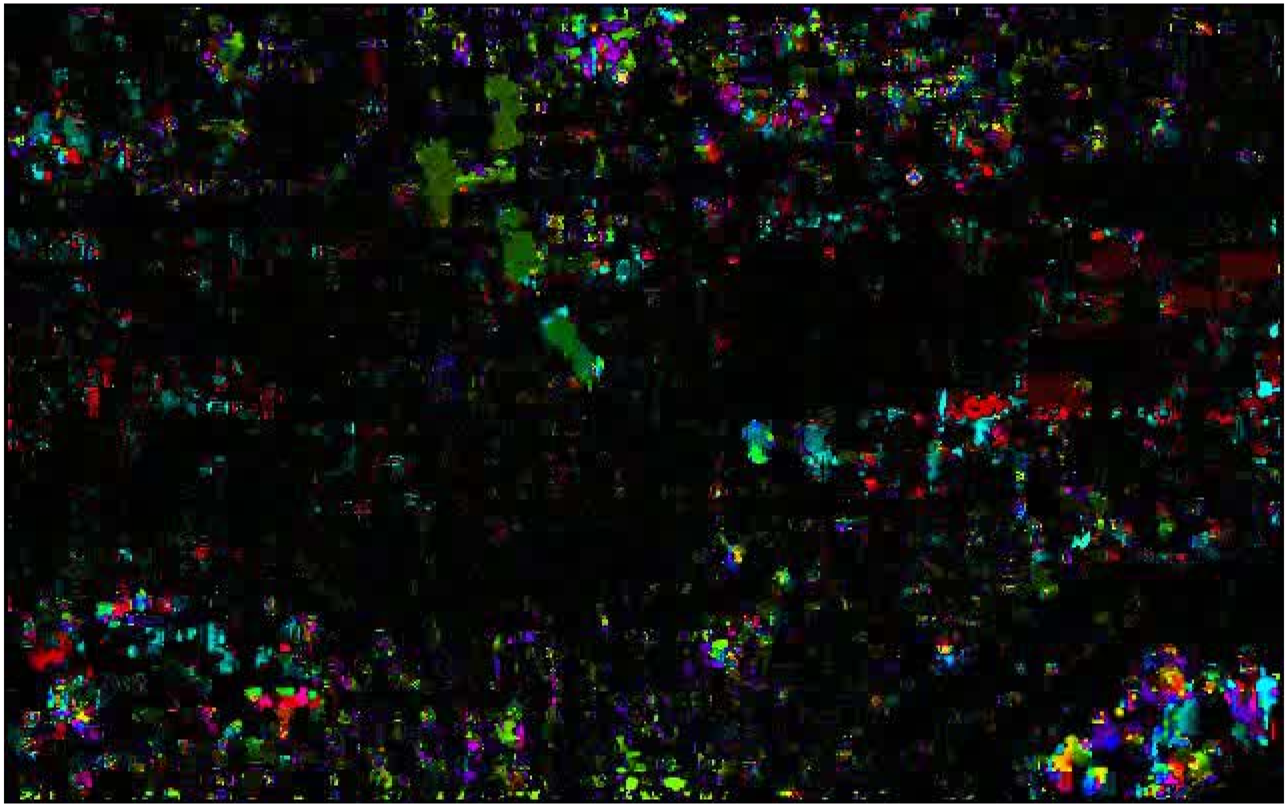


Videos

Color Coded Optical Flows

Optical Flow





Optical Flow

- Applications
  - Motion based segmentation
  - Structure from Motion(3D shape and Motion)
  - Alignment (Global motion compensation)
    - Camcorder video stabilization
    - UAV Video Analysis
  - Video Compression

Horn&Schunck Optical Flow

Brightness constancy assumption

$$f(x, y, t) = f(x + dx, y + dy, t + dt)$$

Taylor Series

$$f(x, y, t) = f(x, y, t) + \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial t}dt$$

