**COT 4210: Discrete Structures II**

**Exam #2**

**March 22, 2012**

**Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Lecturer: Arup Guha**

**(Directions: Please justify your answer to each question. No answer, even if it is correct, will be given full credit without the proper justification.)**

1) (15 pts) Let L be a language that is Turing Recognizable but NOT Turing Decidable. Prove that it is impossible to create an enumerator E for L that enumerates L in lexicographical order.

2) (20 pts) A Turing machine with a doubly infinite tape is similar to an ordinary Turing machine except that its tape is infinite to the left as well as the right. The tape is initially filled with blanks except for the spots with the input and the tape head starts on the left-most square with input. Show that this type of Turing machine is equivalent in power to a regular Turing machine.

3) (15 pts) The language USELESSDFA is defined as follows:

 USELESSDFA = { <D, q> | D is DFA and q is an unreachable state in D. }

Prove that USELESSDFA is decidable.

4) (15 pts) Let X be the set of real numbers with no more than 10 digits after the decimal (in their decimal representation). Determine, with proof, whether or not X is countable.

5) (15 pts) Show that the PCP is undecidable over a binary alphabet ({0,1}). (Remember, the proof in class did not show this, since it only worked with what was potentially a greater length alphabet.)

6) (15 pts) Let SSTM = { <M1, M2> | M1 and M2 are Turing Machines with L(M1) $⊆$ L(M2). }. Show that SSTM is not decidable by showing that if you had a decider for SSTM, you could build a decider for ATM.

7) (5 pts) After which famous golfer is the Arnold Palmer Invitational, which takes place this weekend in Orlando, named?

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**Scratch Page – Please clearly mark any work on this page you would like graded.**