**COT 4210: Discrete Structures II**

**Exam #2**

**March 21, 2013**

**Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

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**(Directions: Please justify your answer to each question. No answer, even if it is correct, will be given full credit without the proper justification.)**

1) (15 pts) Consider the following formal definition of a Turing Machine M:

$$Σ=\left\{0,1,\#\right\}, Γ=\left\{0,1,\#,\$,B\right\}, start state=q\_{0}$$

δ(q0, #) = (q1, #, R) δ(q3, $) = (q3, $, R) δ(q5, $) = (q1, $, R)

δ(q1, 0) = (q2, $, R) δ(q3, 1) = (q4, $, L) δ(q5, #) = (qaccept, #, R)

δ(q1, 1) = (q12, $, R) δ(q3, 0) = (qreject, $, L) δ(q12, 0) = (q12, 0, R)

δ(q1, #) = (qaccept, #, R) δ(q4, $) = (q4, $, L) δ(q12, 1) = (q12, 1, R)

δ(q2, 0) = (q2, 0, R) δ(q4, #) = (q5, #, L) δ(q12, #) = (q13, #, R)

δ(q2, 1) = (q2, 1, R) δ(q5, 0) = (q5, 0, L) δ(q13, $) = (q13, $, R)

δ(q2, #) = (q3, #, R) δ(q5, 1) = (q5, 1, L) δ(q13, 0) = (q4, $, L)

 δ(q13, 1) = (qreject, $, L)

(a) Show the steps this Turing Machine executes with the input #011#100. Use the notation from the textbook.

(b) Give a brief description in English of the set of strings this machine accepts.

(c) What is similar about the states q2 and q12? What is the key difference between the two states.

2) (10 pts) Consider a Turing Machine model that is the same as a standard machine, but is never allowed to move in the same direction twice. How many different languages over the input alphabet {0,1} can be specified using this model? Justify your answer.

3) (12 pts) Let the language L be as follows: L = { <G, k> | G is an undirected, connected graph that can be colored with k or fewer colors. } Note: Remember that a valid coloring of a graph is one where each vertex is assigned a color such that no two vertices connected by an edge are assigned the same color. Prove that L is decidable.

4) (15 pts) Consider the following enumerator of the positive composite integers greater than 1:

 1) List all multiples of 2.

 2) List all multiples of 3.

 3) etc.

What is the flaw?

Use this same principle (listing multiples of different primes) to describe how to create a valid enumerator for the positive composite integers greater than 1. Remember that an enumerator may print the same value more than once, but it must guarantee to eventually print out any particular item in the language.

5) (15 pts) Let L = { <M1, M2> | M1 and M2 are TMs such that L(M1) $∩$ L(M2) $\ne $ $∅$ }. Prove that L is undecidable.

6) (15 pts) Let SSTM = { <M1, M2> | M1 and M2 are Turing Machines with L(M1) $⊆$ L(M2). }. Show that SSTM is mapping reducible to EQTM. Namely, Given an input <M1, M2> describe an algorithm to compute an ordered pair <M1', M2'> such that if and only if <M1, M2> $\in $ SSTM, <M1', M2'> $\in $ EQTM.

7) (15 pts) Let L = { <M, s> | M is a TM with state s such that, M never enters state s on any possible input.} Is L decidable? Prove your answer.

8) (3 pts) Which company runs the website maps.google.com? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Scratch Page – Please clearly mark the work on this page you would like graded.**