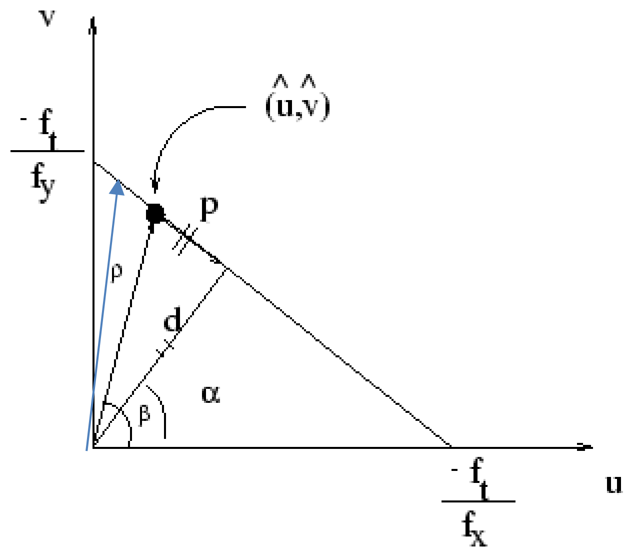
$$f_x dx + f_y dy + f_t dt = 0$$

$$f_x u + f_y v + f_t = 0$$

Interpretation of optical flow eq

$$f_x u + f_y v + f_t = 0$$

$$v = -\frac{f_x}{f_y}u - \frac{f_t}{f_y}$$



d=normal flow

p=parallel flow

$$d = \frac{f_t}{\sqrt{f_x^2 + f_y^2}}$$

Equation of st.line

Horn&Schunck (contd)

$$\iint \{(f_x u + f_y v + f_t)^2 + \lambda(u_x^2 + u_y^2 + v_x^2 + v_y^2)\}dxdy$$

Brightness constancy

Smoothness constraint

min

$$(f_x u + f_y v + f_t) f_x + \lambda(\Delta^2 u) = 0$$

$$(f_x u + f_y v + f_t) f_y + \lambda((\Delta^2 v) = 0$$

$$\Delta^2 u = u_{xx} + u_{yy}$$

Derivative Masks (Roberts)

$$\begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix} \text{first image}$$

$$\begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix} \text{second image}$$

f_x

$$\begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix} \text{first image}$$

$$\begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix} \text{second image}$$

f_y

$$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \text{first image}$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \text{second image}$$

f_t

Apply first mask to 1st image
Second mask to 2nd image
Add the responses to get *f_x*, *f_y*, *f_t*

Laplacian

$$\begin{matrix} 0 & -\frac{1}{4} & 0 \\ -\frac{1}{4} & 1 & -\frac{1}{4} \\ 0 & -\frac{1}{4} & 0 \end{matrix}$$

$$f_{xx} + f_{yy}$$

$$f_{xx} + f_{yy} = f - f_{av}$$

Horn&Schunck (contd)

$$\int\int\{(f_x u + f_y v + f_t)^2 + \lambda(u_x^2 + u_y^2 + v_x^2 + v_y^2)\}dxdy$$

min

$$(f_x u + f_y v + f_t) f_x + \lambda(\Delta^2 u) = 0$$

$$(f_x u + f_y v + f_t) f_y + \lambda((\Delta^2 v) = 0$$

discrete version

$$(f_x u + f_y v + f_t) f_x + \lambda(u - u_{av}) = 0$$

$$(f_x u + f_y v + f_t) f_y + \lambda((v - v_{av}) = 0$$

variational calculus

$$u = u_{av} - f_x \frac{P}{D}$$

$$v = v_{av} - f_y \frac{P}{D}$$

$$P = f_x u_{av} + f_y v_{av} + f_t$$

$$D = \lambda + f_x^2 + f_y^2$$

$$\Delta^2 u = u_{xx} + u_{yy}$$

Algorithm-1

- k=0
- Initialize $u^K \quad v^K$
- Repeat until some error measure is satisfied (converges)

