

COT 4210 Quiz #4 Part A: Class P 4/22/2021

Regular Start Time: 1:30 pm (EST)

Regular End Time: 2:10 pm (EST)

Regular Late Time: 2:20 pm (EST)

1) (10 pts) Here is an algorithm to determine if two integers, a and b , represented in binary, are relatively prime or not:

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for each integer,  $x$ , in the range from 2 to  $\min(a, b)$ :  
    if both  $a$  and  $b$  are divisible by  $x$ :  
        return false  
return true
```

(a) Explain why this algorithm does NOT run in polynomial time in the size of the input.

(b) Explain why this algorithm is NOT proof that the language RELPRIME, (the set of pairs of positive integers that share no common factors) does NOT belong to the class P.

2) (10 pts) Give an algorithm to prove that the following language is in the class P:

NUMPARTS = { $\langle G, k \rangle$ | G is an undirected, unweighted graph with precisely k connected components }

You must make sure that (a) your algorithm completely solves the problem, and (b) give its run time in terms of $|V|$ the number of vertices in the graph G and $|E|$ the number of edges in the graph G .

3) (5 pts) Prove that the following language is in the class P:

TRIANGLE: { $\langle G \rangle$ | G is an undirected, unweighted graph with a CLIQUE of size 3 or greater. }

COT 4210 Quiz #4 Part B: Class NP 4/22/2021

Regular Start Time: 2:10 pm (EST)

Regular End Time: 2:50 pm (EST)

Regular Late Time: 3:00 pm (EST)

1) (10 pts) Let $4\text{COLOR} = \{ \langle G \rangle \mid G \text{ is colorable with 4 colors.} \}$. It can be proven that 4COLOR is an NP-Complete language.

Specifically, we can show that $3\text{COLOR} \leq_P 4\text{COLOR}$. One the reduction is shown, the results follows.

For this question, show the polynomial time reduction between the two problems. (Hint: This reduction isn't as difficult as it may initially seem.)

2) (5 pts) The same device used to do the reduction in question 1 can be used to show that $2\text{COLOR} \leq_P 3\text{COLOR}$. But, even if this reduction is successfully done, it does NOT prove that 2COLOR is NP-Complete. Why?

3) (5 pts) Let the language $\text{UNEVENPART} = \{ \langle S, x, y \rangle \mid S \text{ is a set of positive integers such that a subset } T \text{ of } S \text{ sums to } x \text{ and the set } S - T \text{ sums to } y \}$

Prove that UNEVENPART is NP-Complete.

4) (5 pts) In what city is Orlando International Airport?