

COT 4210 Quiz #6: Classes PSPACE, NPSPACE
Date: 12/1/2015

Name: _____

1) (25 pts) Class P

In the game PAC-MAN-LITE, Pac Man starts in the top left corner of the playing grid and is only allowed to move right or down, and he must finish his path at the bottom right corner of the playing grid. Each square is worth a number of points and Pac Man's score is the sum of the scores of the squares he visits. For example, for the playing grid shown below, Pac Man could achieve a maximum score of 65 utilizing the shaded path shown.

<u>3</u>	4	6	8	9	1
<u>2</u>	<u>18</u>	1	1	1	1
2	<u>1</u>	<u>5</u>	<u>13</u>	<u>3</u>	<u>7</u>
6	5	12	3	1	<u>6</u>

Define $\text{PAC-MAN-LITE} = \{ \langle A, k \rangle \mid A \text{ is a two dimensional array of positive integers representing a Pac Man grid with a maximal path of value } k \text{ from the top left corner to the bottom right corner only moving right and down.} \}$

Prove that PAC-MAN-LITE is in the class P. In order to do so, you must devise an algorithm that correctly solves the problem, prove the correctness of the algorithm and prove that the algorithm you devised runs in polynomial time of the size of the input.

2) (15 pts) NPSPACE

a) A corollary of Savitch's Theorem is that NPSPACE = PSPACE. State Savitch's Theorem.

b) A failed attempt at proving Savitch's Theorem is simply attempting to simulate each possible branch of a non-deterministic Turing machine that utilizes polynomial space, similar to how it was shown that non-deterministic Turing machines recognize the exact same class of languages that deterministic Turing machines. Why does this avenue of proof fail? Give a detailed response.

3) (10 pts) TQBF

With proof, determine whether or not

$$\exists w \forall x \exists y \exists z [(w \forall \bar{y}) \wedge (\bar{w} \forall z) \wedge (w \forall \bar{x}) \wedge (\bar{w} \forall x) \wedge (y \forall z)] \in TQBF.$$