**COT 4210: Discrete Structures II**

**Exam #1**

**June 8, 2011**

**Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

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**(Directions: Please justify your answer to each question. No answer, even if it is correct, will be given full credit without the proper justification.)**

1) (14 pts) Create a DFA over the alphabet {a, b} that contains exactly all non-empty strings that begin and end with the same letter. (For example, b, abbaba and bbab should be accepted, while ab, aaab and baba should be rejected.) Provide both a drawing and the full formal definition of your DFA.

2) (15 pts) Let A and B be regular languages. Provide a complete construction proof (without making use of any of the closure results provided in class) to show that the language $\overbar{A}∩\overbar{B}$ is also regular. In particular, given a DFA that accepts A and a DFA that accepts B, describe how to create a DFA to accept $\overbar{A}∩\overbar{B}$ and justify that this DFA accepts this language.

3) (15 pts) Given the following DFA, consider running the algorithm in the textbook that converts this DFA to an equivalent regular expression. Run the portion of the algorithm that adds the new start, accept state, associated transitions, and rips out state 2. Show your work and draw the resulting GNFA with four states.



4) (15 pts) Let L be the language over the alphabet { ( , ) } of strings of well-formed parentheses. Formally, in any string of well-formed parentheses, the number of open parentheses equals the number of close parentheses and in no substring of a string in the language starting from the beginning of the string does the number of close parentheses exceed the number of open parentheses. For example, the strings () , (()) , and (()())()((()))() are in the language, but ()) , (() , and (()())()) are not. Using the pumping lemma for regular languages prove that this language is not regular.

5) (10 pts) Design a PDA to accept the language from question 4.

6) (15 pts) Convert the following grammar with the start symbol S and terminals 0, 1 to Chomsky Normal Form:

S → ABC | C

A → 0A0 | 1

B → BS | 00 | ε

C → 11 | C0 | ε

7) (15 pts) Use the DFA minimization algorithm shown in class to create the DFA with the fewest number of states that accepts the same language as the DFA described below:

Σ = {0, 1}

Q = {q0, q1, q2, q3, q4, q5}

Start state = q0

F = {q0 , q3, q5}

Transition function:

|  |  |  |
| --- | --- | --- |
| Input State | Input Letter | Output State |
| q0 | 0 | q5 |
| q0 | 1 | q1 |
| q1 | 0 | q4 |
| q1 | 1 | q5 |
| q2 | 0 | q4 |
| q2 | 1 | q0 |
| q3 | 0 | q0 |
| q3 | 1 | q2 |
| q4 | 0 | q1 |
| q4 | 1 | q4 |
| q5 | 0 | q3 |
| q5 | 1 | q2 |

Provide your answer with a precise drawing of the minimized DFA.

8) (1 pt) Who wrote George W. Bush’s autobiography, Decision Points? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Scratch Page – Please clearly mark any work you would like graded on this page.**