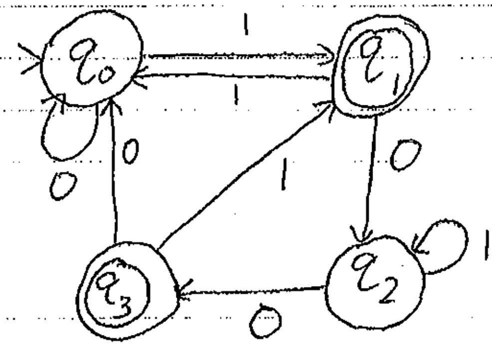
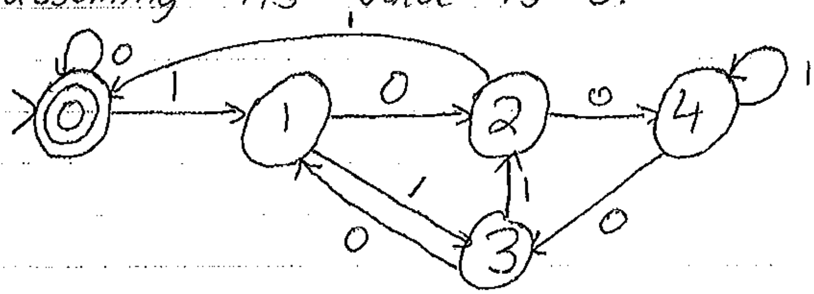
COT 4210 HMK #1 SOLUTIONS (SUM '12)

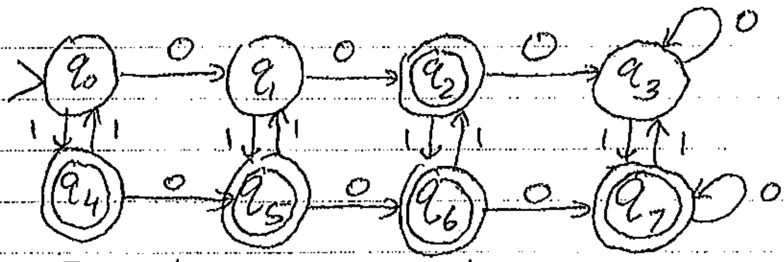
1)



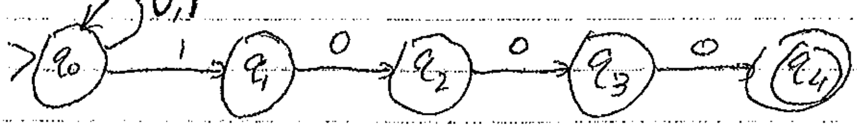
2) States are 0, 1, 2, 3, 4, representing 0 mod 5, 1 mod 5, 2 mod 5, 3 mod 5 and 4 mod 5. $\Sigma = \{0, 1\}$. In this solution, we'll accept ε, assuming its value is 0.



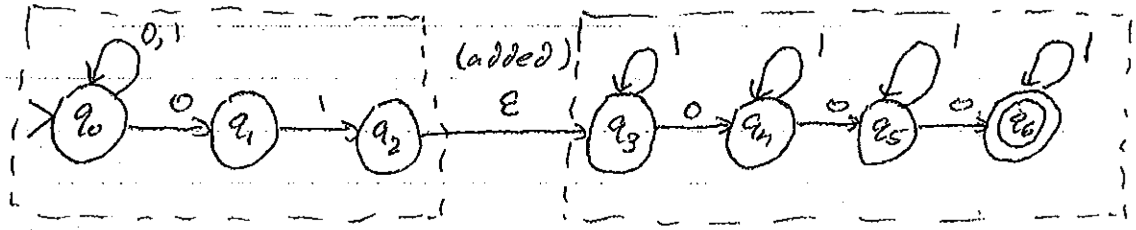
3)



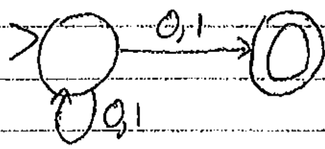
The basic idea here is that the top row represents an even # of 1s and the bottom row represents an odd # of 1s. The state number mod 4 represents how many zeroes the string has, with states q_3, q_7 representing "3 or more 0s".

