**COT 4210: Discrete Structures II**

**Final Exam**

**April 24, 2012**

**Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Lecturer: Arup Guha**

**(Directions: Please justify your answer to each question. No answer, even if it is correct, will be given full credit without the proper justification.)**

1) (10 pts) Regular Languages

(a) (5 pts) Draw an DFA over the alphabet {0, 1, 2, 3, 4, 5, 6, 7, 8, 9} that accepts all non-empty strings without any leading zeros that represent positive integers that have a value divisible by 3. (Note: for example, 3, 27, 84 and 101121 should be accepted but ε, 0, 0036 and 17 should not be accepted.)

(b) (5 pts) Let L be the language over the alphabet {a, b, c, d} that is the set of strings that have an even number of a's. (For example, bccdc, aababcdab and dbbcdabacd are all in L, but bdcabcd and badcaa are not.) Write down a regular expression for L.

2) (10 pts) Context-Free Languages

(a) (4 pts) Consider all of the arithmetic expressions that could be created by the variable x, the operator + and both open and close parentheses. Create a Context Free Grammar with the alphabet { x, +, (, ) } that generates all such valid expressions.

(b) (6 pts) If *C* is a Context-Free Language and *R* is a regular language, then $C∩R$ is a Context-Free Language. (This is true and you don't need to prove it for this question. Rather, you'll need to use this fact to answer the following question.) Consider the language A defined over the alphabet {a, b, c} as follows: A = { w | w contains an equal number of a's, b's and c's}. Prove that A is NOT a Context-Free Language.

3) (10 pts) Let L = { <M> | M is a Turing Machine such that L(M) only contains even-length strings}. Prove that L is undecidable without using Rice's Theorem.

4) (10 pts) Utilizing the polynomial-time reduction of 3SAT ≤P SUBSET-SUM shown in the text book, show the result of that reduction on the 3SAT instance shown below:

$$(x\_{1}∨\overbar{x\_{2}}∨x\_{3})∧(x\_{2}∨\overbar{x\_{1}}∨\overbar{x\_{3}})∧(x\_{3}∨\overbar{x\_{4}}∨x\_{2})∧(x\_{1}∨\overbar{x\_{3}}∨x\_{4})∧(\overbar{x\_{1}}∨\overbar{x\_{2}}∨x\_{4})∧(\overbar{x\_{2}}∨\overbar{x\_{3}}∨\overbar{x\_{4}})$$

In particular, your answer should be a set of integers and a target value, T.

5) (15 pts) The Amazing-Race problem is as follows:

You are given a trek of n segments, si (1 ≤ i ≤ n), for each segment, you may choose two possible paths, one which takes ai minutes and another which takes bi minutes. The goal of the Amazing-Race is to choose one of the two possible paths for each segment such that the total amount of time it takes to traverse each of these paths is exactly equal to a target time T minutes. The input to the Amazing-Race problem can be represented as an ordered list of n ordered pairs (ai, bi) and a target value T where each value is a positive integer, 0 < ai < bi, and $\sum\_{i=1}^{n}a\_{i}$ ≤ T ≤ $\sum\_{i=1}^{n}b\_{i}$. An instance of n ordered pairs (ai, bi) and a target value T belongs in Amazing-Race if and only if there exists a sequence of values t1, t2, t3, ..., tn where ti $\in $ {ai, bi} for all i, 1 ≤ i ≤ n and $\sum\_{i=1}^{n}t\_{i}=T$.

Prove that the Amazing-Race problem is NP-Complete by reducing Subset Sum to it.

6) (5 pts) How many calories are in a single serving of Nabisco 100 calorie snap packs? \_\_\_\_\_\_\_