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Using the Process of Norm Emergence to Model Consensus Formation

Christopher D. Hollander
Department of Modeling and Simulation
University of Central Florida
Orlando, USA
chris.hollander@gmail.com

Annie S. Wu
Department of Elec. Eng. and Comp. Sci.
University of Central Florida
Orlando, USA
aswu@cs.ucf.edu

***Abstract*—Every agent in a society initially possesses a set of personal norms. *Group norms* emerge when agents interact with one another and exchange information in such a way that multiple agents begin to acquire the same personal norm. This emergence is the result of information transmission, social enforcement, and internalization. If a population contains a single group norm, as a result of every agent in the population acquiring the same personal norm, then it can be said that a consensus has been reached by the population. We model the formation of consensus *in silico* by adapting a recently developed model of norm emergence to a multi-agent simulation. A screening experiment is conducted to identify the significant parameters of our model and verify that our model is capable of producing a consensus. The experimental results show that our model can attain consensus as well as two additional states of information equilibrium. The results also indicate that both network structure and agent behavior play an important role in the formation of consensus. In addition, it is shown that the formation of consensus is sensitive to the simulation parameter settings, and certain values can prevent its formation entirely.**

***Keywords*-Emergence; Self-Organization; Norms; Computational Social Science; Complex Systems; Agent-Based Model; Simulation**

I. Introduction

The computational study of norms is a subdomain of computational social science [1]–[3]. Current research on the computational study of norms is largely concerned with normative systems [**?**], [4]–[7] and multi-agent models. Though this area is still young, early results have seen norms and normative systems successfully applied to many areas of computer science and sociological theory [7], [8].

In previous work [7], we constructed a theoretical model of the norm life-cycle as an evolutionary process. This model aggregates many of the discoveries and ideas from the past decade of norms research. Personal norms are created by agents in response to conflict. Once created, personal norms spread from agent to agent via transmission processes until a group norm emerges. As agents are exposed to the personal norms of others, a decision is made whether or not to adopt the new norm. This decision depends on internal and external pressures. If the decision is made to adopt a new norm, then the agent goes through a social learning process that realigns its preferences. When a sufficient number of agents possess the same personal norm, then it is said that a group norm has emerged. Over time, personal norms evolve as conflicts change. This leads to a shift in the group norms.

The portion of the norm life-cycle in which group norms emerge from the spreading and internalization of personal norms is called *norm emergence* [**?**], [4]–[6], [9]–[12]. Because the process of norm emergence is a process of information spread, it is conceivable that it can be used to solve the *consensus problem*. The consensus problem poses the question of how consensus can be achieved in asymptotic time given an initial set of information states and a network topology that models an agent interaction network [13]–[15]. The answer to this question has serious implications to multi-agent coordination [16]–[18], when many agents need to possess the same information if the system is to operate properly.

Our current research uses the emergence portion of our life-cycle model to investigate the emergence of *group norms* as a possible model of consensus formation. *Consensus* occurs with every agent in a multi-agent system possesses the same information state [15]. Like norms, consensus can be thought of as emerging from a population of individuals as a result of social interactions and information exchange. If norm emergence can produce consensus, then the components of norm emergence can be used to develop *consensus algorithms* (solutions to the consensus problem). Norm emergence based consensus algorithms may provide better solutions than traditional consensus algorithms, such as the average consensus algorithm [13], [14], under circumstances that require a great deal of social interaction. However, in order for our model to have real world applications, it must first be verified and validated to ensure that the output is both technically and practically correct.

This paper discusses the initial results of our on-going investigation. In particular, it presents verification that our model is sufficient to produce a consensus. This verification is accomplished by first implementing the model *in silico* and then conducting a screening experiment to isolate the relevant factors of the model and gain an initial understanding about the behavior of the parameters. It is shown that under our model, the interactions between social agents can result in the organization of information into multiple equilibrium states, of which consensus is one possible outcome.

We begin by describing our model of group norm emergence. Then, we introduce a multi-agent simulation built on top of the emergence model. Next, we describe a screening experiment on the simulation that is aimed at identifying the significant factors that contribute to the emergence of group norms and consensus formation. Finally, we discuss the results of our experiment and discuss the direction of our future research in this area.

II. A Model for the Emergence of Group Norms

To investigate the formation of consensus via the processes involved in the emergence of group norms, we adapt a model of norm emergence that was developed during our previous research on normative systems [7]. This model was designed after a thorough survey of the existing state of the art, and constructed to account for a number of commonalities between the various models and their supporting theories. Specifically, the emergence of group norms is the result of the *transmission*, *enforcement*, and *internalization* of personal norms over a social network.

Our model consists of multiple agents, each of which is a simple social entity that possesses a set of behaviors and has the capability to store two norms. The behaviors of an agent correspond to the processes of transmission, enforcement, and internalization. At any given time, the norms possessed by an agent can be in either an *internalized state* or a *learning state*. Norms in the internalized state represent information the agent has successfully learned about in the past. Norms in the learning state represent information that is currently being learned through the internalization process. Norms can have their state changed from the learning state to the internalized state, but not vice versa. An agent can only have one norm in the internalized state and one norm in the learning state at any given time.

