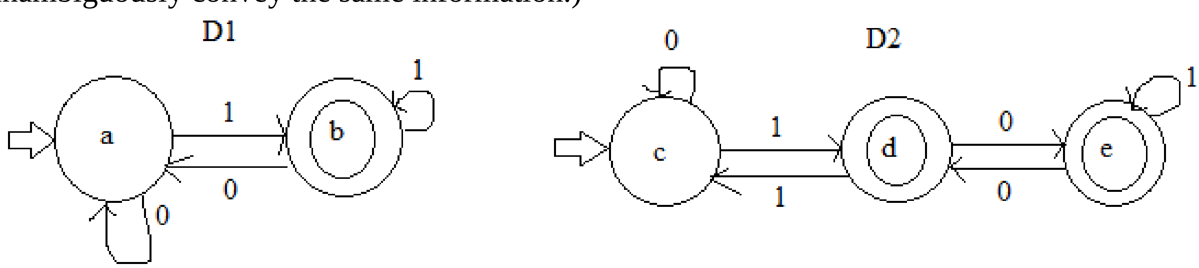
<u>COT 4210 Final Exam Part A: Regular Languages 5/4/2021</u>

Regular Start Time: 1:00 pm (EDT)
Regular End Time: 1:40 pm (EDT)
Regular Late Time: 1:50 pm (EDT)

1) (5 pts) Design a DFA over the alphabet $\Sigma = \{a, b\}$ that accepts all strings that have no more than 2 a's in a row, and no other strings. Either draw the resulting DFA or give the formal result clearly specifying each portion of the 5-tuple definition clearly typed.

2) (10 pts) Below are the drawings of two DFAs: D1 and D2, both over the alphabet {0, 1}. Let L(D1) and L(D2) represent the language described by D1 and the language described by D2, respectively. Use the construction algorithm mentioned in class to design a DFA for the language $\overline{L(D1)} \cap L(D2)$. The states in D1 are {a, b} and the states in D2 are {c, d, e}. Please label the states in your answer as {ac, ad, ae, bc, bd, be}. You may represent your answer as either a drawing or in the formal manner with each item in the 5-tuple definition clearly typed. (Both forms must unambiguously convey the same information.)



3) (10 pts) Let $\Sigma = \{\begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}, \begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix}\}$. Define L = { w | the binary string read left to right of the top of the tiles is the reverse of the binary string on the bottom of the tiles }. For example, $\begin{bmatrix}1\\1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}0\\0\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix} \in L$, since the top of the tiles reads "11001" and the bottom of the tiles reads "10011". If we reverse "11001", we obtain "10011". Prove, via the pumping lemma for regular languages, that L is not regular.

<u>COT 4210 Final Exam Part B: Context Free Grammars 5/4/2021</u>

Regular Start Time: 1:40 pm (EDT)
Regular End Time: 2:20 pm (EDT)
Regular Late Time: 2:30 pm (EDT)

1) (5 pts) A context free grammar over the alphabet {a, b} with variables {S, A, B, C} and the start symbol S is given below:

$S \rightarrow A \mid BA$
$A \rightarrow CC \mid AC \mid \varepsilon$
$B \rightarrow CAC \mid aa \mid b$
$C \rightarrow a \mid BB$

Give a derivation for the string aaaabb in this grammar.

2) (7 pts) Design a PDA for the following language L, over the alphabet {a,b,c}:

$$L = \{ a^x b^y c^z \mid x, y, z \text{ are non-negative integers such that } x > y+z.\}$$

Please give either a drawing of your design or a text description with the 6-tuple of the formal definition of the PDA. If you make a drawing, recall that a transition between states labeled as

a, b → c, means that you are reading the character a, popping character b from the stack and pushing character c onto the stack.

3) (5 pts) In the last step of creating a Chomsky Normal Form grammar from any regular grammar, extra variables and extra rules are introduced so that no rule maps a variable to more than 2 variables. Show an application of this step on the following single rule:

$S \rightarrow ABCDEFG$

You may add additional variables that are English capital letters, H or later in the alphabet.

4) (8 pts) Let $\Sigma = \{a, b, c\}$ and L = {w | w has the same number of a's, b's and c's}. Prove that L is not context free via the Pumping Lemma for Context Free Grammars.

COT 4210 Final Exam Part C: TMs, Decidable/Undecidable Languages 5/4/2021

Regular Start Time: 2:20 pm (EDT)
Regular End Time: 3:00 pm (EDT)
Regular Late Time: 3:10 pm (EDT)

1) (8 pts) Let L_1 and L_2 be Turing Recognizable languages. Prove that $L_1 \cup L_2$ is also Turing Recognizable. In order to earn full credit, your proof must be detail oriented.

2) (8 pts) Prove that the following language C, is a decidable language:

C = { (<G>, k, m) | <G> is a Context Free Grammar such that at least k distinct strings are generated by applying m or fewer rules starting from the start variable. Both k and m are given positive integers.}

Applying a single rule means picking a single variable in the current derivation and substituting it with the right hand side of a rule for that variable. Please give your algorithm to decide membership in this language at a high level without Turing Machine level details and prove that your algorithm must terminate and return the correct result.

3) (9 pts) Let L = { <M_1, M_2, k> | M_1 and M_2 are Turing Machines for which there exists a string s such that $|s| = k$ and both M_1 and M_2 accept s. } Prove that L is undecidable from first principles (without using Rice's Theorem or any other short cut).

COT 4210 Final Exam Part D: Classes P/NP 5/4/2021

Regular Start Time: 3:00 pm (EDT)
Regular End Time: 3:50 pm (EDT)
Regular Late Time: 4:00 pm (EDT)

1) (8 pts) A subsequence of a string s is a subset of the letters of s in the same order as they appear in s. For example, if s = PO**L**YNOMI**A**LTI**ME**, then t = **LAME** is a subsequence of s, since we can create t using the 3^{rd}, 9^{th}, 13^{th} and 14^{th} characters of s, in that order. (Characters highlighted in red.) Define the language L as follows:

L = { (s, t) | s and t are strings and t is a subsequence of the string s }.

Prove that L belongs to the class P.

2) (15 pts) In class it was shown that 3-SAT $\leq_P$ 3-COLOR. For this question, show that the polynomial time reduction can be done in the opposite order, namely that 3-COLOR $\leq_P$ 3-SAT. Note that looking at the proof that was shown in class is unlikely to help you on this question. (I am just trying to help you because I think people's natural instinct would be to look at the details of that proof, but those would not be helpful for this question and I want to save everyone the hassle of looking at that proof and losing time because of it.) Thus, here is what you must do:

(a) Start with an input, G, to the 3-COLOR problem, which is an arbitrary unweighted, undirected graph (**with no colors assigned to any vertices!!!)**

(b) Provide an algorithm to use G to create a Boolean formula φ, in 3-CNF form.

(c) Prove, that if the graph G is 3 colorable, then the corresponding Boolean formula, φ that your algorithm generated is satisfiable.

(d) Prove, that if the output formula φ is satisfiable, then then corresponding input G that created it must be 3 colorable.

(e) Prove that your algorithm to calculate φ from G runs in polynomial time of the input, G.

In the natural description of the reduction, quite a few of the clauses one would create would be the or of two variables, not three. Note that any clause of the form $(a \vee b)$ can easily be transformed into an equivalent clause with three terms as follows: $(a \vee b \vee b)$. There is no need to describe this in your reduction. Your algorithm may produce clauses of two or three terms.

3) (2 pts) What type of fruit is used to make a Pumpkin Pie?