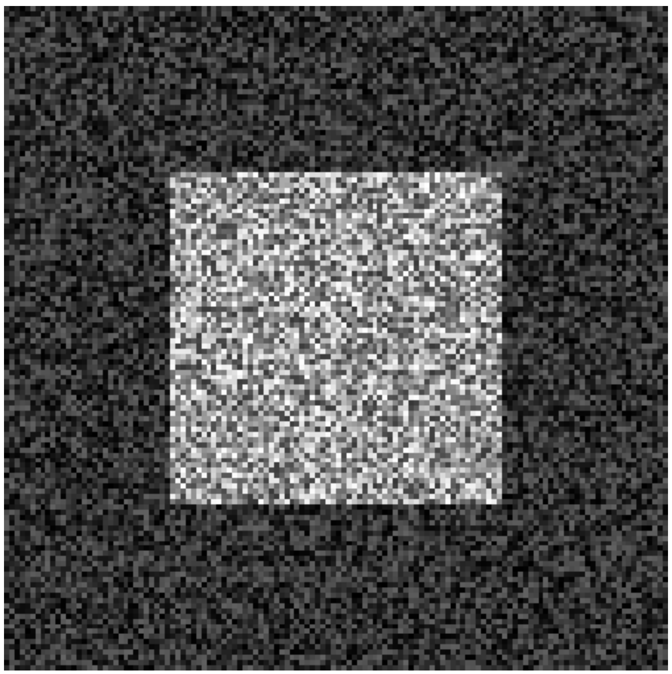
$$u = u_{av} - f_x \frac{P}{D}$$

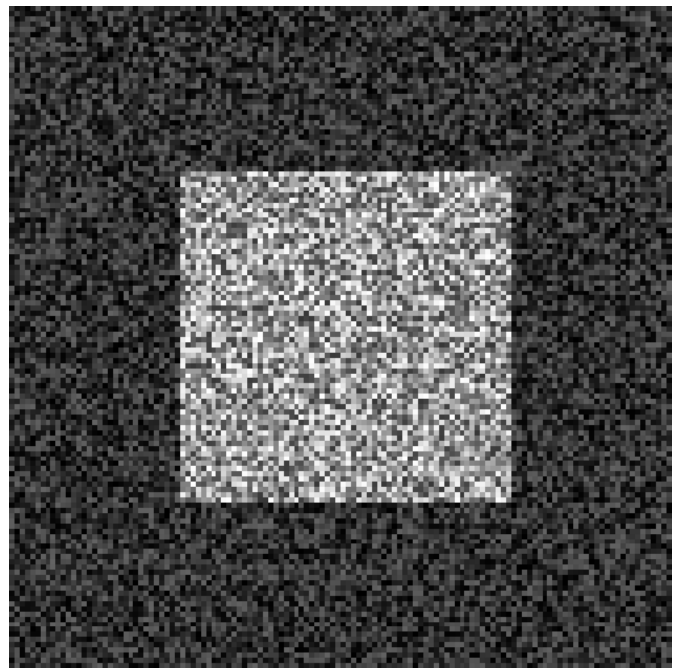
$$v = v_{av} - f_y \frac{P}{D}$$

$$P = f_x u_{av} + f_y v_{av} + f_t$$

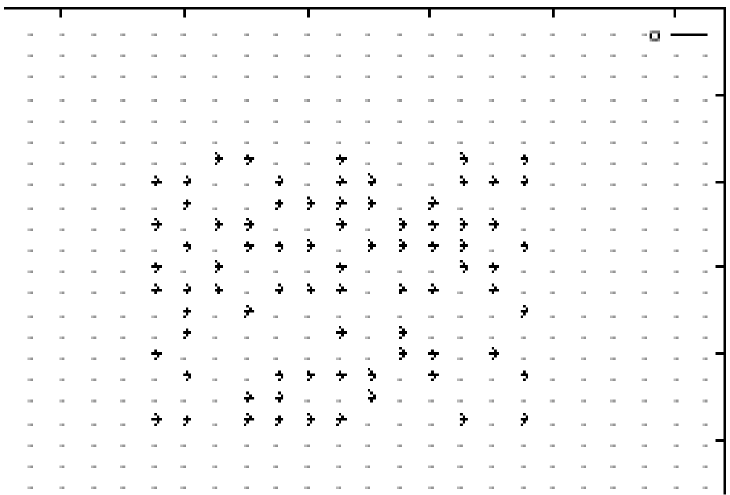
$$D = \lambda + f_x^2 + f_y^2$$

Synthetic Images

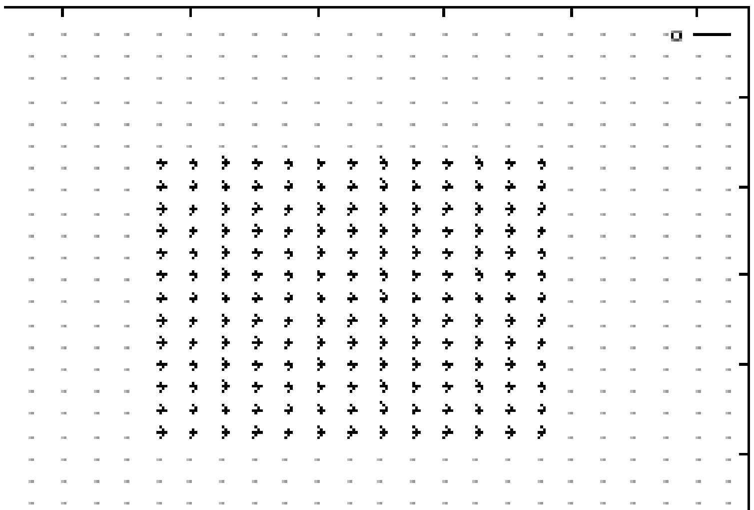




Horn & Schunck Results



One iteration



10 iterations

Lucas & Kanade (Least Squares)

- Optical flow eq

$$f_x u + f_y v = -f_t$$

- Consider 3 by 3 window

$$f_{x1} u + f_{y1} v = -f_{t1}$$

$$\vdots$$

$$f_{x9} u + f_{y9} v = -f_{t9}$$

$$\begin{bmatrix} f_{x1} & f_{y1} \\ \vdots & \vdots \\ & \\ f_{x9} & f_{y9} \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} -f_{t1} \\ \vdots \\ \\ -f_{t9} \end{bmatrix}$$

$$\mathbf{Au} = \mathbf{f_t}$$

Lucas & Kanade

$$\mathbf{Au} = \mathbf{f_t}$$

