**COT 4210: Discrete Structures II**

**Exam #1**

**February 7, 2013**

**Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

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**(Directions: Please justify your answer to each question. No answer, even if it is correct, will be given full credit without the proper justification.)**

1) (12 pts) Design a DFA to accept all ternary strings over the alphabet {0, 1, 2} that are equivalent to 1 mod 4 or 2 mod 4. Remember that a ternary string is in base 3, so the value 21023 = 2x33 + 1x32 + 0x31 + 2x30 = 65 in base 10, which is part of the language described.

2) (12 pts) Use the DFA Minimization algorithm shown in the Sudkamp text to minimize the following DFA described below.

Q = {s, a, b, c, d, e, f}

Σ = {0, 1}

q0 = s

F = {d, e, f}

|  |  |  |
| --- | --- | --- |
| δ | 0 | 1 |
| s | c | f |
| a | d | b |
| b | e | a |
| c | d | c |
| d | a | d |
| e | c | e |
| f | f | e |

3) (15 pts) Let L= {0a1b | a, b > 0 and gcd(a, b) > 1}. Prove via the pumping lemma or otherwise, that L is not regular.

4) (12 pts) Convert the NFA formally described below into a DFA. Please provide either a formal description or a drawing of the converted DFA. Please use the algorithm shown in class.

Q = {a, b, c}

Σ = {0, 1}

q0 = a

F = {b}

|  |  |  |  |
| --- | --- | --- | --- |
| δ | 0 | 1 | ε |
| a | {a} | {a,b} | {} |
| b | {c} | {} | {} |
| c | {b,c} | {} | {a} |

5) (12 pts) Consider the process of deriving a regular expression for the language described by the 3-state DFA below. In this process, the DFA gets converted into a 5 state GNFA and then states get ripped out. Show the 4 state GNFA that occurs in this process when q1 is the first state ripped out of the 5 state GNFA.

Q = {q0, q1, q2}

Σ = {0, 1}

Start state = q0

F = {q1, q2}

|  |  |  |
| --- | --- | --- |
| δ | 0 | 1 |
| q0 | q0 | q1 |
| q1 | q2 | q1 |
| q2 | q1 | q0 |

6) (12 pts) A parabolic word is one that can be partitioned into two parts so that the first part is in alphabetic order and the second part is in reverse alphabetic order. For example, aaabbccccbaa is a parabolic word since it can be partitioned into aaabbccc and cbaa. (Note that multiple valid partitions may exist.) Design a context free grammar over the alphabet {a, b, c} that precisely describes the language of all parabolic words. (Note: ε should be in the language you design.) Briefly justify why your grammar produces the desired language.

7) (8 pts) Give an example of two languages $L\_{1}$ and $L\_{2}$ such that neither language is regular but both $L\_{1}∪L\_{2}$ and $L\_{1}∩L\_{2}$ are regular. Briefly justify why the languages you have chosen for $L\_{1}$ and $L\_{2}$ are not regular but why $L\_{1}∪L\_{2}$ and $L\_{1}∩L\_{2}$ are regular.

8) (15 pts) Use mathematical (strong) induction to show that if G is a CFG in Chomsky Normal Form, then for any string w $\in $ L(G) of length n (n ≥ 1), exactly 2n – 1 steps are required for any derivation of w.

9) (2 pts) Who created the Chomsky hierarchy of languages? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_