**COT 4210: Discrete Structures II**

**Final Exam**

**December 6, 2011**

**Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

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**(Directions: Please justify your answer to each question. No answer, even if it is correct, will be given full credit without the proper justification.)**

1) (10 pts) Consider the set of strings that do not contain the substring 001 over the alphabet {0,1}. Give a regular expression for this set of strings.

2) (10 pts) Convert the following Context Free Grammar into an equivalent PDA using the algorithm described in class.

S → A | B

A → AB1 | 11 | BB

B → BBB1 | 0 | BB0

3) (15 pts) Use the DFA Minimization Algorithm shown in class to minimize the DFA below. Draw the resulting minimal DFA and put a box around your answer.



4) (15 pts) Let L = {0p | p is a prime number}. Prove that L is not Context Free using the Pumping Lemma for Context Free Languages.

5) (10 pts) Let L = { <M, w> | M is a Turing Machine such that when M runs on w, it attempts to move its tape head to the right of the end of the input string.} Determine whether L is decidable or undecidable. If the former is the case, give an algorithm that decides membership in L. If L is NOT decidable, prove that this is the case by reducing a known undecidable problem to it.

6) (20 pts) Let SET-SPLITTING = {<S,C> | S is a finite set and C = {C1 , C2 , C3 , … , Ck}} is a collection of subsets of S, for some k > 0, such that the elements of S can be colored red or blue so that no Ci has all of its elements colored with the same color. Show that SET-SPLITTING is NP-Complete. (Hint: Reduce from NAE-SAT to SET-SPLITTING.)

7) True/False (18 points – 2 points each) Circle the Correct Answer

a) ADFA is a Context-Free Language True False

b) ALLCFG is an undecidable language True False

c) If we restrict the input values in the set for the SUBSET SUM True False

 problem to be positive integers less than 10, then the problem

 is solvable in polynomial time.

d) It is impossible for an NFA and a DFA with the same number True False

 of states to accept the exact same language.

e) A standard PDA allows for non-determinism True False

f) If we changed the PCP to not allow the repeated use of dominos, True False

 this edited version of the problem would be decidable.

g) All languages that can be enumerated are decidable. True False

h) If we can reduce problem A to problem B, then we can decide True False

 problem A.

i) The brute force solutions to all NP-Complete problems run in True False

 O(2n) time where n represents the size of the input.

8) (2 pts) In the children’s game Four Square, how many different squares can one hit the ball?

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**Scratch Page – Please clearly label any work on this page you would like graded.**