<u>COT 4210 Quiz #3 Part A: Decidability, Coountability 3/25/2021</u>

Regular Start Time: 1:35 pm (EST)
Regular End Time: 2:10 pm (EST)
Regular Late Time: 2:20 pm (EST)

1) (6 pts) Prove that the following language, L_1, is a decidable language:

L_1 = { (<D>, k, m) | <D> is the encoding of a DFA which accepts at least k strings that are length m or less}

2) (9 pts) Prove that the following language L_2, is a decidable language:

L_1 = { (<D>, k, m) | <D> is the encoding of a DFA which accepts at least k strings that have a length greater than m}

3) (10 pts) Prove that the set of ordered pairs (x, y) where x and y are both positive integers with $x < y$ is countable. To make sure you just don't copy a similar proof shown in class, in order to get full credit for this one, you need to do the following:

(a) Give an English description of how you would order the ordered pairs, making sure that each one gets listed exactly once. (So, based on your description, I should be able to design a computer program that starts running and spits out ordered pairs listed above without ever repeating one and such that it would eventually spit out any specified ordered pair.)

(b) Given the ordering you've described, what is the rank of the ordered pair (100, 200) in your ordering. **Please leave your answer as an arithmetic expression containing only integers and usual mathematical operations or functions (+, -, *, /, combos and powers).**

(c) Explain why this ordering: (1, 2), (1, 3), (1, 4), … , (2, 3), (2, 4), (2, 5), …, (3, 4), (3, 5), (3, 6), … is insufficient to show that the ordered pairs described are countable.

COT 4210 Quiz #3 Part B: Undecidability, Reducibility 3/25/2021

Regular Start Time: 2:10 pm (EST)
Regular End Time: 2:45 pm (EST)
Regular Late Time: 2:55 pm (EST)

4) (10 pts) Prove that the following language is undecidable:

L_4 = { (<M>, k) | <M> is the encoding of a Turing Machine which accepts all strings of length k}

5) (12 pts) Consider the two following problems:

SAFE-CLASSES = { (S, E) | S is a list of students, and E is a list of pair of students who are not allowed to be in the same class, and there exists a way to split all students in S into two non-empty classes such that the requirements given by E are satisfied }

TWO-COLORABLE = { G | G is an undirected graph such that each vertex in G can be assigned one of two colors, Red or Blue, such that no edge in G connects two vertices that have the same color. }

SAFE-CLASSES is mapping reducible to TWO-COLORABLE. To do the reduction, given an input of students and pairs of students who can't be in the same class, you can create a graph G that is two-colorable if and only if there's a way to split the students in S into two classes satisfying the restriction.

(a) Give an unambiguous algorithmic description of how to take (S, E) and convert it to a corresponding G that completes the mapping reduction successfully.

(b) Use your mapping reduction to create a graph for the following input for the problem SAFE-CLASSES:

S = {Ariel, Binh, Carthik, Deanna, Eduardo, Freddy}
E = { (Ariel, Carthik), (Binh, Carthik), (Binh, Eduardo), (Carthik, Freddy), (Deanna, Eduardo) }

(c) Using your graph or otherwise, give a valid listing of two classes of students which proves that the input above belongs to the language SAFE-CLASSES

7) (3 pts) Who invented the idea of the Turing Machine?