$$\mathbf{A^T A u} = \mathbf{A^T f_t}$$

$$\mathbf{u} = \boxed{(\mathbf{A^T A})^{-1}\mathbf{A^T}}\mathbf{f_t}$$

Pseudo Inverse

$$\min \sum_i (f_{xi}u + f_{yi}v + f_t)^2$$

Least Squares Fit

Lucas & Kanade

$$\min \sum_i (f_{xi}u + f_{yi}v + f_t)^2$$

$$\sum (f_{xi}u + f_{yi}v + f_{ti}) f_{xi} = 0$$

$$\sum (f_{xi}u + f_{yi}v + f_{ti}) f_{yi} = 0$$

Lucas & Kanade

$$\sum (f_{xi}u + f_{yi}v + f_{ti})f_{xi} = 0$$

$$\sum (f_{xi}u + f_{yi}v + f_{ti})f_{yi} = 0$$

$$\begin{aligned}\sum f_{xi}^2 u + \sum f_{xi} f_{yi} v &= -\sum f_{xi} f_{ti} \\ \sum f_{xi} f_{yi} u + \sum f_{yi}^2 v &= -\sum f_{yi} f_{ti}\end{aligned}$$

$$\boxed{\begin{bmatrix} \sum f_{xi}^2 & \sum f_{xi} f_{yi} \\ \sum f_{xi} f_{yi} & \sum f_{yi}^2 \end{bmatrix}} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} -\sum f_{xi} f_{ti} \\ -\sum f_{yi} f_{ti} \end{bmatrix}$$