Each agent is situated in a simple environment represented by a finite two dimensional lattice with three possible types of interaction neighborhoods: *Moore*, *von Neumann*, and *complete*. In a Moore neighborhood, an agent can interact with other agents to the north, south, east, west, northwest, northeast, southwest, and southeast of its current location. In a von Neumann neighborhood, an agent is only able to interact with other agents to the north, south, east, and west of its current location. In a complete neighborhood, each agent is able to interact with every other agent. Each site on the lattice may or may not be populated. Agents do not move. A lattice that is not fully populated may have multiple *clusters* of agents. If the lattice is viewed as a network where edges represent "is a neighbor of" relationships, then a cluster is a set of connected agents. Furthermore, we can refer to the number of neighbors an agent has as the *degree* of that agent. The interpretation of the lattice as a simple network allows certain concepts to be expressed in a clear and concise manner. The choice to use a lattice, as opposed to a random, scale-free, or small world network, was made in an effort to keep things simple for our initial test phase. Subsequent experiments will examine alternative network topologies.

Interaction between agents occurs at a one-to-one ratio and only with an agent's immediate neighbors.

A. Emergence Processes

Transmission is the process in which norms (the *transmission*) are selected and sent from one agent (the *sender*) to another (the *receiver*). The transmitted data is represented as a binary string that is composed of a number of bits equal to the *information length* (IL) parameter.. The transmission process involves the sub processes of *receiver selection* and *transmission selection*.

- *Receiver Selection* is the process where the sender identifies an agent to be the receiver. In the current model, receiver selection is done by selecting one agent in the local neighborhood of the sender from a uniform distribution.
  Let i be any site on the lattice and j be a neighboring site of i. Let $agent_i$ be the sender with N_i neighbors and $agent_j$ be the selected receiver. In the stochastic emergence algorithm, $j \in Uniform(N_i)$.
- *Transmission Selection* is the process where the sender selects the norm that it will transmit to the receiver. This selection process is stochastic. A norm is always transmitted and the selection of the specific norm is based on a parameter called the *learning transmission probability*. The learning transmission probability (LTP) is the probability that an agent will transmit the norm it is current learning, as opposed to the internalized norm. Let $x \in Uniform(1.0)$, then $x \leq LTP \implies$ *the norm being learned is transmitted* and $x > LTP \implies$ *the internalized norm is transmitted*. The learning transmission probability changes over time with $LTP_0 = 0.0$ and $LTP_{t+1} = LTP_t + k \cdot IT + SI$, where $0.0 \leq LTP_t \leq 1.0$ is the learning transmission probability at time t, k is a scaling factor, $IT > 1.0$ is the total time it takes to learn a new norm (the *internalization time*), and $-1.0 \leq SI_t \leq 1.0$ is the impact of sanctions.

Enforcement is the process in which a sender's learning transmission probability is adjusted in response to the most recent norm that was transmitted and the state of the local neighborhood the agent is situated in. Enforcement involves sub-processes that identify the local norm (*local norm identification*), check to see if the sender is in violation of the local norm (*violation checking*), and sanction the sender if a violation is detected (*sanctioning*). In the current model, enforcement is internal.

- *Local Norm Identification* is the process where an agent's local neighborhood is examined and the norm content of the neighborhood is used to create a frequency distribution. Agents are not able to detect the internal state of each other, so the sender must ask each of its neighbors what their internalized norm is. Neighbors are able to lie to the agent, but in the current model it is assumed that they are truthful. The norm with the highest frequency is labeled the *local norm*. If there are multiple states of information with the same frequency then there is no local norm. In the initial mode, agents cannot be sanctioned if there is no local norm.

Let D be the frequency distribution of norms in N_i. Then the local norm is $d = \max(D)$

- *Violation Checking* is the process where a sender's transmission and state are checked against the local norm. If the sender's transmission does not match the local norm, but the norm the sender did not transmit does, then the sender is in violation of the local norm and is sanctioned. However, if the sender's transmission and the norm that the sender did not transmit are both different from the local norm, then the agent is said to be ignorant of the local norm and is not in violation. Ignorance of the local norm means that the agent is incapable of transmitting the proper norm since it does not possess it.

  Let α the internalized norm of the sender, β the norm currently being learned by the sender, and $O \in \alpha, \beta$ be the sender's last transmission, then $(O = \alpha, \alpha \neq d, \beta = d) \wedge (O = \beta, \beta \neq d, \alpha = d) \implies$ *the sender is in violation*, $(O = \alpha, \alpha = d) \wedge (O = \beta, \beta = d) \implies$ *the sender is not in violation*, and $\alpha \neq d, \beta \neq d \implies$ *the sender is ignorant of the local norm*.
- *Sanctioning* is the process in which a sender adjusts its learning transmission probability to favor the current local norm in the next time step. If the agent transmitted its internalized norm when it should have transmitted the norm it is learning, then the learning transmission probability increases. If the agent transmitted the norm it is learning when the internalized norm should have been transmitted, then the learning transmission probability decreases. When sanctioning occurs, adjustments are only made if the sender is in violation. In the current model, the sanctioning process itself is stochastic, with a probability of occurring equal to the *sanctioning probability* (SP) parameter.

  Let $0.0 \leq SP \leq 1.0$ be the sanctioning probability, $0.0 \leq SI < 1.0$ be the *sanction impact*, and $x \in Uniform(1.0)$, then $x \leq SP \implies$ *the sender sanctions itself if a violation is detected*. If a sanction occurs, $(O = \alpha, \alpha \neq d, \beta = d) \implies LTP_{t+1} = LTP_t + SI$ and $(O = \beta, \beta \neq d, \alpha = d) \implies LTP_{t+1} = LTP_t - SI$.