4) 

We assume that all strings in L must have at least one 1.

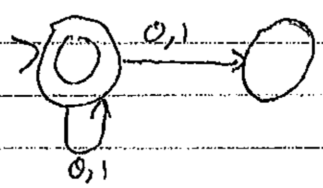
5) 

6) Consider the following NFA:



The language of this NFA is $\Sigma^* - \varepsilon$.

Using Tommy's transformation, the NFA for the language $\overline{L}$ would be



But, this NFA accepts Σ^*, ~~wo~~ which is NOT $\overline{\Sigma^* - \varepsilon}$. Specifically the string 1 is accepted by both machines, proving that the second does NOT accept the complement language of the first.

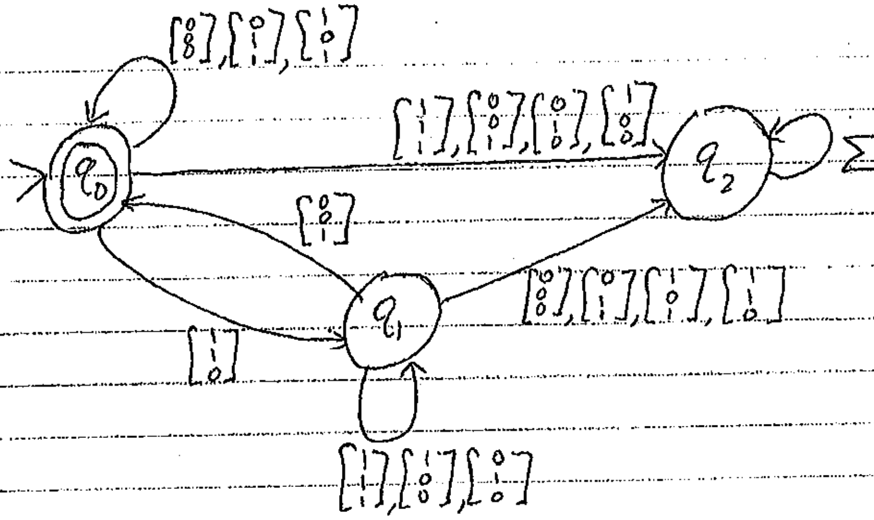
7) We will show that B is regular by creating a DFA that accepts B^R. Our DFA will have 3 states representing these points in our computation:

q_0: What we have read in so far adds up correctly.

q_1: What we have read in matches all the answer bits, BUT we have a carry bit we are expecting in the next character to read in.

q_2: There is already a mistake in the answer bits. Nothing in this state can be accepted, regardless of what is read in, in the future.

7 cont.)



Explanation of transitions:

$q_0 \to q_0$: In all 3 of these, the first two bits add to the third, maintaining equality

$q_0 \to q_1$: The parity of the bottom bit is the same as the sum of the top two, but a carry bit is produced, so the current string is no longer valid.

$q_0 \to q_2$: The parity of the answer bit is incorrect. All strings with this prefix are <u>NOT</u> in the designated language.

$q_1 \to q_0$: This fixes the previous ~~pari~~ carry bit problem.

$q_1 \to q_1$: The carry bit has cascaded to the next bit.

$q_1 \to q_2$: The current bit is now incorrect, so all strings with this prefix are <u>NOT</u> in the language.