Lucas & Kanade

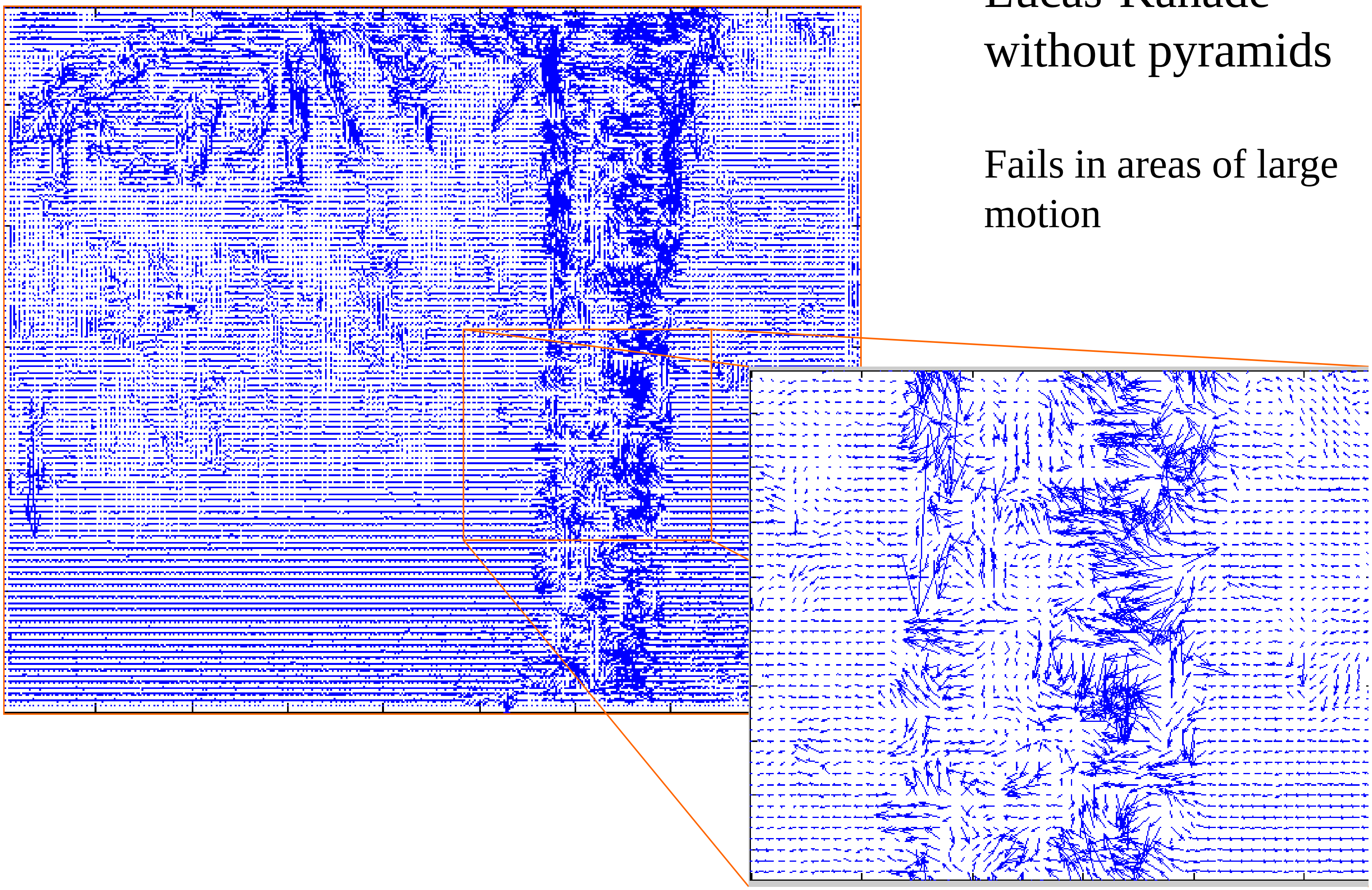
$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \sum f_{xi}^2 & \sum f_{xi} f_{yi} \\ \sum f_{xi} f_{yi} & \sum f_{yi}^2 \end{bmatrix}^{-1} \begin{bmatrix} -\sum f_{xi} f_{ti} \\ -\sum f_{yi} f_{ti} \end{bmatrix}$$

$$\begin{bmatrix} u \\ v \end{bmatrix} = \frac{1}{\sum f_{xi}^2 \sum f_{yi}^2 - (\sum f_{xi} f_{yi})^2} \begin{bmatrix} \sum f_{yi}^2 & -\sum f_{xi} f_{yi} \\ -\sum f_{xi} f_{yi} & \sum f_{xi}^2 \end{bmatrix} \begin{bmatrix} -\sum f_{xi} f_{ti} \\ -\sum f_{yi} f_{ti} \end{bmatrix}$$

Lucas & Kanade

$$u = \frac{-\sum f_{yi}^2 \sum f_{xi} f_{ti} + \sum f_{xi} f_{yi} \sum f_{yi} f_{ti}}{\sum f_{xi}^2 \sum f_{yi}^2 - (\sum f_{xi} f_{yi})^2}$$