Internalization is the process in which a norm is transferred from one agent to another and transformed from the learning state to the internalized state. When a sender transmits a norm to a receiver, it may be *modified* during transmission. Norms are modified as a result of error due to the sender, the receiver, or the environment. Upon receiving the information about a norm, the receiver makes a decision whether or not to *accept* the norm. If the receiver accepts, it begins to *learn* the norm.

- *Modification* is the process in which a transmission is altered to account for noise. Modification occurs prior to the decision of an agent to accept or reject a transmission. In the current model, modification is treated as a bitwise mutation operator on the transmission string. A *mutation probability* (MP) parameter controls the mutation operator.

  Let $0.0 \leq MP \leq 1.0$ be the mutation probability, O be the original transmission string, O' be the modified transmission, $O_b, O'_b \in 0, 1$ be the bit at position b in the respective transmission string, and $x \in Uniform(1.0)$. Then $x \leq MP, O_b = 0 \implies O'_b = 1$ and $x \leq MP, O_b = 1 \implies O'_b = 0$.
- *Acceptance* is the process where an agent determines whether or not it wants to accept an incoming transmission from a sender. The decision to accept a transmission is probabilistic and based on the *acceptance probability* (AP) parameter; but only if the incoming incoming norm is new to the agent. Incoming norms that are already known to the agent are automatically rejected. In the current model, acceptance of a new norm implies that the norm currently being learned is forgotten.

  Let $0.0 \leq AP \leq 1.0$ be the acceptance probability and $x \in Uniform(1.0)$, then $x \leq AP \implies$ *the incoming norm is accepted.*
- *Learning* is the process in which the learning transmission probability is adjusted independent of enforcement. In the current model, this adjustment is given by $LTP_{t+1} = LTP_t + \frac{1}{IT}$. Once the learning transmission probability exceeds 1.0, the internalized norm is forgotten and replaced by the norm that was being learned. The learning transmission probability is then reset to 0.0 and the agent is free to begin learning a new norm. When an agent is not learning a new norm, it will always transmit its internalized norm. The learning process is the only way that an agent can replace its internalized norm; though the norm being learned is replaced whenever an agent accepts a new norm.

The decision to use a stochastic approach for this initial model comes from the idea that all events can be translated into a set of probabilities. This allows the emergence model to be as general as possible, while still proving a framework that can be used to create more specific algorithms by replacing the stochastic elements with deterministic ones. Future experiments will examine the impact of more realistic transmission, enforcement, and learning processes.

III. Norm Emergence Simulation

The Norm Emergence Simulation (NESIM) is a multi-agent simulation that enables us to investigate whether or not our model is sufficient to allow the formation of consensus. For the current experiment, the model parameters are homogeneous and the initial norm that is assigned to each agent is heterogeneous. The parameters are homogeneous so that we can maintain experimental control in order to systematically verify whether or not the formation of consensus is possible. The norms are heterogeneous because consensus occurs when all agents possess the same internalized norm. To detect consensus and the formation of group norms, the simulation is engineered to check for multiple equilibrium states. The

