**COT 4210: Discrete Structures II**

**Exam #2**

**July 11, 2012**

**Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Lecturer: Arup Guha**

**(Directions: Please justify your answer to each question. No answer, even if it is correct, will be given full credit without the proper justification.)**

1) (10 pts) Give the formal definition of a Turing Machine as a 7-tuple.

2) (10 pts) An "Indecisive Turing Machine" is one that never moves right or left more than two moves in a row. Sketch out a high-level, informal proof (in one direction only) to show that any standard Turing Machine can be rewritten as an "Indecisive Turing Machine" that accepts the same language.

3) (10 pts) A Linear Bounded Automata (LBA) is a Turing Machine where the only portion of the tape that can be used for computation is the portion of the tape with the input. Consider a Polynomial Bounded Automata-k (PBA), where the amount of tape that can be used is exactly nk tape squares, where n represents that input size and k is a fixed positive integer.

Let APBA-k = { <M, w> | M is a PBA-k that accepts w. }

In proving that APBA-k is decidable, we must simply count the total number of configurations the machine M can be in and show that it’s finite. Once we show this, our algorithm to decide membership in the language simply entails simulating M running on w until it halts or until it runs the calculated total plus one number of steps. If the machine M had |Q| states, |Γ| tape characters (including the blank), and w is n characters long, calculate precisely the total number of configurations that M could possibly be in, in terms of these variables.

4) (8 pts) In the undecidability proof of the Post Correspondence Problem (PCP), one of the groups of tiles added to the instance of the problem created are tiles of the form $\left[\frac{a}{a}\right]$, for each symbol a in the tape alphabet. What is the purpose of these tiles?

5) (15 pts) ) Let SSDFA = { <D1, D2> | D1 and D2 are DFAs such that L(D1) $⊆$ L(D2). } Is SSDFA decidable? Prove your answer.

6) (15 pts) Let SSCFG = { <G1, G2> | G1 and G2 are CFGs such that L(G1) $⊆$ L(G2). } Is SSCFG decidable? Prove your answer.

7) (15 pts) Let S equal the set of real numbers of the form a + b$\sqrt{2}$, where a and b are positive rational numbers. Is the set S countable? Prove your answer.

8) (15 pts) Let L = { <M, w> | M is a Turing Machine, when run on w, reaches at least 50% of the states defined in M.} Prove that L is undecidable.

9) (2 pts) What type of food can one purchase at Pizza Hut? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Scratch Page – Please clearly mark any work you would like graded on this page.**