8) L_1: $1(0 \cup 1)^*0$

L_2: $(0 \cup 1)^*0101(0 \cup 1)^*$

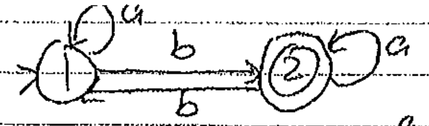
L_3: $(0 \cup 1 \cup \varepsilon)(0 \cup 1 \cup \varepsilon)(0 \cup 1 \cup \varepsilon)(0 \cup 1 \cup \varepsilon)(0 \cup 1 \cup \varepsilon)$

L_4: $\varepsilon \cup 1(01 \cup 11)^*(0 \cup 1 \cup \varepsilon)$

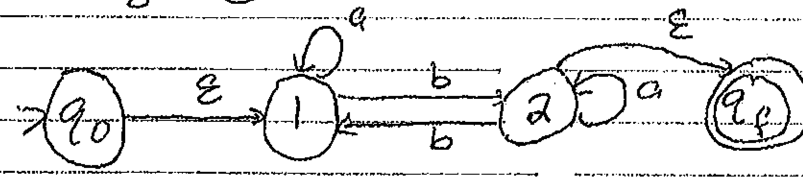
L_5: $(0^*10^*)(0^*10^*10^*)^* \cup 1^*01^*01^*$

Note: For each of these, it's important to include <u>ALL</u> possible strings that fit the given criteria

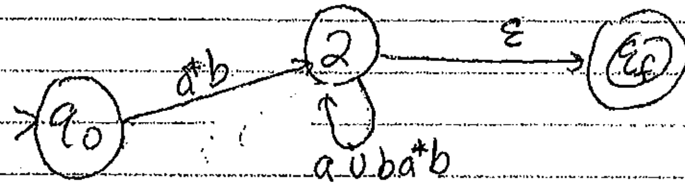
9) D_1:



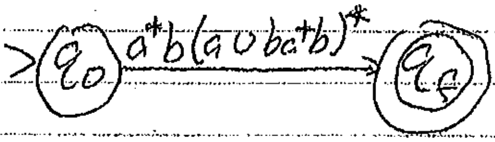
GNFA-4:



RIP out 1:

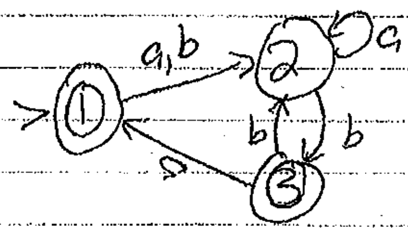


9 cont) RIP out 2:

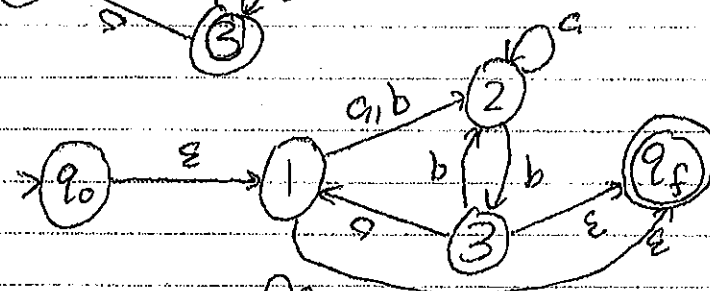


Thus a R.E. that expresses the same language as D_1 is $\boxed{a^*b(a \cup ba^*b)^*}$

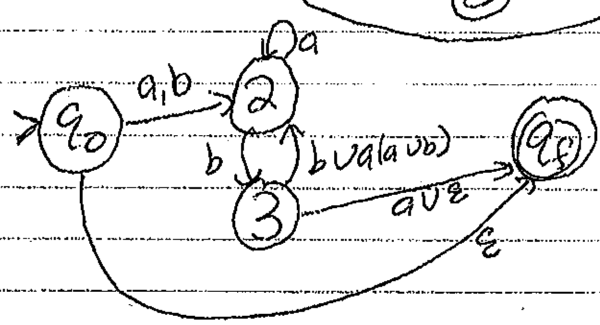
D_2:



