Fall 2015 COT 4210 Final Exam
Date: December 15, 2015

First Name: _____ Last Name: _____

1) (12 pts) Let L = $\{0^{n^3} | n \in Z^+\}$, the language over the alphabet $\{0\}$ of strings of lengths that are positive perfect cubes. Use the pumping lemma to show that L is not regular.

2) (12 pts) Let G be a context free grammar in Chomsky Normal Form. Using induction on n, prove that for any string $w \in L(G)$ of length n, exactly 2n-1 steps are required for any derivation of w.

3) (10 pts) Let S be the set of circles with centers at lattice points on the Cartesian plane with positive integer length radii. (Note: A lattice point is one with integers for both its x and y coordinates.) Is the set S countable? Provide proof of your answer. Please show a detailed proof and don't simply say that this is a specific case of a more general type of set for which we learned the result in class. (I am grading you on the mechanics of the detail of the proof for this specific question.)

4) (10 pts) Let L = { <M, w, k> | M is a Turing machine, that when run on string w, visits at least k states.} Is L decidable? Provide proof of your answer.

5) (10 pts) Let L = { <M, w> | M is a Turing machine, that when run on string w, never moves its tape head beyond the |w| leftmost squares of the tape.} Is L decidable? Provide proof of your answer.

6) (10 pts) We say that a set of *n* integers S dominates a set of *n* integers T, if there exists at least one permutation of the set S and one permutation of the set T such that the i^{th} item of the permutation of S is strictly greater than the i^{th} item of the permutation of T, for all i, $1 \leq i \leq n$. For example, the set {3, 6, 2} dominates the set {5, 1, 1}, since 6 > 5, 3 > 1 and 2 > 1, but the set {4, 4, 5} does not dominate the set {5, 1, 1}. Write an efficient method in Java to determine whether or not a given input set S dominates an input set T. You may assume that the length of both input arrays to your function is the same. You may use the Java API method described below. Fill in the prototype given.

Java Doc for method in java.util.Arrays

```
// Sorts the specified array into ascending numerical order.
public static void sort(int[] a);
```

```
public static boolean dominates(int[] S, int[] T) {
```

```
}
```

7) (10 pts) In class you were shown a polynomial time reduction from 3-SAT to Independent Set. Show the result/output of this reduction on the following input instance of 3-SAT:

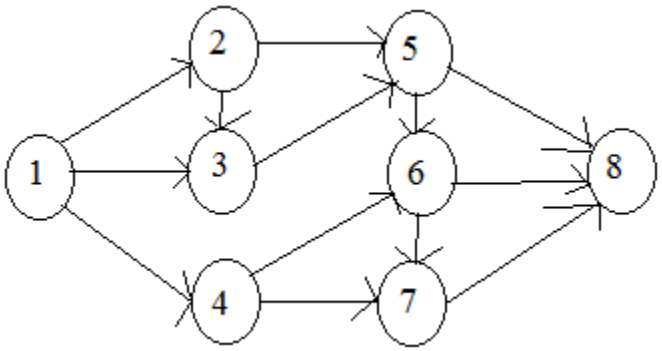
$$(a \vee \bar{b} \vee \bar{c}) \wedge (\bar{a} \vee \bar{a} \vee b) \wedge (\bar{b} \vee c \vee c) \wedge (a \vee b \vee b)$$

8) (15 pts) A subset of the nodes of a graph G is a dominating set if every other node of G is adjacent to some node in the subset. Let

DOMINATING-SET = { <G, k> | G has a dominating set of k nodes }

Show that DOMINATING-SET is NP-Complete by giving a reduction from VERTEX-COVER.

9) (10 pts) With proof, determine whether or not the following input graph and vertex 1 belongs in the language GEOGRAPHY GAME. Recall that GEOGRAPHY GAME is the set of graph vertex pairs such that players alternate turns, taking edges from the previous vertex (without repeating vertices) until one of the players is stuck. A graph and vertex are in the language if and only if the first player has a winning strategy.



10) (1 pt) Though they have broadened to serve many types of items, what's the signature breakfast item served at Dunkin' Donuts?

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Scratch Page - Please clearly mark any work on this page you would like graded.