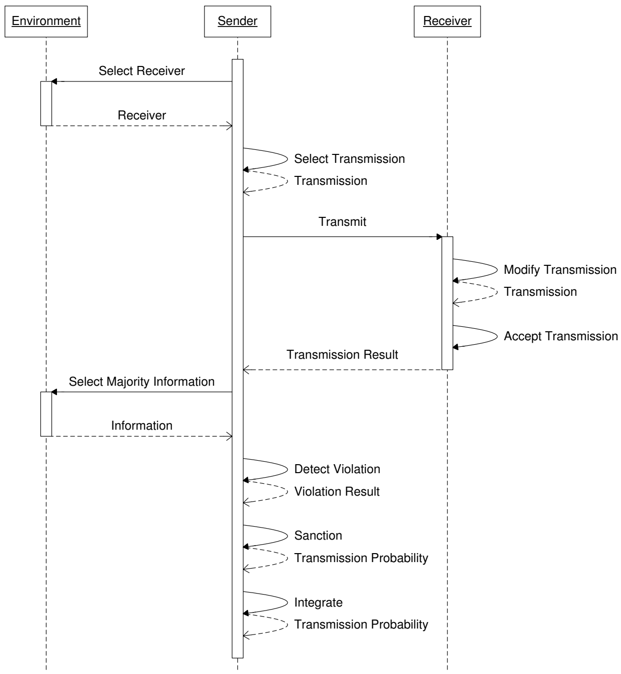


Fig. 1. NESIM process execution sequence

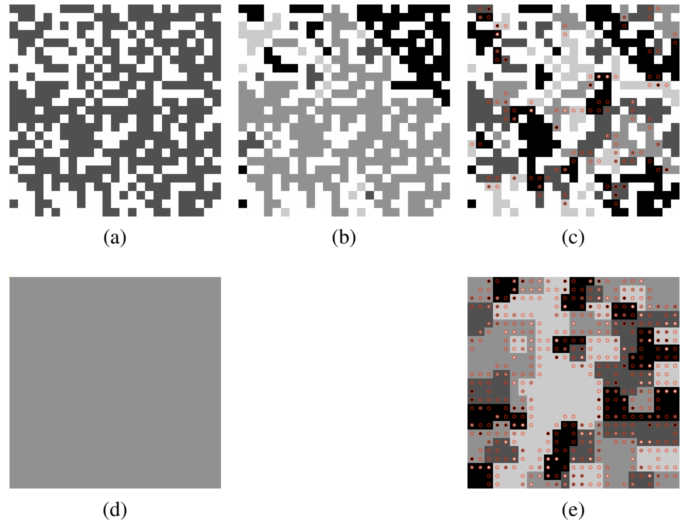


Fig. 2. Stable equilibrium states on a 25x25 von Neumann lattice with four possible norms. The stability tolerance is 100 steps. Agents are represented by grayscale squares that indicate their internalized norm. White space denotes an empty lattice site. Circles over an agent represent a norm being learned by that agent. Panels a, b, and c have an agent density of 0.6. Panels d and e have an agent density of 1.0. (a) and (d) are an example of global convergence. The same norm is internalized by all agents in the population. (b) is an example of local convergence. Each cluster has converged to its own norm. (c) and (e) are an example of dynamic equilibrium. No agent has changed its internalized norm in over 100 time steps, but each agent continues to try and (unsuccessfully) learn new norms.

presence of these states, along with the number of steps executed by the simulation, make up the simulation outputs.

A. Simulation Overview

In NESIM, every agent executes the processes of transmission, enforcement, and internalization in a sequential order (Figure 1), once per time step, until one of two halting conditions is reached: either the model enters an equilibrium state, at which point group norms may emerge (Figure 2), or the simulation executes more than 21,000 time steps[1]. Consensus occurs only when the agent population contains a single group norm. An agent's norm is indicated by its grayscale coloring in simulation space[2].

The simulation is able to recognize three states of stable equilibrium: *global convergence*, *local convergence*, and *dynamic equilibrium*. Cycles are also possible, but the simulation does not detect them.

- *Global convergence* (Figure 2a, 2d) occurs when every agent in the population has the same internalized norm, and, if the agent is learning a new norm, that norm is equal to the internalized norm (So that even if learning is successful, it will not change an agent's internalized norm). Additionally, the internalized norm state of every agent must have remained constant for a number of time steps greater than the *stability tolerance*, where the stability tolerance is a simulation parameter.
- *Local convergence* (Figure 2b) occurs when every agent in a cluster has the same internalized norm, and, if the agent is learning a new norm, that norm is equal to the internalized norm. Additionally, the internalized norm of every agent must have remained constant for a number of time steps greater than the stability tolerance. Local convergence can only occur on a disconnected network where agents are broken into multiple clusters. The recognition of local convergence signifies that while that entire population has not converged to a single norm, each cluster has.
- *Dynamic Equilibrium* (Figure 2c, 2e) occurs when the internalized norm of every agent in the simulation has remained constant and unchanged for a length of time greater than the stability tolerance, but the norm being learned is not considered. The appearance of a dynamic equilibrium implies that the norms of the agents in the model have become organized, but it does not imply that local or global convergence has been achieved. Because dynamic equilibrium does not require agents to be finished learning, it is possible to have multiple norms in the same cluster. The existence of multiple norms is an important sign of theoretical soundness because it matches the results observed by other researchers [19]–

[1]One time step elapses after every agent in the simulation has run and all of the statistics for the current step have been collected. The value 21,000 was chosen because initial runs showed that emergence tends to happen in the first 10,000 time steps.

[2]For this current experiment, norms only control the color of an agent. As such, each norm can be thought of as specifying a particular type of clothing to be worn.

[21].

Equilibrium states may co-exist with one another. When the model has reached a state of global convergence, the conditions for local convergence and dynamic equilibrium are also met. When the model has reached a state of local convergence, the conditions for dynamic equilibrium are also met. However, local convergence does not imply that there is also global convergence. When dynamic equilibrium is reached, nothing can be implied about the possibility of global or local convergence.

The spread of a norm is measured as a percentage of the population. Consensus can only occur when this value reaches 100%. However, as a model of norm emergence, a norm becomes a group norm when the measure of spread exceeds a simulation parameter called the *emergence threshold*. Group norms may emerge from any of the three equilibrium conditions, and when the emergence threshold is set to a low value (below 0.5), it is possible for multiple group norms to exist. In addition to determining when all agents have internalized the same norm, the emergence threshold provides a way to measure the type and amount of deviance in a population. However, it should be stressed that this is a simulation-level measure. The agents themselves currently have no way to know when they are obeying a group norm versus a deviant norm.

B. Simulation Parameters and Experimental Factors

In order to verify that our model is capable of reaching an equilibrium state and producing a consensus, we test the significance of 10 factors that should directly influence the simulation output (Table I). We will refer to these 10 parameters as "experimental factors," or "factors", in order to be consistent with the statistical literature. These 10 factors can be split into two categories, environmental and behavioral. Environmental factors specify the structure of the agent environment, including the interaction network and population size. Behavioral factors control the actions executed by the agent in each time step. Testing the factors related to environment and agent behavior simultaneously allows me to identify the interactions that may exist between and within the two groups.

