**COT 4210: Discrete Structures II**

**Final Exam**

**December 7, 2010**

**Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

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**(Directions: Please justify your answer to each question. No answer, even if it is correct, will be given full credit without the proper justification.)**

1) (10 pts) Let L be the language over the alphabet {0, 1} that contains precisely the strings that end in 101 or 0010. Draw an NFA that accepts the language L. Clearly mark all relevant parts of the NFA.

2) (10 pts) Consider the algorithm shown in the text to convert a DFA into a regular expression. In this algorithm, an n-state DFA is converted into a (n+2) state GNFA. From there, states are successively ripped out until only two states remain. The formal description of a DFA D with 4 states, a, b, c and d, is given below. Show the 5 state GNFA that results during this process after only ripping out state c.

Q = {a, b, c, d}

Σ = {0, 1}

Start state = a

F = {b, d}

|  |  |  |
| --- | --- | --- |
| δ | 0 | 1 |
| a | b | c |
| b | c | a |
| c | c | d |
| d | c | b |

3) (10 pts) Use the procedure shown in the textbook to convert the grammar shown below into a PDA that accepts the exact same language. (Note: The terminals in this grammar are 0 and 1, and the start symbol is S.)

S → A | B11B

A → 1A0 | B

B → 0B0 | BB | 1 | ε

4) (10 pts) Let L = {<D1 , D2> | L(D1) $⊆$ L(D2) }, where D1 and D2 are representations of DFAs. Show that L is decidable.

5) (10 pts) Let L’ = {<M1 , M2> | L(M1) $⊆$ L(M2) }, where M1 and M2 are representations of Turing Machines. Show that L’ is undecidable.

6) (10 pts) For each of the languages described in the text shown below, determine the category to which it belongs. If a language belongs to more than one category, then circle the left-most category which applies. (For example, if a language is regular AND decidable, you must circle regular.)

Code R = Regular

 C = Context-Free

 D = Decidable

 TR = Turing-Recognizable

 N = None of the previous designations

a) 0\*1\* R C D TR N

b) ELBA R C D TR N

c) ANFA R C D TR N

d) $\overbar{A\_{TM}}$ R C D TR N

e) ATM R C D TR N

f) {0n1n | n $\in $ Z, n ≥ 0} R C D TR N

g) ALBA R C D TR N

h) {$0^{2^{n}}$| n $\in $ Z, n ≥ 0} R C D TR N

i) PCP R C D TR N

j) {0n1n2n | n $\in $ Z+ } R C D TR N

7) (10 pts) Consider the problem of determining the number of paths in a graph G from a given start vertex to a given end vertex of a length n. Show that this problem is in the class P. (Note: The obvious solution is NOT in the class P, so something more clever has to be done. Hint: Consider the process of exponentiating the adjacency matrix of G and the meaning of the numbers this new matrix stores.)

8) (10 pts) In class, a polynomial-time reduction from Vertex Cover to Subset Sum was given. Perform this reduction on the Vertex Cover instance shown below. Your answer should be a set of numbers and a corresponding target value. (Note: To aid grading, label the vertices and edges in the order in which they are listed below for the reduction.)

G = (V, E), where V is the set of vertices and E is the set of undirected edges in G.

V = {v1, v2, v3, v4, v5}

E = { (v1, v2), (v1, v3), (v1, v4), (v2, v5), (v3, v4), (v3, v5), (v4, v5) }

k = 2 (We are asking whether or not G has a vertex cover of size 2.)

9) (19 pts) n people live in a house and wish to share their expenses equally. Their respective expenses before settling are x1, x2, …, xn. Assume that all of these are greater than 0. They agree to write each other checks so as to make each person’s expenses equal the average cost. Naturally, they want to minimize the number of checks written. Formalize this as a decision problem and prove that it is NP-Complete.

10) (1 pt) The temperature in Orlando hit 32 degrees Fahrenheit last night. After whom is the Fahrenheit temperature scale named?

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