COT 4210 Quiz #5: Classes P, NP
Date: 11/17/2015

Name: _____

1) (15 pts) Let CS = { $<S_1, S_2, k>$ | S_1 and S_2 are sequences of integers with a common subsequence of length k}. For example, S_1 = {2, 8, 3, 9, 4, 1, 7, 12, 5, 4, 2}, S_2 = {6, 15, 2, 12, 9, 3, 4, 7, 7, 8, 9, 12, 3, 4, 2, 5} and k = 6 would be an element of CS, since S_1 and S_2 have a common subsequence of length 6. (Note: A subsequence is a subset of integers in a sequence in the same relative ordering that they appear in the original sequence. A common subsequence is an identical subsequence contained in two separate sequences.) Prove that CS $\in$ P.

2) (15 pts) In class you were shown a proof that Vertex Cover is NP-Complete. This proof involved a polynomial time reduction from 3-SAT to Vertex Cover. Show the output of the reduction function given the following 3-SAT input:

$$(a \vee \bar{b} \vee a) \wedge (c \vee \bar{a} \vee \bar{a}) \wedge (b \vee \bar{c} \vee \bar{a}) \wedge (b \vee a \vee c)$$

Recall that your answer should include an undirected, unweighted graph and an integer k.

3) (20 pts) PARTITION = {<S> | S is a set of integers that can be partitioned into two subsets, S_1 and S_2 such that the sum of the elements of S_1 is equal to the sum of the elements of S_1}. For example, the set {13, 8, 2, 3, 9, 6, 7} is an element of PARTITION because the subset {13, 8, 3} sums to 24, which is equal to the sum of the remaining elements, {2, 9, 6, 7}. Prove that PARTITION is NP-Complete by reducing it from SUBSET-SUM.