The environmental factors consist of the:

- *Information Length* (IL) specifies the number of distinct information states that a transmission can represent.
- *Grid Width* and *Grid Height* determine the horizontal and vertical dimensions of the environment and make up the *grid size* (GS).
- *Edge behavior* (EB) determines whether or not the lattice wraps from one side to another to form a torus.
- *Neighborhood Type* (NT) specifies the degree of the local neighborhood. The current version of NESIM allows for *von Neumann neighborhoods*, *Moore neighborhoods*, and *Complete neighborhoods*.
- *Agent Density* (AD) specifies the ratio of populated to unpopulated sites on the lattice. The higher this value the larger the population size will be.

TABLE I
PARAMETERS OF THE NORM EMERGENCE SIMULATION

Symbol	Range	Low Value	High Value
GS	$[3, \infty)$	5	10
EB	$\{Cut, Wrap\}$	Cut	$Wrap$
IL	$[1, \infty)$	1	3
NT	$\{vonNeuman,$ $Moore,$ $Complete\}$	$vonNeumann$	$Complete$
AD	$[0.0, 1.0]$	0.4	1.0
AP	$[0.0, 1.0]$	0.05	1.0
MP	$[0.0, 1.0]$	0.0	0.05
IT	$[1.0, \infty)$	1.0	10.0
SP	$[0.0, 1.0]$	0.0	1.0
SI	$[-1.0, 1.0]$	0.05	1.0

The behavioral factors consist of the:

- *Acceptance Probability* (AP) specifies the probability that an agent acting as a receiver will accept an incoming transmission.
- *Modification Probability* (MP) specifies the probability that a transmission will be modified before the receiver decides whether or not to accept it.
- *Internalization Time* (IT) specifies the total number of time steps that it takes an agent to learn new information.
- *Sanctioning Probability* (SP) specifies the probability that an agent will be sanctioned if it violates the local norm.
- *Sanction Impact* (SI) specifies the amount that the learning transmission probability will be modified by if an agent is sanctioned.

The range of valid values for each parameter can be found in Table I.

C. Response Variables

In the statistical literature, a response variable is the output of a system. For the experiments discussed in this paper, we are interested in the effect of the environmental and behavioral factors on four response variables: global convergence, local convergence, dynamic equilibrium and *halting time*.

- Global convergence, local convergence, and dynamic equilibrium are conditions on the norm frequencies of the simulation. Global convergence implies that a single personal norm has been internalized by every agent in the population. Local convergence and dynamic equilibrium imply that more than one personal norm has been internalized by the population.
- *Halting time* measures the number of time steps it takes for the simulation to reach a state of equilibrium. If an equilibrium state is not reached within 21,000 time steps, then we consider the associated factor permutation unable to converge.

For all but the halting time, the response variables are recorded as either a 0 or a 1. This allows each response variable to be averaged over multiple replications so that a probability can be determined for each specific set of factor values.

IV. Experimental Design

The experiment described in this paper is intended to identify and explore the basic dynamics of our current model of norm emergence. The primary hypotheses of this experiment is that in the norm emergence simulation, one personal norm to spread to every agent, resulting in a pattern of internalization that can be classified as consensus. Furthermore, there will be parameter configurations that allow deviant agents to resist the adoption of the personal norms of others. Because of this, consensus formation will not always be possible; it will depend on the initial conditions of the emergence algorithm.

Because this experiment is focused on proving the existence of consensus behavior in a model of norm emergence, it is conducted as a screening experiment backed by a factorial design [22]–[24]. In a factorial design, the model inputs are referred to as *factors*, and their specific values are called *levels*. The output of the model is a *response*, and a *response variable* refers to a variable that holds the output, such as a flag representing the appearance of global convergence. The purpose of using a screening experiment based on a factorial design is to identify the measure of significance that each factor has on each response variable, so that the model can be optimized and insignificant components removed. A factor is labeled as statistically significant if an ANOVA analysis assigns it a p-value less than some desired threshold. However, statistical significance does not imply practical significance. The specific design used in this initial experiment is a 1/8 fractional factorial design on 10 factors with 2 levels per factor and full fold-over. This yields a resolution VI design with a total of 256 factor permutations. There are 5 replications at each factor permutation. This produces a design matrix that allows for a total of 1280 data points per response variable. Because of the nuances that appear in constructed simulations [24], [25] only the effects that have a p-value less than 0.1 and a normalized effect value (in the range of $[0, 1]$) higher than 0.05 are considered significant. The analysis of the data generated from the simulation is done using Minitab 16.1.0.

The use of a factorial design allows us to easily explore the parameter space of our model by sampling a subset of all possible solutions that can be obtained by permuting the inputs between their high and low levels (Table I). Additionally, because the inputs are varied one by one, using a factorial design allows the significance of both the first-order (main effects) and second-order effects (interaction effects) to be measured with regard to a particular response variable. The significance of an input is critical to understanding the dynamics of the system.

V. Experimental Results and Analysis

The experiment described in this paper is intended to identify and explore the basic dynamics of our current model of norm emergence. In order to do this, a multi-agent simulation is created to implement the emergence model. The primary purpose of this simulation is to generate data that can be used to understand the dynamics of the emergence of group norms. To generate the required data, a screening experiment is conducted on the norm emergence simulation in order to identify which parameters have a significant effect on the response variables that represent equilibrium states of global convergence, local convergence, and dynamic equilibrium, as well as the halting time. Because social models are substantially different from the industrial models found in the engineering disciplines [24], [25], the analysis of the experimental results focuses on the magnitude of the effects more so than the p-values. This approach is taken because, given enough replications, all factors in a simulation model should appear statistically significant [25]. In particular, emphasis is placed on effects that have a p-value less than 0.10 and an effects magnitude larger than 5%. The experimental results discussed in this section depend on the assumption that the agent behaviors (AP, MP, IT, SP, and SI) are homogeneous. Future experiments will need to be conducted to consider the heterogeneous case, but we suspect that the results will be similar.

