**COT 4210: Discrete Structures II**

**Exam #3**

**August 3, 2011**

**Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Lecturer: Arup Guha**

**(Directions: Please justify your answer any proof type question. A majority of the points in these questions are awarded for the justifications given, instead of the final answer.)**

1) Categorizing Languages (10 points)

Please list which category most accurately describes the language in question. In particular, pick the first set on the ordered list below to which each language belongs. Assume that the alphabet for each language is {0, 1}, unless otherwise stated. Please use the given abbreviations for each set:

1. Reg – Regular Languages

2. CFG – Context Free Grammars

3. TD – Turing Decidable Languages

4. TR – Turing Recognizable Languages

5. NTR – Not Turing Recognizable Languages (the complement of TR)

L1 = { w | the length of w is a prime number } \_\_\_\_\_\_\_\_

L2 = { w | w contains an equal number of 0s and 1s } \_\_\_\_\_\_\_\_

L3 = { w | w contains an equal number of substrings 01 and 10 } \_\_\_\_\_\_\_\_

L4 = { w | w contains an equal number of substrings 00 and 11 } \_\_\_\_\_\_\_\_

L5 = { <M, w> | M is a Turing machine that accepts w } \_\_\_\_\_\_\_\_

L6 = $\overbar{L\_{5}}$ \_\_\_\_\_\_\_\_

L7 = { <D, w> | D is a DFA that accepts w } \_\_\_\_\_\_\_\_

L8 = { <M, w> | M is a Turing Machine and does not accept w } \_\_\_\_\_\_\_\_

L9 = { <M, w> | M is a Turing Machine and rejects w } \_\_\_\_\_\_\_\_

L10 = { <M, w> | M is a LBA that accepts w } \_\_\_\_\_\_\_\_

2) Regular Languages (8 points)

Assume, unless otherwise stated that the alphabet for each language is {0, 1}.

(a) (2 pts) Draw a DFA that accepts all strings with exactly one or three 0’s in them and any number of 1s, and no other strings.

(b)(2 pts) Write down a regular expression for the language described in part (a).

(c) (2 pts) Draw an NFA that accepts all strings that end in 010, and no other strings.

(d) (2 pts) Consider converting a NFA with 20 states into an equivalent DFA. What is the maximum number of states that DFA could have?

3) Turing Machines (10 points)

(a) (5 pts) Describe in English, the set of strings the following Turing Machine accepts.

Q = { q0, q1, q2, q3, q4, qaccept, qreject } Σ = {0, 1} Γ = {0, 1, x, y, blank}

Start state = q0 Accept state = qaccept Reject state = qreject

δ is described below. Note, if a transition doesn’t exist, it goes to qreject.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Input State | Output State | Read | Write | Move |
| q0 | qaccept | blank | blank | R |
| q0 | q2 | 0 | y | R |
| q0 | q3 | 1 | y | R |
| q1 | qaccept | blank | blank | R |
| q1 | q1 | x, y | x, y | R |
| q1 | q2 | 0 | x | R |
| q1 | q3 | 1 | x | R |
| q2 | q2 | 0, x | 0, x | R |
| q2 | q4 | 1 | x | R |
| q3 | q3 | 1, x | 1, x | R |
| q3 | q4 | 0 | x | R |
| q4 | q4 | 0, 1, x, blank | 0, 1, x, blank | L |
| q4 | q1 | y | y | R |

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(b) (5 pts) What is the role of each of the states in the Turing Machine described in part(a)?

q0 : \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

q1 : \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

q2 : \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

q3 : \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

q4 : \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

4) Undecidability (10 points)

(a) (5 pts) Let X be the set of complex numbers where each component (real and imaginary) is integral. Determine, with proof, whether or not X is countable.

(b) (5 pts) Let L = { <M, x> | M is a Turing Machine that accepts at least one string of length x }. Prove that L is undecidable via a reduction from any undecidable problem we studied in class.

5) NP-Complete Problems (20 points)

(a) (10 pts) The independent set problem is as follows: Given a graph G and an integer k, determine whether or not there are k vertices in G such that no two vertices out of the k share the same edge. Prove that the set INDEPENDET-SET is NP-Complete via a reduction from a known NP-Complete problem.

(b) (10 pts) In class, a proof was shown reducing Vertex Cover to Subset Sum in polynomial time. Use the construction in that proof to create the corresponding subset sum instance to the following Vertex Cover instance:

The adjacency matrix for the graph G is as follows:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | a | b | c | d | f |
| a |  | e1 | e2 | e3 |  |
| b | e1 |  | e4 |  | e5 |
| c | e2 | e4 |  |  |  |
| d | e3 |  |  |  | e6 |
| f |  | e5 |  | e6 |  |

The value of k for this instance of the problem is k = 3.

Note: The names of the vertices are a, b c, d and f. The names of the edges are e1, e2, …, e6.

In addition, for this example, determine a vertex cover of size 3 in this graph and then find the corresponding subset that adds up to the appropriate target.

6) (2 pts) How many songs are selected each week on Ryan Seacrist’s show, “American Top 40”?

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**Scratch Page – Please clearly mark any work on this page you would like graded.**