

**COT 4210 Homework #2: Undecidability and Reducibility**  
**Due Date: Monday July 9, 2012 (in class)**

- 1) Find a match in the following instance of the PCP:  $\left\{ \left[ \frac{ab}{abab} \right], \left[ \frac{b}{a} \right], \left[ \frac{aba}{b} \right], \left[ \frac{aa}{a} \right] \right\}$ .
- 2) Show that  $A_{TM}$  is not mapping reducible to  $E_{TM}$ .
- 3) Let  $S = \{ \langle M \rangle \mid M \text{ is a TM that accepts } w^R \text{ whenever it accepts } w \}$ . Show that  $S$  is undecidable.
- 4) Consider the problem of testing whether a Turing machine  $M$  on an input  $w$  ever attempts to move its head left at any point during its computation on  $w$ . Formulate this problem as a language and show that it's decidable.
- 5) Let  $L' = \{ \langle M_1, M_2 \rangle \mid L(M_1) \subseteq L(M_2) \}$ , where  $M_1$  and  $M_2$  are representations of Turing Machines. Show that  $L'$  is undecidable.
- 6) Show that the PCP is decidable over a unary alphabet, that is, over the alphabet  $\Sigma = \{1\}$ .
- 7) Show that the PCP is undecidable over a binary alphabet, that is, over the alphabet  $\Sigma = \{0, 1\}$ .  
Note: In class we proved that the PCP was undecidable over a larger, fixed sized alphabet. You may use this result in your proof.