We examined the first and second-order interactions between the experimental factors and identified multiple significant third-order interactions. Unfortunately, the software we used for analysis (Minitab) was not able to handle all of the data that the simulation produced. As a consequence, we are not confident in the results for these third-order interactions. However, many of the third-order interactions we examined appear to involve the same effects as the second-order interactions, so the identification of significant factors is not affected. We did not examine forth and higher order interactions for the current analysis, but because of aliasing they do exhibit a small impact on our results. The range of parameters should be reduced if future experiments are to conduct a more in-depth parameter exploration.

A. Global Convergence

Global convergence occurs when a single norm is internalized by every agent in the population (figure 2a, 2d). The experimental results indicate that the modification probability is the most significant factor with regards to this response variable. This is to be expected since high values of the modification probability increase the chance that transmissions will be modified. If transmissions are modified often enough, then the population will be unable to converge to a single information state because they will constantly change the norm they are trying to learn before the learning process completes, and learn nothing as a result.

The effect levels of the neighborhood type, agent density, and grid size suggest that the environment also plays an important role in determining whether or not the simulation will reach a state of global emergence. Specifically, if the underlying network is disconnected it is impossible for agents to transmit across clusters, and so it is only by chance that global emergence will occur. This is in agreement with results from both consensus theory [13], [14] and percolation theory [26].

The significant interactions detected by the screening experiment also support the idea that modification and the environment are largely responsible for global convergence. The internalization time and sanctioning probability also have an effect, but they are small relative to the environmental factors. These results suggest that it would be interesting to run additional experiments looking at the significance of agent behaviors when the environmental factors are held constant.

The acceptance probability, sanction impact and information length do not appear to have a significant effect on global emergence.

B. Local Convergence

Local convergence occurs when every cluster is taken over by a single norm (figure 2b). The network structure has the most significant effect on the appearance of local emergence. This is not surprising since connected networks should yield global emergence over local emergence. As with global emergence, the modification probability also has an effect on local emergence; though to a much lesser degree.

If we consider the combined data from the global and local convergence responses, it appears that as long as the modification probability and internalization time are low, the simulation will eventually reach an equilibrium state. If the underlying network is sufficiently disconnected, then the equilibrium state will be local convergence. If the underlying network is connected, then the equilibrium state will be global convergence. However, if agents are unable to change their internalized norms because they are never able to finish learning a new norm (due to high values of internalization time), then the equilibrium state will be neither global convergence nor local convergence. It will result in a dynamic equilibrium.

C. Dynamic Equilibrium

Dynamic equilibrium occurs when all agents in the system continue to learn, but are unable to finish the internalization process. This produces a geometric clustering of internalized norms (figure 2c, 2e), but the prevents the system from converging either globally or locally. The internalization time is the most significant factor for dynamic equilibrium. This is because agents with a high internalization time are insulated from changing their internalized norm. In order for the internalized norm of an agent to change, the learning transmission probability must reach 1.0. The larger the internalization time, the longer this takes. If an agent is constantly accepting new norms, then it will be never able to finish learning. This situation is most likely to occur on the border of two clusters with different internalized norms. Dynamic equilibrium results because no border agent changes its internalized norm, but at least one agent is always trying to learn a new one. The significance of the sanctioning probability and sanction impact supports this line of reasoning, as both factors work in conjunction with the internalization time to adjust the learning transmission probability.

Unlike with global or local convergence, dynamic equilibrium is primarily dependent on the behavior of the agents and not the environment, although the network structure cannot be ignored completely since it affects the shape of the intra-cluster norm borders. In particular, the acceptance probability is fairly significant for dynamic equilibrium. This is because low values give agents time to finish internalizing a norm, where as high values result in greater degrees of information exchange and more frequent resetting of the learning process.

One particular feature we noticed in the raw data was that the internalization time is at the high value in every single occurrence of a dynamic equilibrium. This indicates that non-instantaneous learning is necessary to produce this specific equilibrium state.

D. Halting Time

The significant main effects suggest that the agent behavior plays the primary role in determining how long it will take for an equilibrium state to be reached. In particular, the modification probability is the primary factor, with a high value resulting in a longer emergence time. The internalization time, sanctioning probability, and sanction impact are able to counter the effects of modification, but only within a currently unknown boundary. In addition, the acceptance probability acts as a time-scaling factor and helps stabilize agent learning.

VI. Discussion

This experiment identified and explored the basic dynamics of our model of norm emergence. The primary hypotheses of this experiment was that the norm emergence simulation, driven by our model, would allow one norm to spread to every agent, resulting in a pattern of internalization that resembles consensus. Furthermore, it was assumed that there would be parameter configurations that allowed deviant agents to resist the adoption of the norms of others. Because of this, consensus formation would not always be possible; it would depend on the initial conditions of the emergence algorithm.

