COT 4210 Quiz #3: Class P, NP

Date: 4/14/2015

Directions: Please answer each question on your own paper. Write your name on the top right corner of each sheet of paper you use. Clearly mark your answers and include justifications as needed. Staple your paper in the top left corner and turn your answers in. Keep this sheet for reference.

1) (10 pts) Modified Pac Man (MPM) is played on a 2 dimensional integer array. Pac Man always starts at the top left corner of the array, at index (0, 0) and ends at the bottom right corner of the array, at index (r - 1, c - 1), where r is the number of rows in the array and c is the number of columns in the array. He can only move down and to the right, namely, he can either increment his row number of column number with each movement. Pac Man's score is simply the sum of the squares he visits on his path. For example, in the grid shown below, for the highlighed path, Pac Man's score is $2 + 6 + 2 + 9 + 4 + 7 + 6 + 9 + 4 + 7 = 56$.

2	3	9	1	4	7
6	1	1	4	8	1
2	9	2	3	4	5
9	4	7	1	4	1
8	3	6	9	4	7

Define the language MPM = { (A, n) | A is a 2 dimensional integer array for which Pac Man can score at least n points.} Prove that MPM ∈ P.

2) (5 pts) Let CRT-Lite = { (S, n) | S is set of modular equivalence classes with at least one non-negative integer solution less than n}. As an example, S = { 2 mod 3, 3 mod 4, 5 mod 6 } with n = 10 has no solution (this can be verified via brute force) but the same set with n = 15 does, since $11 \equiv 2$ mod 3, $11 \equiv 3$ mod 4 and $11 \equiv 5$ mod 6. Prove that the language CRT-Lite ∈ NP.

3) (10 pts) An independent set in a unweighted, undirected graph is a set of vertices within the graph such that no two of the vertices are connected by an edge. Given that language CLIQUE is NP-Complete, prove that the language IS = { (G, k) | G is an undirected unweighted graph with an independent set of size k.} is also NP-Complete.