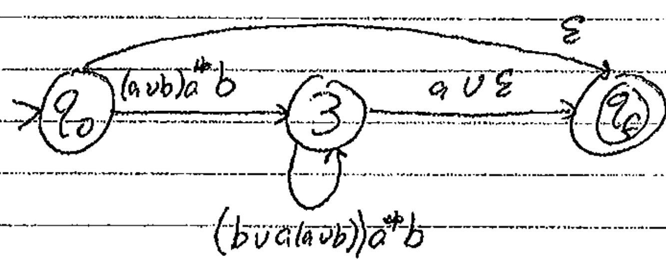
GNFA-5:



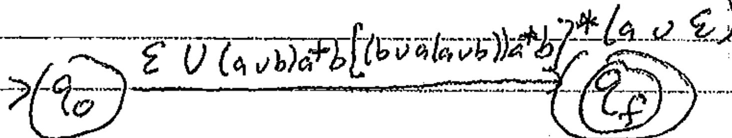
RIP out 1:



RIP out 2:



RIP out 3:



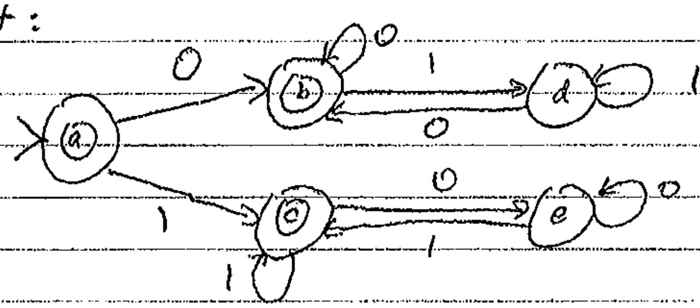
Final R.E.: $\boxed{\varepsilon \cup (a \cup b)a^*b\left[(b \cup a(a \cup b))a^*b\right]^*(a \cup \varepsilon)}$

10) We will show that L_1 is not regular by showing that it does not satisfy the pumping lemma.
Let p be the pumping length for L_1, assuming it's regular. Consider $s = 0^p 1^p 2^p$. The pumping lemma claims that $s = xyz$ with $|xy| \le p$, $|y| > 0$.
Then $x = 0^i$, $y = 0^j$, $z = 0^{p-i-j} 1^p 2^p$.
Consider $xy^2z = 0^{p+j} 1^p 2^p \notin L_1$ since $j > 0$.
This contradicts the pumping lemma. Thus L_1 is NOT regular.

We will show that L_2 is not regular by showing that it does not satisfy the pumping lemma.
Let p be the pumping length for L_1, assuming it's regular. Consider $s = 0^p 1 0^p 1$. The pumping lemma claims that $s = xyz$ with $|xy| \le p$, $|y| > 0$.
Then $x = 0^i$ $y = 0^j$ $z = 0^{p-i-j} 1 0^p 1$.
Consider $xy^3z = 0^{p+2j} 1 0^p 1$. The length of this string is $2(p+j+1)$. Since $p+j+1 \le p+j+j = p+2j$, it follows that the first half of this string is ALL 0s while the second half is not. Thus, $xy^3z \notin L_2$, contradicting the pumping lemma, proving L_2 isn't regular.

We will show that L_3 is not regular by showing that it does not satisfy the pumping lemma. Let p be the pumping length and consider the string $s = 0^{2^p}$. We must have $s = xyz$ with $x = 0^i$, $y = 0^j$, $z = 0^{2^p - i - j}$ and $1 \le j \le p$. Consider $xy^2z = 0^{2^p + j}$.
We claim that $xy^2z \notin L_3$. Note that $2^p + j > 2^p$.
The next smallest string in L_3 has length 2^{p+1}.
We must show that $2^p + j < 2^{p+1}$. $2^p + j \le 2^p + p < 2^p + 2^p = 2^{p+1}$. It follows that $xy^2z \notin L_3$ and L_3 is not regular.