The results of this experiment confirm the correctness of both hypothesis. The data shows that by varying the initial conditions, our emergence model can produce a consensus. In addition, the results suggest that the behavioral parameters may have a larger impact on the ability of norms to spread than the underlying network topology; however the this may not hold true for non-lattice networks. Furthermore, because the simulation was able to reach a dynamic equilibrium, we suggest that our emergence model is also capable of producing solutions in which multiple group norms can co-exist. If each group is the considered independently, it can be said that our model produces multiple consensus, or allows the self-organization of a population into multiple teams. Taken together, all of the experimental results paint a tapestry that describes the basic dynamics of the norm emergence simulation and its underlying stochastic emergence model. They also suggest that further experimentation and refinements are required before emergence can be proved to be a solution to the consensus problem.

A. Basic Model Dynamics

The modification probability is the key factor in determining whether or not the simulation will reach an equilibrium state;

low values of modification can produce cycles of convergence, where the internalized norms are unstable and change over time from one state to another. High values of the modification probability prevent any equilibrium state from being reached. Further experiments are required to detect the boundaries of this behavior, and identify where any tipping points are that switch the system from convergent to divergent.

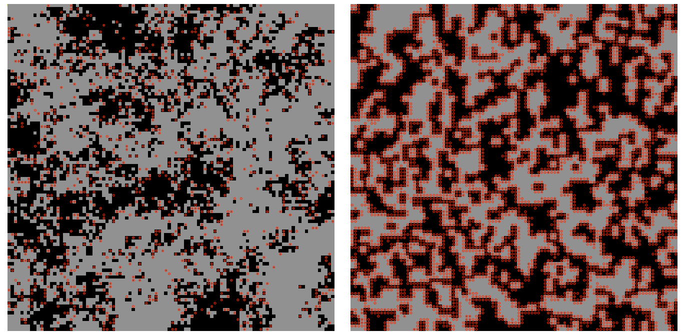
One way to counter the behavior of modification is to use large values for the internalization time. Using large values for the internalization time is shown to reduce, and even eliminate, the effects of modification. Although, when the internalization time of the agents is so high that learning cannot be accomplished (because the learning transmission probability is constantly being reset), then the model is forced into dynamic equilibrium.

The internalization time also has an impact on the visual patterns that appear in the simulation. Values larger than 1.0 cause well-formed clusters of internalized norms to appear with smooth continuous borders, as seen in Figure 3. We believe that this occurs because, in cases of non-instantaneous learning, agents with the same norms are able to reinforce one another and easily replace new norms before they can be learned.

Sanctions can help agents overcome long internalization times by moving their learning transmission probabilities closer to the 1.0 level required to complete learning, but only if the sanctioning impact is sufficiently high. The real impact of sanctions is that they move the norms of agents closer to the local norm, thereby causing intra-cluster formations of homogeneous norms. This assists in the self-organization of the system.

The acceptance probability appears to control the speed at which norms can spread. Low values give agents more time to try and learn new norms without being disrupted, while high values can cause the rapid back and forth transmission of norms that prevents learning.

The neighborhood type, agent density, and edge behavior control how connected the agents are. The neighborhood type and edge behavior determine the degree of the agents. The agent density controls the clustering of the agent population. It appears that the more connected the underlying lattice is, the higher the chance for convergence if the mutation probability is low. This behavior is is expected in light of the findings from percolation theory [26]. The connectivity of the agents also determines which equilibrium state will emerge (Figure 2). When agents are highly connected such that there is only a single cluster, the simulation will halt in a state of either global convergence or dynamic equilibrium. The simulation can only halt in a state of local convergence when the agents form multiple clusters. It is particularly odd that the network structure does not appear to significantly affect the halting time, but this may be a consequence of the level choices for the factors or the result of using a lattice and not examining a wider variety of structures such as rings and power-law networks. As a comparison, some experiments comparing small world to lattice networks [13] have shown that norms converge much faster on small world networks. Other's [6], [27] have also found that the topology has an impact on the time it takes for a norm to emergence.



(a) (b)

Fig. 3. The effect of different internalization times after 500 steps on a 100x100 cut von Neumann lattice with two possible norms (black or gray) using a random seed of 0. Squares represent internalized norms, circles represent norms being learned. (a) has an internalization time of 1.0. (b) uses an internalization time of 10.0.

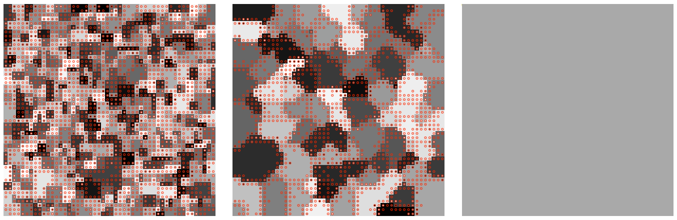
The number of norms initially in the system does not have a practically significant impact on any of the equilibrium state or the halting time. This is because the norms in the initial population are either quickly reduced to only a handful, or unable to spread and cause dynamic equilibrium as a result. However, information length does matter when there is modification. When transmissions are modified, the information length affects the resulting transmission. Large information lengths increase the innovation capacity of the system by allowing a transmission a wider range of norms to mutate into.