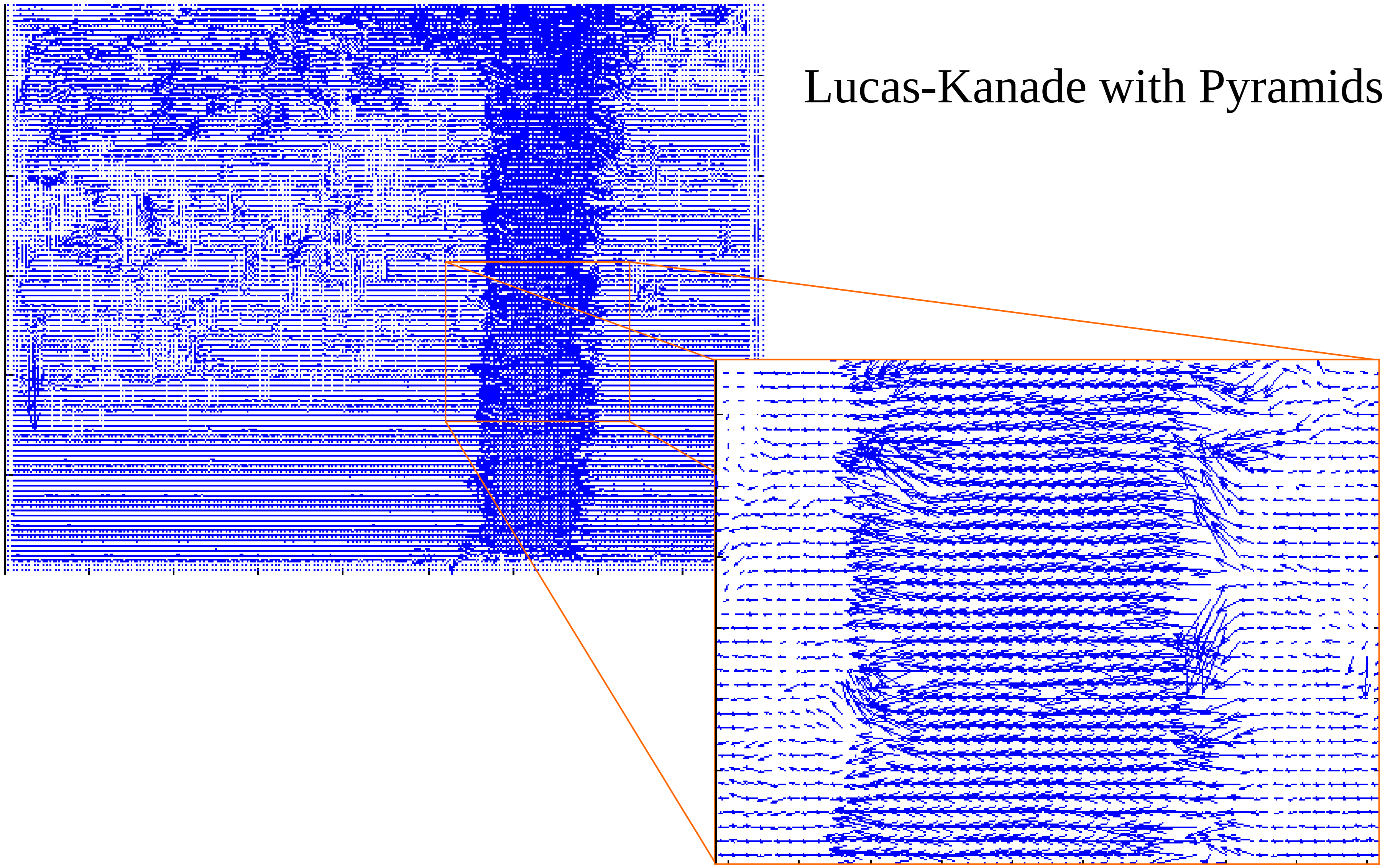
$$v = \frac{\sum f_{xi} f_{ti} \sum f_{xi} f_{yi} - \sum f_{xi}^2 \sum f_{yi} f_{ti}}{\sum f_{xi}^2 \sum f_{yi}^2 - (\sum f_{xi} f_{yi})^2}$$



Lucas-Kanade without pyramids

Fails in areas of large motion

Lucas-Kanade with Pyramids



Comments

- Horn-Schunck and Lucas-Kanade optical methods work only for small motion.
- If object moves faster, the brightness changes rapidly,
  - 2x2 or 3x3 masks fail to estimate spatiotemporal derivatives.
- Pyramids can be used to compute large optical flow vectors.