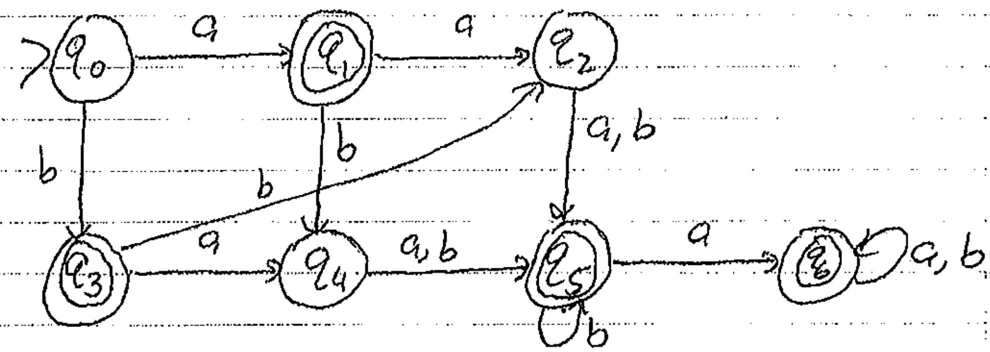
11) L_1 is regular. Here is a DFA that accepts it:



State a represents that nothing has been read in.
States b and c represent that the string read in so far satisfies the given criteria and started with 0 or 1, respectively.
State d ~~repest~~ represents a string starting with 0 and ending with 1.
State e represents a string starting with 1 and ending with 0.
Basically, any string starting and ending with the same character is accepted, since it contains an equal number of 01 and 10 "transitions".

L_2 is not regular. We will show that it does not satisfy the pumping lemma. Consider the string $S = (00)^p(11)^p$, where p is the pumping length. $S \in L_2$. Let $S = xyz$, ~~wh~~ with $x = 0^i$, $y = 0^j$, $z = 0^{2p-i-j}(11)^p$. Consider $xy^2z = 0^{2p+j}1^{2p}$. $xy^2z \notin L_2$ because it has $2p+j-1$ occurrences of 00 and $2p-1$ occurrences of 11. Since $j > 0$, $2p+j-1 \neq 2p-1$ and $xy^2z \notin L_2$. It follows that L_2 is NOT regular.

12) Here is a drawing of the original DFA:



<u>Algorithm</u>

[0,2] $\quad S[1,5] = \{[0,2]\}$, on a
$\quad S[3,5] = \{[0,2]\}$, on b

[0,4] $\quad S[1,5] = \{[0,2],[0,4]\}$ on a
$\quad S[3,5] = \{[0,2],[0,4]\}$ on b

[1,3] $\quad S[2,4] = \{[1,3]\}$ on a, b

[1,5] $\quad D[1,5] = 1$ by a.
$\quad D[0,2] = 1$ recursively
$\quad D[0,4] = 1$ recursively

[1,6] $\quad D[1,6] = 1$ by a.

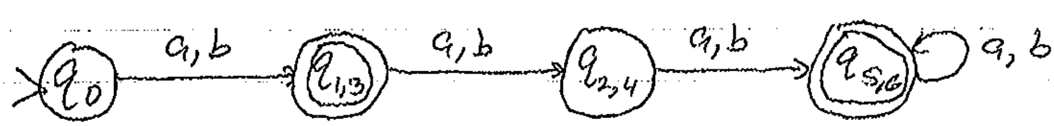
[2,4] $\quad$ no action

[3,5] $\quad D[3,5] = 1$ by a.

[3,6] $\quad D[3,6] = 1$ by a.

[5,6] $\quad$ no action

Thus the states that can "stay together" are $\{q_1, q_3\}$, $\{q_2, q_4\}$, and $\{q_5, q_6\}$.
Here is the minimized DFA:



This is the language of the set of strings with length 1, or 3 or more.