Based on the data collected from our screening experiment and observations of selected runs, we make the following additional observations about our emergence model:

- As the degree of the agents increase, the number of norms that can be maintained during a state of dynamic equilibrium appears to decrease. This is illustrated in Figure 4.
- Most of the norm exchange between agents occurs at the borders of norm clusters (Figure 3b). This is most likely a result of the current implementation automatically rejecting a transmission if it contains a norm that is already known.
- If the stability tolerance is low (< 100), there are multiple parameter configurations that can can prevent the spread of information and result in the model reaching dynamic equilibrium almost immediately. This suggests that the stability tolerance should be kept to a high value, but, because the current model is stochastic, there can never be any certainties that a state of dynamic equilibrium will remain forever.

B. Factor influence on the Response Variables

It is not enough to understand the overall dynamics of normative information emergence. To guide future experiments,



(a) (b) (c)

Fig. 4. The effect of neighborhood type on norm diversity at dynamic equilibrium 50x50 cut lattice with 256 possible norms. To force dynamic equilibrium, we set $IT = 10.0$, $SP = 1.0$, $SI = 1.0$. (a) uses a von Neumann neighborhood, giving each agent 4 neighbors. (b) uses a Moore neighborhood, giving each agent 8 neighbors. (c) uses a Complete neighborhood, connecting every agent to every other agent. Each agent has 624 neighbors.

such as comprehensive sensitivity analysis, the relationship between significant factors and the response variables must also be understood. A summary of these relationships for the most significant factors is described below. However, care must be taken in the literal application of these results. For instance, an extremely large internalization time will lead to dynamic equilibrium by preventing any agent from ever learning new information.

- The likelihood of reaching global convergence is enhanced when the agents are connected, the agent density of the environment is high, and the modification probability is low.
- The likelihood of reaching local convergence is enhanced when the agents are disconnected and the agent density, modification probability, acceptance probability and internalization time are low.
- The likelihood of reaching dynamic equilibrium is enhanced when the the acceptance probability, internalization time, sanctioning probability, and sanction impact are high, and the modification probability is low in comparison to the internalization time.
- The likelihood of reaching either global or local convergence is enhanced when the modification probability and internalization time are low.
- The likelihood of reaching any equilibrium state is enhanced when the acceptance probability, internalization time, sanctioning probability, and sanction impact are high, and the modification probability is low.
- The time it takes to reach an equilibrium state is low when the acceptance probability and internalization time are high, and the modification probability is low.

C. Next Steps

The norm emergence simulation, which implements our emergence model, is able to produce conditions that match those of a consensus. However, a careful consideration of the results and the problem space suggests that a finite lattice may not be the ideal representation on which to test our model. Instead, we propose that a general network representation should be used. This approach will allow us to reduce the factors being considered and provide explanations of the algorithm dynamics that are easier to understand when compared against than those that can be arrived at under a finite lattice representation. A more general network representation will also allow us to compare results across different topologies, enabling us to generate evidence that can be used to prove when a consensus can be reached using the processes of our model.

The set of factors that we will use with a general network model are the agent count (AC), agent degree (AD), information length(IL), acceptance probability (AP), internalization time (IT), modification probability (MP), and sanction impact (SI). We will also investigate the utility of adding a raw transmission probability (TP) to enhance the behavior-space of the agents and allow for reputation and other non-guaranteed-transfer mechanisms. The main difference between a finite lattice representation and a more general network representation is that the grid size and agent density will be replaced with a discrete agent count (AC) parameter, the neighborhood type and edge behavior will be replaced with an agent degree (AD) parameter, and the sanction probability will be set to 1.0.

Using the new network representation, We will rerun a screening analysis to ensure the model is consistent with expectations. Once we have established a base line for parameter bounds, we will begin to explore additional parameter settings and alternative implementations of the normative processes as an alternative to the current stochastic methods. As part of a richer parameter exploration, we will investigate the effects of *blockers* [28] and memory [10]; where blockers are agents that have an acceptance probability of 0.0. An initial look at their effect shows that a handful of blockers on a disconnected network are enough to prevent global or local emergence and encourage dynamic equilibrium.

VII. Conclusion

In this paper we have verified that the emergence model introduced by Hollander and Wu [7], and implemented in our current research, is sufficient to allow the emergence of group norms and the formation of consensus. We have also identified and interpreted the effects of the significant factors involved in the various equilibrium states associated with the model.

Although our initial investigation was not meant to fully explore the parameter space of our model or identify critical patterns of behavior, we discovered that multiple norms can self-organize into distinct groups within the same lattice structure. The trivial case occurs when the agent clusters are disconnected. In this scenario information is not able to pass between clusters, and as a result norms able to converge independent of the disconnected agents. The more interesting case occurs when the agents are connected to one another, either directly or indirectly. In a connected network, multiple norms are able to self organize (Figure 4a and 4c, Figure 3b) into distinct groups with highly active borders that hold firm even after thousands of time steps. Even more interesting is that these self-organized groups can be resilient to high levels of mutation.

We have identified a basic relationship between the average degree of the agents and the long-term norm diversity in the system (Figure 4). As the degree of each agent increases, the total number of norms that are able to survive in a system decreases. This is an important characteristic that may have many practical applications in the design of multi-agent systems, particularly with regard to the self-organization of groups or consensus formation, as required by many coordination problems.

Based on the success of our experiment, it has been decided that while a finite lattice representation of our model is sufficient, a more generalizable network-based representation would be preferred. Implementing this change is our next immediate step, followed by a detailed sensitivity analysis of the model. Our long term goal is continue investigating how the processes of norm emergence can be applied to consensus formation and multi-agent coordination.

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